Nonlinear H_∞ controller for flexible joint robots with using feedback linearization

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Abstract -This paper proposes a new approach to feedback linearization of flexible link robots which have uncertain modeling. The flexibility of joints is performed by use of the solenoid nonlinear springs, which have damper property. The simplified nonlinear H_∞ controller is used to control linearized flexible link robots. The new continues and smooth model of frictions is used for modeling the dynamics of flexible robots. The effect of parameters tolerance, external disturbances and also other nonlinearities are expressed by a nonlinear independent term in linearized dynamics. Simulation results demonstrate that the proposed feedback linearization with simplified nonlinear H_∞ controller has ability to control flexible joint robots with a good performance. Also it is more efficient than the NL-H_∞ based controller with nonlinear dynamics. This technique can be used to control the other nonlinear systems which have the dynamics similar to flexible joint robots.

Keywords: nonlinear H_∞ - flexible joint – robust –feedback lionization

1. Introduction

In the last years, a large range of robots have been developed and used to perform special type of tasks, such as aerospace and high hazardous chemical activities, to interact with humans in the industrial projects, household activities as assistance to elder or physically challenged people. The most important necessities for the specifications of robots in the human environment are safety and reliability of the robotic system [1][2][9]. These necessities limit and prohibit the use of standard industrial robots for the cooperation with humans. Often ordinary industrial robot systems are designed with rigid links that implies to high weight link. In order to have more safety and reliability, joint flexibility is present in many current industrial robots. Flexible Joint Robot (FJR) systems with elastic specifications are usually used in the chemical, military, integrated circuits manufacturing and many other industrial processes in order to have properties with high sensitivity and accuracy [17]. During the last decades, nonlinear systems and control theory have had major development. In general, the use of linear and traditional controllers for FJRs is limited to proximity of equilibrium operation point with low accuracy [6]. Therefore using most excellent controllers such as nonlinear and intelligent classes are justified because, industrial projects usually require accuracy, repeatability and simplicity in the realization of the control law. In many robotics applications, the joint flexibility cannot be neglected. On the other hand, it is well known that ignoring the modeling of elastic coupling between the actuators is a major source of oscillatory problems and the robot joints can lead to instability, high frequency vibrations and reduced performance in some extreme cases [16][3],[6],[11]. Joint flexibility in robots can be caused by naturally inherent of used material in structure of robots or human made that is applied intentionally. When harmonic drives, belts or long shafts are used as motion transmission elements, a dynamic displacement is introduced between the position of the driving actuators and that of the driven links, which is the output to be controlled. Flexible behavior can be changed to rigid with stiffness going to infinity, then the dynamic model of robots with elastic joints lend itself to a singularly perturbed format. Based on the natural two-time-scale nature of the FJR dynamics, several approximate tracking controllers have been proposed. Performance of these methods is actually acceptable when the joints are sufficiently rigid [6]. It is confirmed that the FJRs have non-linear dynamic, therefore a high-accurate controlling for FJRs requires advanced controllers with special design [6] So the joint flexibility should be considered in any practical robotic modeling. From a modeling view, until now several models for FJR have been proposed and developed which the Spong model is used the most commonly [16]. This model is based on the following assumptions: first the kinetic energy of each rotor is due only to its own rotation, and second the joint flexibility behaves as a linear spring. The first assumption is valid only when the velocities of the previous links and rotors are very small. Other problem in robots especially FJRs is friction modeling. De Wit showed that the friction had to be modeled correctly and could not be ignored [13]. In control theory, feedback linearization as one of the most active research areas, is a powerful technique for control of nonlinear systems, where it has been extensively applied to many electromechanical systems such as, rigid and flexible joint robots [4][18]. Feedback linearization converts a nonlinear system into an equal controllable linear system by using state feedback. Despite this remarkable theory, the actual design of a state-feedback linearization is still very difficult and it has been performed only on simple systems for example rigid robots with up to three joints. Recently, it has been proven that the dynamic models of FJRs are invertible with no zero dynamics [5][6]. Even though researches on the design of controllers to achieve a linear input-output has been done well, but the conventional input-output linearization techniques will perform very unsuccessfully when the converted linear system has the unstable and unobservable internal dynamics [7]. Hence the input-output linearized system must be have stable zero dynamics for internal stability[7]. The performance and robustness of linearized systems are decreased by model uncertainties, and also the estimation of unknown dynamics seems to be difficult.

In this paper, we consider FJRs which have some rigid arms and joints with high flexibility. Also parametric uncertainties have been considered on modeling. We generalize the applicability of feedback linearization and decoupling control techniques to the uncertain n-link FJRs. We will see that, in the presence of uncertainties, static state feedback may or may not be enough to obtain full-state linearization and input–output decoupling. So a robust linearized feedback is proposed to robustly control an uncertain FJR system around an operating point. The first objective of this paper is to develop a new comprehensive uncertain n-link FJR model, the new structure for torque coupling. The second objective of this paper is to design a robust nonlinear controller using a simplified NL-H_∞ and feedback linearization techniques. The controller will be designed for a single-link flexible joint robot manipulator as a case study. Simulations are carried out to test the performance of the proposed control approach.

2. Modeling

Until now several models are presented for expression of dynamics of rigid robots. One of the most used models in robotic control has been proposed based on Lagrange formulation [19]. In addition to Lagrange method, there are two less known methods based on classical mechanic theory. Despite the difference in how to create equations, all models lead to the unique structure.

2.1 Model of Rotary Serial Robots

Lagrange method provides system dynamics describing equations in mechanical systems by using the kinetic $K(q,\dot{q})$ and potential $V(q)$ energy which are expressed as a function of links position vector as below:

$$q = \begin{bmatrix} q_1 & q_2 & \dots & q_n \end{bmatrix}^T \qquad (1)$$

Links position in robots with rotary joints is selected by joints angle. Therefore, the system kinetic and potential energy can be expressed as follows:

$$\begin{aligned} K(q,\dot{q}) &= \tfrac{1}{2}\dot{q}^T M(q)\dot{q} \\ V(q) &= \sum_{i=1}^{n} m_i g^T r_{ci} \end{aligned} \qquad (2)$$

where $M(q)$, g and r_{ci} are positive inertia matrix, the direction of gravity vector and the coordinates of the center of mass of i^{th} link vector respectively. Then by using $L = K - V$ and applying in the following differential equation system dynamics equations is created.

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{q}_j} - \frac{\partial L}{\partial q_j} = T_j \qquad (3)$$

Also T_j which is as the actuator torque in the mechanical systems, is called generalized force. It is also assumed that the links, joints and gearboxes are all rigid type in rigid class robots. Thus, the equations of the rigid robot by using Lagrange formulation are expressed as follows [19]:

$$\begin{aligned} M(q,\theta_{rb},\theta_{kin})\ddot{q} + C(q,\dot{q},\theta_{rb},\theta_{kin}) \\ + g(q,\theta_{rb},\theta_{kin}) + f_f(q,\dot{q}) = T \end{aligned} \tag{4}$$

Where $M(.) \in \mathbb{R}^{N\times N}$ is a symmetric positive matrix and can be expressed as $M(.) = M_a(.) + M_m(.)$. Also $M_a(.)$ and $M_m(.)$ are configuration dependent and compact gear-boxed motor inertia matrixes on the link side. Also $C(.) \in \mathbb{R}^N$ and $g(.) \in \mathbb{R}^N$ represent torques of coriolis-centrifugal and gravity. $T \in \mathbb{R}^N$ is system actuator vector and $f_f(.) \in \mathbb{R}^N$ is the vector of frictions such as coulomb and viscose [17]. θ_{rb} and θ_{kin} are expression rigid body and kinetic parameters for i^{th} link which are expressed as follows [19]:

$$\begin{aligned} \theta_{rb}^i &= \begin{bmatrix} m_i & \zeta^i & J_{xx}^i & J_{yy}^i & J_{zz}^i & J_{xy}^i & J_{xz}^i & J_{yz}^i \end{bmatrix} \\ \theta_{kin}^i &= \begin{bmatrix} \ell_x^i & \ell_y^i & \ell_z^i \end{bmatrix} \end{aligned} \tag{5}$$

Where ℓ^i, ζ^i and J^i are link length vector, link mass center and inertia tensor of mass center that are defined as follows:

$$\begin{aligned} \zeta^i &= \begin{bmatrix} \zeta_x^i & \zeta_y^i & \zeta_z^i \end{bmatrix} \\ J^i &= \begin{bmatrix} J_{xx}^i & J_{xy}^i & J_{xz}^i \\ J_{xy}^i & J_{yy}^i & J_{yz}^i \\ J_{xz}^i & J_{yz}^i & J_{zz}^i \end{bmatrix} \end{aligned} \tag{6}$$

2.2 Flexible Joint Robots modeling

In the previous section the main subject was modeling of robots with rigid joints and links. But in practice, joints are made by a specific type of gearbox with elastic property so called Harmonic Drive. This property is usually modeled by a combination of spring - damper pair. Unlike the rigid robots, the generated torque in compact gear-boxed motor is transmitted through a spring - damper pair to the link. Therefore position and velocity of gearboxes shaft and links are always not equal in FJRs. So FJRs dynamics for a robot with n-link consists of two dependent separate differential equation with 2n Degree Of Free (DOF). According to spong modeling summarized equations for FJRs can be written as follows:

$$\begin{aligned} M_a(q_a)\ddot{q}_a + C(q_a,\dot{q}_a) + g(q_a) + f_a(\dot{q}_a) \\ = K(q_m - q_a) + D(\dot{q}_m - \dot{q}_a) \\ M_m\ddot{q}_m + K(q_m - q_a) + D(\dot{q}_m - \dot{q}_a) \\ + f_m(\dot{q}_m) = T_m \end{aligned} \tag{7}$$

Where $q_m \in \mathbb{R}^N$ and $q_a \in \mathbb{R}^N$ are the position vector of gearboxes shaft and links. T_m is affective output torque which is generated by compact gear-boxed motor. $K \in \mathbb{R}^N$ and $D \in \mathbb{R}^N$ are diagonal matrixes related to the coefficients of dampers stiffnesses. In this equations, parameters of rigid body and kinetic are expressed by θ_{rb} and θ_{kin} that for simplicity in notation were omitted. f_a and f_m are the frictions vector for the link and motor sides. Usually f_m has non-linear dynamics so that is reduced efficiency of the robot controller. In most cases f_a can be ignored because of its low effect. A simple friction model, which includes viscous and coulomb frictions, is expressed as follows:

$$f(\dot{q}_m) = f_v.\dot{q}_m + f_c.\mathrm{sgn}(\dot{q}_m) \tag{8}$$

This model is not used because of non-smooth form. Instead of (8) the other most used equation, which was proposed by Feeny and Moon is used as follows **Error! Reference source not found.**:

$$f(\dot{q}_m) = f_v.\dot{q}_m + f_c\left(\mu_k + (1-\mu_k)\cosh^{-1}(\beta.\dot{q}_m)\right)\tanh(\alpha.\dot{q}_m) \tag{9}$$

Other several dynamics were also proposed for friction by researchers [12],[8]**Error! Reference source not found.**.

3. Feedback linearization and FJR dynamics

Linearization techniques can be used to create a linear dynamics of nonlinear systems such as FJRs. Often feedback linearization as a fine and most used linearization technique is used for nonlinear cases. By using this method, nonlinear equations are converted to linear form so that a simple linear controller can be used to control. Usually in FJRs, the final controlled system has low stability and performance because of external disturbance inputs and parameter uncertainty so that feedback linearization can be used in neighborhood of equilibrium point. In this research, this problem is solved by using a new structure of robust feedback linearization.

3.1 FJRs Space state equations

At first, to complete the describing equations of FJR, the following equation as a approximate dynamic of actuator is added to (7):

$$T_m = R^{-1}.K_m\left(V_m - K_b\dot{\theta}_2\right) \tag{10}$$

Where $R \in \mathbb{R}^N$, $K_m \in \mathbb{R}^N$ and $K_b \in \mathbb{R}^N$ are the positive diagonal matrixes of ohmic impedance, armature current to torque and speed to back e.m.f voltage conversion constants respectively. This model has a linear structure and also dynamics of the electric section is ignored because of its high speed. By define $f_m(\dot{q}_m) = f_2(\dot{q}_m) + B.\dot{q}_m$, $q_a = \theta_1$ and $q_m = \theta_2$ and by replacing (10) in (7) the equations of FJRs dynamics is rewritten as follows:

$$M_a(\theta_1)\ddot{\theta}_1 + C_a(\theta_1,\dot{\theta}_1) + g_a(\theta_1) + f_a(\dot{\theta}_1) = T_C \tag{11.a}$$

$$T_C = K(\theta_2 - \theta_1) + D(\dot{\theta}_2 - \dot{\theta}_1) \tag{11.b}$$

$$M_m\ddot{\theta}_2 + \left(B + R^{-1}K_m.K_b\right)\dot{\theta}_2 + f_2(\dot{\theta}_2) + T_C = R^{-1}K_m.V_m \tag{11.c}$$

Now θ_2 is calculated from (11.a) and (11.b) as follows:

$$\theta_2 = \theta_1 - D(\dot{\theta}_2 - \dot{\theta}_1) + N_1(\theta_1,\dot{\theta}_1,\ddot{\theta}_1) \tag{12}$$

then $\dot{\theta}_2$ and $\ddot{\theta}_2$ are obtained:

$$\begin{aligned}\dot{\theta}_2 &= \dot{\theta}_1 - D(\ddot{\theta}_2 - \ddot{\theta}_1) + N_2(\theta_1,\dot{\theta}_1,\ddot{\theta}_1,\overset{(3)}{\theta}_1)\\ \ddot{\theta}_2 &= \ddot{\theta}_1 - D(\overset{(3)}{\theta}_2 - \overset{(3)}{\theta}_1) + N_3(\theta_1,\dot{\theta}_1,\ddot{\theta}_1,\overset{(3)}{\theta}_1) + K^{-1}M_{a1}\overset{(4)}{\theta}_1\end{aligned} \tag{13}$$

Where N_1 , N_2 and N_3 are as follows:

$$\begin{aligned}N_1 &= K^{-1}(M_a\ddot{\theta}_1 + C_a + g_a + f_a)\\ N_2 &= K^{-1}(M_{a1}\ddot{\theta}_1 + M_a\overset{(3)}{\theta}_1 + C_{a1} + g_{a1} + f_{a1})\\ N_3 &= K^{-1}(M_{a2}\ddot{\theta}_1 + (M_{a1} + M_{a2})\overset{(3)}{\theta}_1 + C_{a2} + g_{a2} + f_{a2})\end{aligned} \tag{14}$$

Also M_{a1} , M_{a2} , C_{a1} , C_{a2} , f_{a1} and f_{a1} are defined as follows:

$$\begin{aligned} M_{a1}(\theta_1,\dot{\theta}_1) &= \frac{dM(\theta_1)}{dt} \\ M_{a2}(\theta_1,\dot{\theta}_1,\ddot{\theta}_1) &= \frac{d^2M(\theta_1)}{dt^2} \end{aligned} \tag{15.a}$$

$$\begin{aligned} C_{a1}(\theta_1,\dot{\theta}_1,\ddot{\theta}_1) &= \frac{dC_a(\theta_1,\dot{\theta}_1)}{dt} \\ C_{a2}(\theta_1,\dot{\theta}_1,\ddot{\theta}_1,\overset{(3)}{\theta}_1) &= \frac{d^2C_a(\theta_1,\dot{\theta}_1)}{dt^2} \end{aligned} \tag{15.b}$$

$$\begin{aligned} g_{a1}(\theta_1,\dot{\theta}_1) &= \frac{dg_a(\theta_1)}{dt} \\ g_{a2}(\theta_1,\dot{\theta}_1,\ddot{\theta}_1) &= \frac{d^2g_a(\theta_1)}{dt^2} \end{aligned} \tag{15.c}$$

$$\begin{aligned} f_{a1}(\dot{\theta}_1,\ddot{\theta}_1) &= \frac{df_a(\dot{\theta}_1)}{dt} \\ f_{a2}(\dot{\theta}_1,\ddot{\theta}_1,\dddot{\theta}_1) &= \frac{d^2f_a(\dot{\theta}_1)}{dt^2} \end{aligned} \tag{15.d}$$

Now by replacing $\dot{\theta}_2$ and $\ddot{\theta}_2$ from (13) in the third equation (11.c) and also with the definition:

$$\begin{aligned} M_z &= \left(M_m K^{-1} M_{a1}\right)^{-1} \\ B_m &= \left(B + R^{-1}K_m.K_b\right) \end{aligned} \tag{16}$$

We will:

$$\begin{aligned} \overset{(4)}{\theta}_1 = {} & \underbrace{N_4(\theta_1,\dot{\theta}_1,\ddot{\theta}_1,\ddot{\theta}_1,\overset{(3)}{\theta}_1)}_{f(\theta_1)} + \\ & \underbrace{N_5(\theta_1,\dot{\theta}_1,\ddot{\theta}_2,\overset{(3)}{\theta}_2)}_{d(\theta_1,\theta_2)} + \underbrace{N_6(\dot{\theta}_1,\ddot{\theta}_1)}_{g(\theta_1)}.\underbrace{V_m}_{u} \end{aligned} \tag{17}$$

Where N_4, N_5 ,and N_6 are defined as follows:

$$\begin{aligned} N_4 &= -M_z \begin{bmatrix} M_m D\overset{(3)}{\theta}_1 + B_m N_2 + \\ (M_m + M_a + B_m D)\ddot{\theta}_1 + \\ B_m\dot{\theta}_1 + C_a + g_a + f_a + f_2 \end{bmatrix} \\ N_5 &= M_z\left[M_m D\overset{(3)}{\theta}_2 + B_m D\ddot{\theta}_2\right] \\ N_6 &= M_z R^{-1} K_m \end{aligned} \tag{18}$$

By define system state vector as follows:

$$\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} = \begin{bmatrix} \theta_1 & \dot{\theta}_1 & \ddot{\theta}_1 & \overset{(3)}{\theta}_1 \end{bmatrix} \tag{19}$$

Equation (17) in state space is written as follows:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f(x) + d(x, \ddot{\theta}_2, \overset{(3)}{\theta}_2) + g(x).u \end{aligned} \quad (20)$$

3.2 Feedback Linearization and Uncertain modeling

In most non-linear systems using feedback linearization leads to simple controller, but the stability of linearized system is strongly dependent on the equilibrium point and instability may occur in system because of parameters uncertainty and disturbance. The uncertain model for 4^{th} equation of (19) can be rewritten as follows:

$$\dot{x}_4 = f(x) + \Delta f(x) + [\Delta g(x) + g(x)].u + d'(x, \ddot{\theta}_2, \overset{(3)}{\theta}_2) \quad (21)$$

Where $\Delta f(x)$ and $\Delta g(x)$ are expression of dynamics of uncertain parameters on the system. $d(x, \ddot{\theta}_2, \overset{(3)}{\theta}_2)$ is replaced with $d'(x, \ddot{\theta}_2, \overset{(3)}{\theta}_2)$ because of the effect of parameters uncertainty. Now by define $w(x, u, \ddot{\theta}_2, \overset{(3)}{\theta}_2)$ as follows:

$$w(x, u, \ddot{\theta}_2, \overset{(3)}{\theta}_2) = \Delta f(x) + \Delta g(x).u + d'(x, \ddot{\theta}_2, \overset{(3)}{\theta}_2) \quad (22)$$

The 4^{th} equation of (19) will be:

$$\dot{x}_4 = f(x) + g(x).u + w(x, u, \ddot{\theta}_2, \overset{(3)}{\theta}_2) \quad (23)$$

Now by choosing u as:

$$u = g^{-1}(x)[v_1 - f(x)] \quad (24)$$

And by replacing (24) in (23), describing equations of n-Link FJR can be rewritten as below:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= v_1 + d(x, u, \ddot{\theta}_2, \overset{(3)}{\theta}_2) \end{aligned} \quad (25)$$

Where v_1 is new input control vector. The final linearized system has a n-decupled linear dynamics and nonlinear dynamics because of disturbances and parameters uncertainty as follows:

$$\begin{aligned} \dot{x}_i &= \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} x_i + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} v_i + \begin{bmatrix} 0 \\ 0 \\ 0 \\ d_i(x, u, \ddot{\theta}_2, \overset{(3)}{\theta}_2) \end{bmatrix} \\ y_i &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x_i \\ & i = 1, 2, \ldots, n \end{aligned} \quad (26)$$

Equations (25) show that the linearized system still has nonlinearity on uncertain dynamics. There are two recommended method to design a robust controller based on H_∞ theory for n-link FJR. The primary technique is using a unique L-H_∞ controller with (25) and the other is using n-single independent NL-H_∞ controllers with (26). However a unique NL-H_∞ based on (26) will have higher performances, but in practical it is not recommend because of complexity of its control law and delay of calculation.

4. NL-H_∞ controller design

By use of (26), an independent controller based on NL-H_∞ theory is designed for any link of n-link FJR so that designed controller for i[th] link is only dependent to x_i vector. Equations (26) must be rewritten as a new form because of compatibility with structure of NL-H_∞ theory. It is also assumed that all states of linearized system are full available. Therefore equations of i[th] link can be described in state space form as follows [14]:

$$\begin{aligned} \dot{x}_i &= A_i\bar{x}_i + g_i(\bar{x}_i)\bar{w}_i + B_i v_i \\ \bar{z}_i &= h_i(\bar{x}_i) + l_i(\bar{x}_i)v_i \end{aligned} \tag{27}$$

Where $\bar{x}_i \in R^4$ is the system state vector, $\bar{w}_i \in R^{m_i}$ is the disturbance input vector, $v_i \in R^1$ is the new control input vector and $\bar{z}_i \in R^{p_i}$ is the penalty vector. Also it is assumed that functions $g_i(x_i), h_i(x_i), l_i(x_i)$ are defined and smooth in the neighborhood of X on R^4.

4.1 uncertain modeling

In this paper, tolerances of mechanical and electrical parameters such as mass and length of link and resistance and inductance of motor are considered as main uncertain sources, because these parameters can be usually changed. It is also assumed that measuring of fixed mechanical and electrical parameters in compact motor and gearbox have been done with sufficient precision. Tolerance of any parameter can be defined as follows:

$$\begin{aligned} &P_{un-ij} = P_{0-ij}(1+\alpha_{ij}w_{ij}) \\ \text{or} \quad &P_{un-ij} = P_{0-ij}(1+\alpha_{ij}w_{ij})^{-1} \end{aligned} \tag{28}$$

If deviation term of any parameter appears as dominator, second equation of (28) can be used. Where α_{ij} are positive numbers as maximum deviations of uncertain parameters. Also w_{ij} are virtual external input signals which are limited by $|w_{ij}| < 1$. Therefore $\bar{w}_i$ is defined as follows:

$$\begin{aligned} \bar{w}_i &= \left[w_{i1}, \dots, w_{ip}, \dots, w_{ijh}, \dots, w_{ijh\dots m}, \dots, x_d\right]^T_{1\times q} \\ &\{j, h, \dots, m\} = 1, \dots, p \end{aligned} \tag{29}$$

Where indexes '**p**' and '**q**' are the number of uncertain parameters and inputs which are as external disturbances. In some equations may be terms that have product of two or more external inputs such as $w_{i1}.w_{i2}$ (or $w_{i1}.w_{i2}...w_{im}$). In these cases, they can be replaced by w_{i12} ($w_{i12\dots m}$) as a new external input. The term $g_i(x_i)\bar{w}_i$ in (27) can be replaced as follows [14]:

$$g_i(x_i)\bar{w}_i = \sum_{j=1}^{q} g_{ij}(x_i)\bar{w}_{ij} \tag{30}$$

Where $g_{ij}(x_i)$ is obtained as follows:

$$g_{ij}(x_i) = \begin{bmatrix} 0 & 0 & 0 & d_{ij}(x_i)|_{\bar{w}_i=\bar{0}\,,\, w_{ij}=1} \end{bmatrix}^T \tag{31}$$

Also in this paper, $h_i(x_i)$ was selected as follow:

$$h_i(x_i) = \begin{bmatrix} q_{i1}x_{i1} & q_{i2}x_{i2} & 0 & 0 & 0 & q_{i6}x_{i6} \end{bmatrix}^T \tag{32}$$

Where q_{i6} can be used as tracking accuracy setting, q_{i1} and q_{i2} can be used to adjust the response of system for having the least overshoot. It is necessary to say that q_{i6}, q_{i1} and q_{i2} are often chosen experimentally. The following assumption is made to simplicity the controller designing process:

$$l_i^T(x_i)l_i(x_i) = R_{i2} \qquad (33)$$

Where R_{i2} is a nonzero constant value. Then NL-H$_\infty$ control law can be calculated as [14]:

$$V_{im} = -R_{i2}^{-1}B_i^T V_{ix}^T \qquad (34)$$

Where V_{ix} is the gradient of $V_i(x_i)$ which is a positive define function of $\bar{x}_i$ so that must satisfy locally the following differential equation in the neighborhood of origin in R^4.

$$V_{ix}A_i x_i + \frac{1}{2}h_i^T h_i + \frac{1}{2}V_{ix}\left(\sum_{j=1}^{q}\frac{g_{ij}g_{ij}^T}{\gamma_{ij}^2} - B_i R_{i2}^{-1}B_i^T\right)V_{ix}^T = 0 \quad (35)$$

Hence attenuation of disturbance signals is done by γ_i scale. Because of non-linear nature of system, there is a low possibility to find a closed analytical solution for equation (35). In most cases, the numerical solution is used instead of the analytical solution.

4.2 NL-H$_\infty$ controller by Taylor series

As it was mentioned, in most cases NL-H$_\infty$ based control law is calculated by Taylor series. It means that $V_i(x_i)$ in (35) has the following structure [15]:

$$V_i(x_i) = \frac{1}{2}x_i^T P_i x_i + \sum_{k=3}^{\infty} P_{ik}x_i^{[k]} \qquad (36)$$

Therefore applied voltage to the motor of i^{th} link can be generated by:

$$V_{im} = Q_{i1}.x_i + \sum_{k=3}^{\infty} Q_{ik}.x_i^{[k]} \qquad (37)$$

Where Q_{i1} and Q_{ik} can be calculated by using P_i, P_{ik} and the proposed equations in [15]. $P_i \in R^{4\times4}$ is a symmetric positive define matrix that is calculated by the following Riccati's equation:

$$A_i^T P_i + P_i A_i + C_i^T C_i - P_i\left(\frac{B_i B_i^T}{R_{i2}} - \sum_{j=1}^{q}\frac{B_{ij}B_{ij}^T}{\gamma_{ij}}\right)P_i = 0 \qquad (38)$$

C_i, B_{ij} are matrix and vector that can be produced by linearization of $h_i(x_i)$ and $g_{ij}(x_i)$ on equilibrium point as follows:

$$\begin{aligned} h_i(x_i) &= C_i x_i + h_i^{[2+]}(x_i) \\ g_{ij}(x_i) &= B_{ij} + g_{ij}^{[1+]}(x_i) \end{aligned} \qquad (39)$$

$h_i^{[2+]}(x)$ consist of the second and higher order terms of $h_i(x_i)$ and $g_{ij}^{[1+]}(x_i)$ consist of the first and higher order terms of $g_{ij}(x_i)$. Also $x_i^{[k]}$ is generated by using the Kroniker product of the state vector of i^{th} link.

5. Single link FJR test-bed

In order to analyze the proposed controller performance, it was simulated on the single link FJR model. In practical this case study has two main sections, electro-mechanic and control.

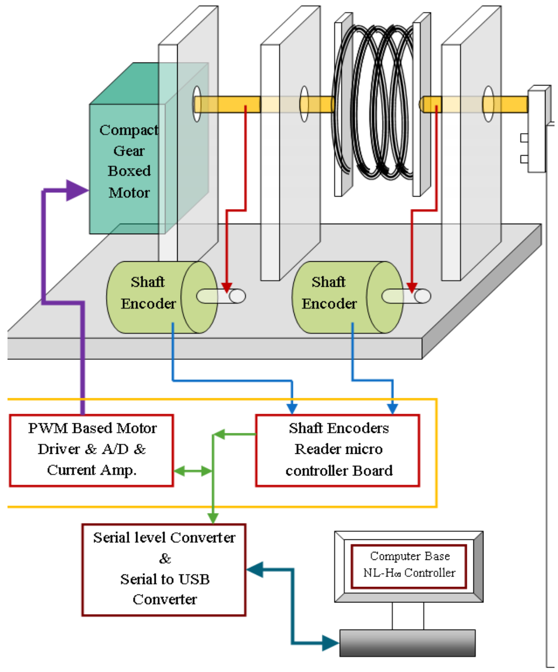

Figure 1. Scheme of a practical single link FJR and controller

Electro-mechanic section is split to compact gear-boxed DC motor as actuator, solenoid spring as torque transformer and lightweight plastic rigid link as arm. Controller section has four high speed micro-controllers. Gearbox-shaft and link positions are measured by high precision shaft encoders as measured output signals.

5.1 System mechanical and electrical parameters

Table (I) shows values of compact gear-boxed DC motor set electrical and mechanical parameters.

TABLE I
COMPACT DC MOTOR AND GEARBOX SET PROPERTIES

Parameters	Nominal values	Tolerance
Coil resistance	R=9 ohm	4%
Torque const.	km=3.3	3%
Back emf const.	kb=5.62	3%
Motor inertia	J=2.15	3%

Also, Table (II) shows values of link and spring mechanical parameters. Viscose friction coefficient is totally presented for system in Table (II).

TABLE II
LINK AND SPRING PROPERTIES

Parameters	Nominal values	Tolerance
Link Length	l=0.53	6%
Link mass	m=0.12	9%
Gravity coefficient	g=9.8	1%
Inertia	I=0.083	4%
FJR Total Viscose const.	Fv=1.53	-
Joint stiffness	K=3.33, D=1.2	

5.2 Single link FJR dynamics

Dynamics of single link FJR is expressed as follows:

$$\begin{aligned} & I\ddot{\theta}_1 + mgl\sin(\theta_1) + f_v\dot{\theta}_1 = U_c \\ & U_c = K(\theta_2 - \theta_1) + D(\dot{\theta}_2 - \dot{\theta}_1) \\ & J\ddot{\theta}_2 + \left(B + \frac{K_m K_b}{R}\right)\dot{\theta}_2 + U_c = \frac{K_m}{R} V_m \end{aligned} \qquad (40)$$

By below assumptions:

$$\left[x_1, x_2, x_3, x_4\right] = \left[\theta_1, \dot{\theta}_1, \ddot{\theta}_1, \dddot{\theta}_1\right] \qquad (41)$$

Equations (40) can be rewritten in state space form as follows:

$$\begin{aligned} & \dot{x}_1 = x_2 \\ & \dot{x}_2 = x_3 \\ & \dot{x}_3 = x_4 \\ & \dot{x}_4 = f_L(x) + f_{NL}(x) + g(x).V_m + d(x, w, \ddot{\theta}_2, \dddot{\theta}_2) \end{aligned} \qquad (42)$$

Where $f_L(x), f_{NL}(x), d(x, w, \ddot{\theta}_2, \dddot{\theta}_2)$ and $g(x)$ are define as follows:

$$\begin{aligned} & f_L(x) = -a_{1N}x_4 - a_{2N}x_3 - a_{3N}x_2 \\ & f_{NL}(x) = -\frac{kmgl}{JI}\sin x_1 - b_{2N}x_2\cos x_1 \\ & \qquad -\frac{mgl}{I}(-x_2^2\sin x_1 + x_3\cos x_1) \\ & d(x, w, \ddot{\theta}_2, \dddot{\theta}_2) = -a_{1N}\alpha_1 x_4 w_1 - a_{2N}\alpha_2 x_3 w_2 \\ & \qquad -a_{3N}\alpha_3 x_2 w_3 - \frac{kmgl}{JI}\beta_1 \sin x_1 w_4 \\ & \qquad -b_{2N}\beta_2 x_2 \cos x_1 w_5 \\ & \qquad -\frac{mgl}{I}\beta_3(-x_2^2\sin x_1 + x_3\cos x_1)w_6 \\ & \qquad +\frac{D}{I}\beta_4 w_7 + c_2\beta_5 w_8 \\ & g(x) = \frac{KK_m}{JIR} \end{aligned} \qquad (43)$$

In (43), $d(x, w, \ddot{\theta}_2, \dddot{\theta}_2)$ can be replaced with $d(x, w)$ by assumption $[\ddot{\theta}_2, \dddot{\theta}_2] = [\beta_4 w_7, \beta_5 w_8]$. Where β_4 and β_5 are define for worst conditions of $\ddot{\theta}_2$ and $\dddot{\theta}_2$ as follows:

$$\begin{aligned} \beta_4 &= Max(\dddot{\theta}_2) \\ \beta_5 &= Max(\ddot{\theta}_2) \end{aligned} \tag{44}$$

Also $a_{1N}, a_{2N}, a_{3N}, b_{1N}, b_{2N}, b_{3N}$ and c_1, c_2 are defined by using system mechanical and electrical parameters (see appendix). Where $\alpha_1, \alpha_2, \alpha_3$ and $\beta_1, \beta_2, \beta_3$ are calculated using Tables (I,II) which their values are also shown in appendix. Now by choosing V_m as below:

$$V_m = g^{-1}(x)(u - f_{NL}(x)) \tag{45}$$

Equation $\dot{x}_4$ of (42) can be converted as semi linear form as follows:

$$\dot{x}_4 = f_L(x) + u + d(x, w) \tag{46}$$

Dynamic of uncertain parameters is expressed by $d(x, w)$ as below:

$$d(x, w) = \sum_{i=1}^{8} d_i(x)_{4\times 1} w_i \tag{47}$$

Where $d_i(x)$ are disturbance vectors which are functions of $a_{iN}, b_{iN}, c_i, \alpha_i, \beta_i$ and system states. All $d_i(x)$ are shown in appendix.

6. Simulation Results

In this section, it is shown that the proposed technique has advantages by using simulation outputs for some input commands $x_d = [90^{'}, 120^{'}, 150^{'}, 160^{'}, 180^{'}]$. Link position, tracking error, motor applied voltage are considered in subsection 6.1. Frequency response and effect of external disturbance and parametric uncertainties are also presented in subsections 6.2-6.3.

6.1 Simplified nonlinear H_∞ and linearized model

In order to show advantages of proposed feedback linearization controller a number of tests were done on single link FJR with m=0.7kg and L=0.55m. Fig. 5 shows response of proposed controller and nonlinear H_∞ controller for $x_d = [90^{'}, 120^{'}, 180^{'}]$. Also it shows link angle was settled to input command at t=5^s by proposed controller, but it has high overshoot for $x_d = [180^{'}]$ and delay for $x_d = [90^{'}]$ with nonlinear H_∞ controller. Tracking error for $x_d = [90^{'}, 180^{'}]$ are shown in fig.6. Tracking error of $x_d = [180^{'}]$ has higher value with nonlinear H_∞ controller because of bad overshoots, but

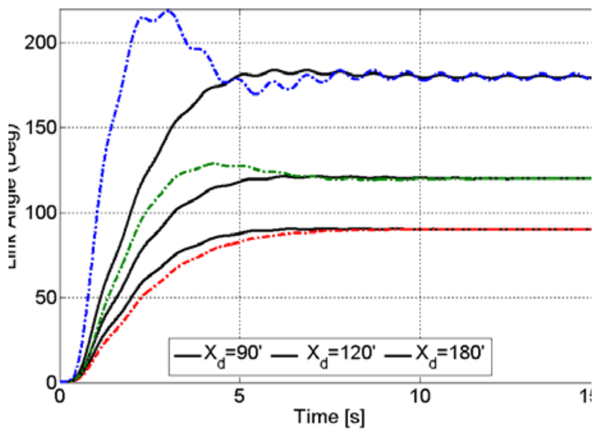

Figure 5. Link position - proposed controller line and N-LH$_\infty$ dashed

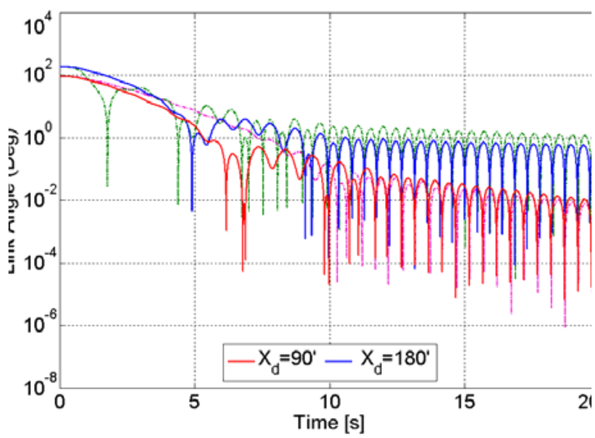

Figure 6. Link position tracking error -proposed controller line and N-LH$_\infty$ dashed

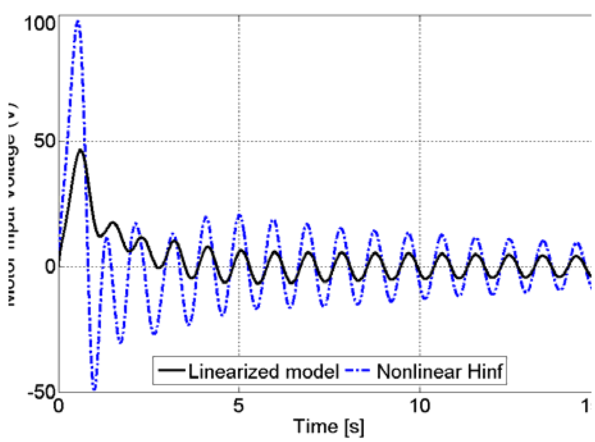

Figure 7. Motor applied voltage for $x_d = [180^{'}]$

for $x_d = [90^{'}]$ has less value because of overdesign view. Motor applied voltage for $x_d = [180^{'}]$ is shown in fig. 7. As it shows, applied voltage has more altering and larger positive and negative peeks with nonlinear H$_\infty$ controller compare to proposed controller. Thereupon, it is seen that proposed controller has more advantages compare to nonlinear H$_\infty$ specially around of link position $180^{'}$.

6.2 disturbance and frequency response

Fig. 8 shows response of controlled system to impact forces, which are applied to link, as external disturbances for $x_d = [60', 90', 180']$. As it shows the effect of impacts are compensated at less than 5^s for $x_d = [60', 90']$ but it be continued for long time for $x_d = [180']$. Certainly it is predictable that high energy disturbances can be generated instability for around of link position 180'. Fig. 9 shows frequency response of linearized system by feedback linearization method. Unlike to theory which linearized system has four repetitive poles.

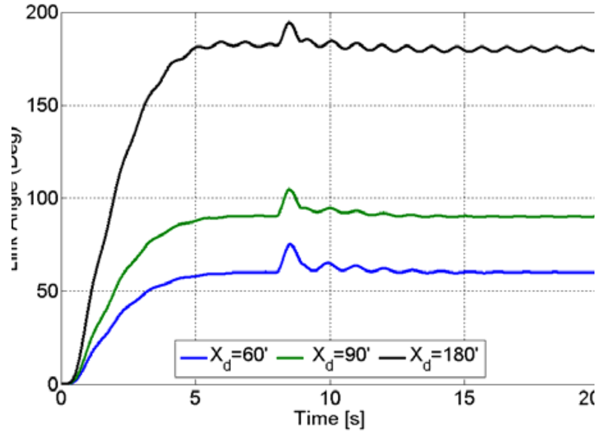

Figure 8. Link response to external disturbances

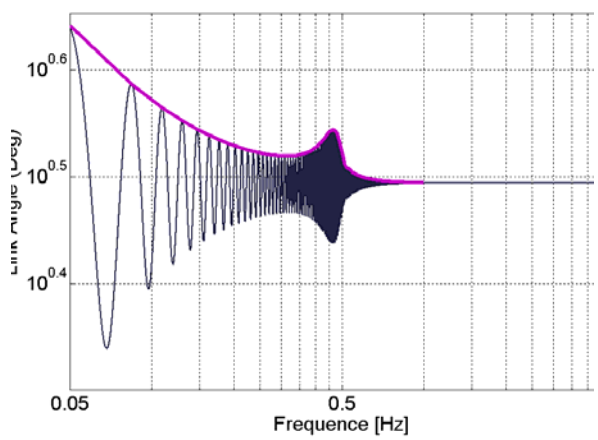

Figure 9. Linearized system frequency response

In addition to four repetitive poles, simulated model presents a number of zeros and poles around of 0.3Hz and 0.45Hz.

6.3 Model parameters uncertainties

Robustness of the proposed controller are verified by a number of simulation with parametric uncertainty on m*l, J and K. Fig. 10 shows the step response of link position to $x_d = [90', 160']$ which link has both mass and length uncertainty. Fig. 10 implies that system has robustness on tolerance of multiply of mass and length at least to ±50% . Fig. 11 shows the step response of controlled system to $x_d = [90', 150']$ with ±20% tolerance of J (motor moment inertia). The effect of uncertainty of the joint stiffness (K) as other significant parameter is shown at fig. 12 for different commands. Figs. 10,11,12 show that controlled system has robustness for ±20% tolerance of mentioned electrical and mechanical parameters.

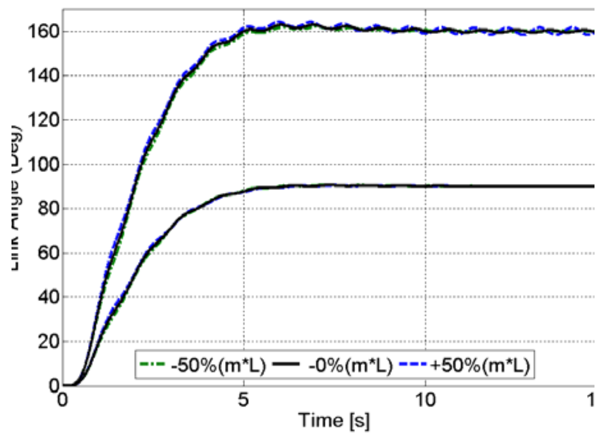

Figure 10. Link position with uncertainty on m*l

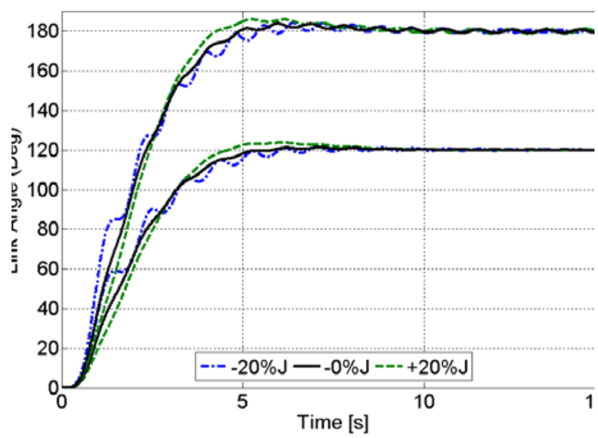

Figure 11. Link position with uncertainty on J

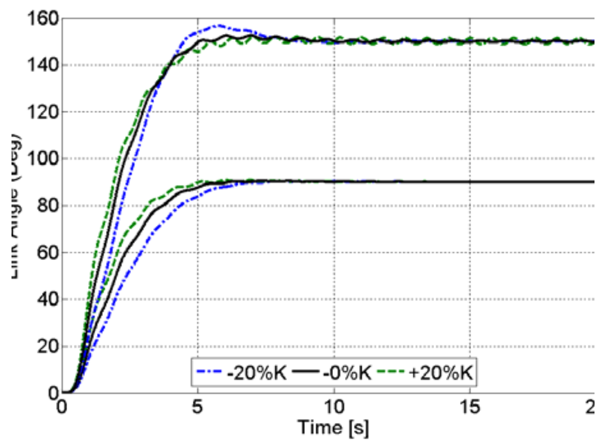

Figure 12. Link position with uncertainty on K

In all tests, link position has more ringing because of low stiffness of coupling spring. The results of Figs. 10-12 show that the proposed controller can guarantee the system stability against parameter tolerances and external disturbances.

7. Conclusion

This paper proposed a new approach to feedback linearization of nonlinear systems which have uncertain modeling such as FJRs. In this paper, the friction model is expressed with a continues and derivable equation. Also damper property of coupling is taken into account. Also in this paper the simplified NL-H_∞ theory based control law was used to control FJRs. The effect of parameters uncertainty and external disturbances were expressed by a nonlinear independent term in proposed linearzied dynamics. The results of simulations showed that single link FJR can be controlled by L-H_∞ theory because of linear structure of final dynamics with extra nonlinear term as parameters uncertainty. Moreover, simulation results of proposed controller on single link FJR showed that feedback linearization approach and simplified NL-H_∞ based controller has ability to stabilize and control even on unstable domain with acceptable performance. Also simulation results showed that proposed approach is more efficient than the NL-H_∞ based controller with nonlinear dynamics. The proposed approach can be applied to any multilink flexible on unstable area with a good performance.

REFERENCES

[1] A. Bicchi, S. Lodi Rizzini, and G. Tonietti, "Compliant design for intrinsic safety: General issue and preliminariey design", In Proc. Of IEEE/RSJ Int. Conf. on Intelligent Robots and Systems, 2001.

[2] A. Bicchi and G. Tonietti, "Fast and soft arm tactics: Dealing with the safety-performance trade-off in robot arms design and control", IEEE Robotics and Automation Magazine, 11(2):22–33, 2004.

[3] A. De Luca and P. Lucibello, "A general algorithm for dynamic feedback linearization of robots with elastic joints", In Proc. IEEE Int. Conf. on Robotics and Automation, pages 504–510 Vol.1, 1998.

[4] A. De Luca, "Decoupling and feedback linearization of robots with mixed rigid/elastic joints", International Journal of Robust and Nonlinear Control, 8(11), 965-977, 1998.

[5] A. De Luca and L. Lanari, "Robots with elastic joints are linearizable via dynamic feedback", 34th IEEE Conf. *on* Decision and Control, New Orleans, LA, 1995, pages 3895–3897.

[6] ALESSANDRO DE LUCA, "DECOUPLING AND FEEDBACK LINEARIZATION OF ROBOTS" Int. J. Robust Nonlinear Control 8, 965–977, 1998.

[7] Abdellah Mokhtari, Abdelaziz Benallegue and Bioubaker Daachi, "ROBUST FEEDBACK LINEARIZATION AND GH$_\infty$ CONTROLLER FOR A QUADROTOR UNMANNED AERIAL VEHICLE", Journal of ELECTRICAL ENGINEERING, VOL. 57, NO. 1, 2006, 20–27

[8] Canudas de Wit, C., Olsson, H., Astrom, K. J., & P.Lischinsky. 1995, "A New Model for Control of Systems with Friction", IEEE transactions on Automatic Control 40, 419-425

[9] B.F. Feeny and F.C. Moon, "Chaos in a forced dry-friction oscillator: experiments and numerical modeling", Journal of Sound and Vibration 170 (1994), pp. 303–323.

[10] G. Tonietti, R. Schiavi, and A. Bicchi, "Design and control of a variable stiffness actuator for safe and fast physical human/robot interaction," In Proc. IEEE Int. Conf. on Robotics and Automation, pages 528–533, 2005.

[11] G. Palli and C. Melchiorri T. Wimb¨ock, M. Grebenstein and G. Hirzinger, "Feedback linearization and simultaneous stiffness-position control of robots with antagonistic actuated joints", IEEE International Conference on Robotics and Automation Roma, Italy, 10-14 April 2007

[12] Armstrong-Helouvry, B., P. Dupont, and C. Canudas de Wit, "A survey of models, analysis tools and compensation methods for control of machines with friction", Automatica, 30, 1083-1138 (1994)

[13] HODA A. ELMARAGHY, TAREK LAHDHIRI. and FRANK CIUCA "Robust Linear Control of Flexible Joint Robot Systems" Journal of Intelligent and Robotic Systems **34:** 335–356, 2002.

[14] Huang, J., & Lin, C. F. "Numerical approach to computing nonlinear H_∞ control laws", Journal of Guidance, Control and Dynamics, 18(5), pp. 989–993, 1995.

[15] JIE HUANG, "An algorithm to solve the discrete HJI equation arising in the L2 gain optimization problem", INT. Journal CONTROL, VOL. 72, NO. 1, pp.49- 57, 1999

[16] M. W. Spong. "Modeling and control of elastic joint robots", Journal of Dynamic Systems, Measurement, and Control, 109(4):310–319, 1987.

[17] Siciliano, B., "Control in Robotics: Open Problems and Future directions," Proc. IEEE Int. Conf. on Control Applications, 1998

[18] S. S. Ge*, Zhendong Sun, T. H. Lee " Nonregular feedback linearization for a class of second-order nonlinear systems " Automatica 37 (2001) 1819-1824

[19] Spong, M., Hutchinson, S., and Vidyasagar, M. (2006). Robot Modeling and Control. Wiley

Appendix:

TABLE III
VALUES OF UNCERTAIN COEFFICIENTS

$\alpha_1 = 0.12$	$\alpha_2 = 0.1$	$\alpha_3 = 0.2$
$\beta_1 = 0.12$	$\beta_2 = 0.1$	$\beta_3 = 0.2$
$\beta_4 = 0.23\,{}^{rad}\!/_{s^3}$	$\beta_5 = 0.43\,{}^{rad}\!/_{s^2}$	

TABLE IV

THE VECTOR OF $d_i(x)$

$d_1(x) = -a_{1N}\alpha_1 x_4$	$d_2(x) = -a_{2N}\alpha_2 x_3$
$d_3(x) = -a_{3N}\alpha_3 x_2$	$d_4(x) = -b_{1N}\beta_1 \sin x_1$
$d_5(x) = -b_{2N}\beta_2 x_2 \cos x_1$	$d_7(x) = c_1\beta_4$ $d_8(x) = c_2\beta_5$
$d_6(x) = -b_{3N}\beta_3(-x_2^2 \sin x_1 + x_3 \cos x_1)$	

TABLE V

THE VECTOR OF a_{iN}, b_{2N}, c_2

$a_{1N} = \dfrac{BR + K_m K_b}{JR} + \dfrac{D}{I}$	$a_{3N} = \dfrac{K(BR + K_m K_b)}{JIR}$
$a_{2N} = \dfrac{(J+I)KR + D(BR + K_m K_b)}{JIR}$	
$b_{2N} = \dfrac{mgl(BR + K_m K_b)}{JIR}$	$c_2 = \dfrac{D(BR + K_b K_m)}{JIR}$