Case Study of Survival Function under Strength Attenuation of System for Exponential Distribution

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Abstract—In this paper an expression for the reliability of a single component system is derived when the strength of the component and the imminent stresses on the system are random and follow non-identical Probability distribution. Tabulated some numerical values for different values of the parameter and also for the system which are presented graphically.

Keywords- Survival function, Exponential distribution, Stress-Strength model, Probability density function.

I. Introduction

Leve (1965) considered Single Component System with constant stress; the strengths of the components are attenuated by specified deterministic factors after successive survivals.

The general Mathematical model is obtained for the survival function of the expression which has been discussed. We deduce the expressions when both stress and strength are taken as Exponential distributions. We obtained 8 particular cases are taken for the different values of ρ and k and the Numerical calculations and the graphs for Exponential distribution are drawn.

The strength of a component is defined as the “minimum stress” required causing the component to fail. If Y is the stress to which a component is subjected and X is the strength of the component, we can write

X = [min y / the component fails when subjected to stress y]

The component operates as long as Y is less than X and the reliability of the component may therefore be defined as $R_c = p\ [y < x]$

II. General Mathematical Model

Consider a single component system α of an initial random strength x_1 with probability density function f(x_1), with the strength attenuation factor K_i^* (i = 1,2,3, ... n). The strength is exposed to independent random stresses y_1 with probability density function g(y_i) which arrives as a Poisson process over time. If $y < x_1$ the component survives the first attack, but with change in its strength the new strength is $x_2 = k_2 x_1$, where k_2 is the corresponding attenuation factor with the modified strength the component faces the Second attack and so on. Thus, if the system has survived ‘r’ attacks its strength on (r+1)th attack will be

$$X_{r+1} = K^*_{r+1} X_1 = (K_1, K_2, K_3 \ldots\ldots K_{r+1}) X_1$$

Where k^*_{r+1} = k_1, k_2, k_3 . . . . . . k_{r+1} is the cumulative attention factor at the (r+1)th attack.

i.e., $X_{r+1} = k^*_{r+1} X_1$ for elegance of expression, the attention factor K_1 is defined as K_1=1.

Hence the Survival function $\Im$(n) is defined as

$$\Im(n) = \int_0^\infty \left[\prod_{r=1}^{n} p\left(y_r < k_r^* x_1 \right) \right] f(x_1) dX_1$$

$$= \int_{o}^{\infty} \prod_{r=1}^{n} G\left(K_r^* X_1\right) f(x_1) dX_1$$

III. SURVIVAL FUNCTION FOR EXPONENTIAL DISTRIBUTION

When the initial strength X_1, and the successively impinging stresses y_1, y_2, y_3 ..... y_n are all independent exponential with parameters μ and $\lambda_1, \lambda_2, \lambda_3, \ldots \lambda_n$ respectively, Then the probability density functions are defined as

$$f(x_i) = \mu\, e^{-\mu x_1} \quad 0 \leq x_1 \leq \infty$$

$$g_i(y_i) = \lambda_t\, e^{-\lambda y} \quad 0 \leq y \leq \infty$$

Hence $G_i(Y_i) = \int_0^{K_i^* x_i} \lambda_i e^{-\lambda_i Y_i} dy_i = 1 - e^{-k_i^* \lambda_i x_i}$

then the survival function $\Im(n)$ is

$$\Im(n) = \int_0^{\infty} \prod_{i=1}^{n} G_i(y_i) f(x_i) dx_i$$

$$= \int_0^{\infty} \prod_{i=1}^{n} \left(1 - e^{k_i^* \lambda_1}\right) \mu e^{-\mu x_i} dx_1$$

$$= \int_o^{\infty} \left[1 - \sum_{i=1}^{n} e^{-k_i^* \lambda_i x_i} + \sum_{i<j=1}^{n} e^{-(k_i^* \lambda_i + k_j^* \lambda_j) x_1} \ldots\ldots\ldots\ldots (-1)^n \sum_{i=1}^{n-1} e^{k_i^* \lambda_i x} \right] \mu e^{-\mu x_1} dx_1$$

$$= \mu \int_o^{\infty} \left[e^{-\mu x_1} - \sum_{i=1}^{n} e^{-(\mu + k_i^* \lambda_i) x_1} + \sum_{i<j=1}^{n} e^{-(\mu + k_i^* \lambda_i + k_j^* \lambda_j) x_1} \ldots\ldots\ldots \right.$$

$$\left. \ldots\ldots\ldots\ldots (-1)^n \sum_{i<j<k}^{n} e^{-\left(\sum_{i=1}^{n-1} e^{k_i^* \lambda_i x}\right) x_1} \right] dx_1$$

$$= \left[1 - \sum_{i=1}^{n} \frac{\mu}{\mu + k_i^* \lambda_i} + \sum_{i<j=1}^{n} \frac{\mu}{\mu + k_i^* \lambda_i + k_j^* \lambda_j} - \sum_{i<j<k}^{n} \frac{\mu}{\mu + k_i^* \lambda_i + k_j^* \lambda_j + k_k^* \lambda_k} \ldots \right.$$

$$\left. ....(-1)^n \sum_{i<j<k}^{n} \frac{\mu}{\mu + k_1^* \lambda_1 + k_2^* \lambda_2 + ... + k_n^* \lambda_n} \right]$$

IV. CASE STUDY

Defining $\ell_i = \dfrac{\lambda_i}{\mu}$

$$\Im(n) = \left[1 - \sum_{i=1}^{n} \frac{1}{1 + k_i^* \ell_i} + \sum_{i<j=1}^{n} \frac{1}{1 + k_i^* \ell_i + k_j^* \ell_j} - \sum_{i<j<k}^{n} \frac{1}{1 + k_i^* \ell_i + k_j^* \ell_j + k_k^* \ell_k} .... \right.$$

$$\left. .... (-1)^n \frac{1}{1 + k_1^* \ell_1 + k_2^* \ell_2 + ... + k_n^* \ell_n} \right]$$

Putting n=1,2,3 and 4 in above equation we get,

$$\Im(1) = 1 - \frac{1}{1 + k_1^* \ell_1}$$

$$\Im(2) = \left[1 - \left\{ \frac{1}{1 + k_1^* \ell_1} + \frac{1}{1 + k_2^* \ell_2} \right\} + \frac{1}{1 + k_1^* \ell_1 + k_2^* \ell_2} \right]$$

$$\Im(3) = \left[1 - \left\{ \frac{1}{1 + k_1^* \ell_1} + \frac{1}{1 + k_2^* \ell_2} + \frac{1}{1 + k_3^* \ell_3} \right\} \right.$$

$$\left. + \left\{ \frac{1}{1 + k_1^* \ell_1 + k_2^* \ell_2} + \frac{1}{1 + k_2^* \ell_2 + k_3^* \ell_3} + \frac{1}{1 + k_1^* \ell_1 + k_3^* \ell_3} \right\} - \frac{1}{1 + k_1^* \ell_1 + k_2^* \ell_2 + k_3^* \ell_3} \right]$$

$$\Im(4) = \left[1 - \left\{ \frac{1}{1 + k_1^* \ell_1} + \frac{1}{1 + k_2^* \ell_2} + \frac{1}{1 + k_3^* \ell_3} + \frac{1}{1 + k_4^* \ell_4} \right\} \right.$$

$$+ \left\{ \frac{1}{1 + k_1^* \ell_1 + k_2^* \ell_2} + \frac{1}{1 + k_2^* \ell_2 + k_3^* \ell_3} + \frac{1}{1 + k_3^* \ell_3 + k_4^* \ell_4} + \frac{1}{1 + k_1^* \ell_1 + k_3^* \ell_3} + \frac{1}{1 + k_1^* \ell_1 + k_4^* \ell_3} \right\}$$

$$- \left\{ \frac{1}{1 + k_1^* \ell_1 + k_2^* \ell_2 + k_3^* \ell_3} + \frac{1}{1 + k_1^* \ell_1 + k_2^* \ell_2 + k_4^* \ell_4} + \frac{1}{1 + k_1^* \ell_1 + k_3^* \ell_3 + k_4^* \ell_4} \right\}$$

$$\left. + \frac{1}{1 + k_1^* \ell_1 + k_2^* \ell_2 + k_3^* \ell_3 + k_4^* \ell_4} \right]$$

Numerical evaluation of $\Im(n)$ for n=1, 2,3and 4 are made. The values of the parameters ℓ and k for which the computations are done are $\ell_i = \ell$, $\ell_i = i\,\ell$ and $k_i^* =$ i, i!, $\frac{1}{i!}$ and $\frac{1}{i}$ for these 8 particular cases numerical calculations have been done by taking the value of ℓ from 0.03 to 2.90.

Case 1:- When $\ell_i = \ell$ and $k_i^* =$ i!

$$\Im(1) = 1-\frac{1}{1+\ell}$$

$$\Im(2)= \left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+2\ell}\right\}+\frac{1}{1+3\ell}\right]$$

$$\Im(3)= \left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+2\ell}+\frac{1}{1+6\ell}\right\}+\left\{\frac{1}{1+3\ell}+\frac{1}{1+8\ell}+\frac{1}{1+7\ell}\right\}-\frac{1}{1+9\ell}\right]$$

$$\Im(4)= \left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+2\ell}+\frac{1}{1+6\ell}+\frac{1}{1+24\ell}\right\}\right.$$
$$+\left\{\frac{1}{1+3\ell}+\frac{1}{1+8\ell}+\frac{1}{1+30\ell}+\frac{1}{1+26\ell}+\frac{1}{1+7\ell}+\frac{1}{1+25\ell}\right\}$$
$$\left.-\left\{\frac{1}{1+9\ell}+\frac{1}{1+27\ell}+\frac{1}{1+31\ell}+\frac{1}{1+32\ell}\right\}+\frac{1}{1+33\ell}\right]$$

Case 2:- When $\ell_i = \ell$ and $k_i^* = \frac{1}{\text{i!}}$

$$\Im(1) = 1-\frac{1}{1+\ell}$$

$$\Im(2)= \left[1-\left\{\frac{1}{1+\ell}+\frac{2}{2+\ell}\right\}+\frac{2}{2+3\ell}\right]$$

$$\Im(3)= \left[1-\left\{\frac{1}{1+\ell}+\frac{2}{2+\ell}+\frac{6}{6+\ell}\right\}+\left\{\frac{2}{2+3\ell}+\frac{6}{6+4\ell}+\frac{6}{6+4\ell}\right\}-\frac{6}{6+7\ell}\right]$$

$$\Im(4)= \left[1-\left\{\frac{1}{1+\ell}+\frac{2}{2+\ell}+\frac{6}{6+\ell}+\frac{24}{24+\ell}\right\}\right.$$
$$+\left\{\frac{2}{2+3\ell}+\frac{6}{6+4\ell}+\frac{24}{24+5\ell}+\frac{24}{24+5\ell}+\frac{6}{6+7\ell}+\frac{24}{24+25\ell}\right\}$$
$$\left.-\left\{\frac{6}{6+10\ell}+\frac{24}{24+37\ell}+\frac{24}{24+29\ell}+\frac{24}{24+17\ell}\right\}+\frac{24}{24+41\ell}\right]$$

Case 3:- When $\ell_i = i\,\ell$ and $k_i^* =$ i!

$$\Im(1) = 1-\frac{1}{1+\ell}$$

$$\Im(2)= \left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+4\ell}\right\}+\frac{1}{1+5\ell}\right]$$

$$\Im(3)=\left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+4\ell}+\frac{1}{1+18\ell}\right\}+\left\{\frac{1}{1+5\ell}+\frac{1}{1+22\ell}+\frac{1}{1+19\ell}\right\}-\frac{1}{1+23\ell}\right]$$

$$\Im(4)=\left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+4\ell}+\frac{1}{1+18\ell}+\frac{1}{1+96\ell}\right\}\right.$$

$$+\left\{\frac{1}{1+5\ell}+\frac{1}{1+22\ell}+\frac{1}{1+114\ell}+\frac{1}{1+100\ell}+\frac{1}{1+19\ell}+\frac{1}{1+97\ell}\right\}$$

$$\left.-\left\{\frac{1}{1+23\ell}+\frac{1}{1+101\ell}+\frac{1}{1+115\ell}+\frac{1}{1+118\ell}\right\}+\frac{1}{1+119\ell}\right]$$

<u>Case 4</u>:- When $\ell_i = i\,\ell$ and $k_i^* = \frac{1}{i!}$

$$\Im(1) = 1-\frac{1}{1+\ell}$$

$$\Im(2)=\left[1-\frac{2}{1+\ell}+\frac{1}{1+2\ell}\right]$$

$$\Im(3)=\left[1-\left\{\frac{2}{1+\ell}+\frac{2}{2+\ell}\right\}+\left\{\frac{1}{1+2\ell}+\frac{4}{1+3\ell}\right\}-\frac{2}{2+5\ell}\right]$$

$$\Im(4)=\left[1-\left\{\frac{2}{1+\ell}+\frac{2}{2+\ell}+\frac{6}{6+\ell}\right\}\right.$$

$$+\left\{\frac{1}{1+2\ell}+\frac{4}{2+3\ell}+\frac{6}{6+4\ell}+\frac{12}{6+7\ell}\right\}$$

$$\left.-\left\{\frac{2}{2+5\ell}+\frac{6}{6+13\ell}+\frac{12}{6+10\ell}\right\}+\frac{6}{6+16\ell}\right]$$

<u>Case 5</u>:- When $\ell_i = \ell$ and $k_i^* = i$

$$\Im(1) = 1-\frac{1}{1+\ell}$$

$$\Im(2)=\left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+2\ell}\right\}+\frac{1}{1+3\ell}\right]$$

$$\Im(3)=\left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+2\ell}+\frac{1}{1+3\ell}\right\}+\left\{\frac{1}{1+3\ell}+\frac{1}{1+5\ell}+\frac{1}{1+4\ell}\right\}-\frac{1}{1+6\ell}\right]$$

$$\Im(4)=\left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+2\ell}+\frac{1}{1+3\ell}+\frac{1}{1+4\ell}\right\}\right.$$

$$+\left\{\frac{1}{1+3\ell}+\frac{2}{1+5\ell}+\frac{1}{1+7\ell}+\frac{1}{1+6\ell}+\frac{1}{1+6\ell}\right\}$$

$$-\left\{\frac{1}{1+6\ell}+\frac{1}{1+7\ell}+\frac{1}{1+8\ell}+\frac{1}{1+9\ell}\right\}+\frac{1}{1+10\ell}\Bigg]$$

Case 6:- When $\ell_i = \ell$ and $k_i^* = \frac{1}{i}$

$$\Im(1) = 1-\frac{1}{1+\ell}$$

$$\Im(2)= \left[1-\left\{\frac{1}{1+\ell}+\frac{2}{2+\ell}\right\}+\frac{2}{2+3\ell}\right]$$

$$\Im(3)= \left[1-\left\{\frac{1}{1+\ell}+\frac{2}{2+\ell}+\frac{3}{3+\ell}\right\}+\left\{\frac{2}{2+3\ell}+\frac{6}{6+5\ell}+\frac{3}{3+4\ell}\right\}-\frac{6}{6+11\ell}\right]$$

$$\Im(4)= \Bigg[1-\left\{\frac{1}{1+\ell}+\frac{2}{2+\ell}+\frac{3}{3+\ell}+\frac{4}{4+\ell}\right\}$$

$$+\left\{\frac{2}{2+3\ell}+\frac{6}{6+5\ell}+\frac{3}{3+4\ell}+\frac{4}{4+5\ell}+\frac{4}{4+3\ell}+\frac{12}{12+7\ell}\right\}$$

$$-\left\{\frac{6}{6+11\ell}+\frac{4}{4+7\ell}+\frac{12}{12+19\ell}+\frac{12}{12+13\ell}\right\}-\frac{12}{12+25\ell}\Bigg]$$

Case 7:- When $\ell_i = i\,\ell$ and $k_i^* = i$

$$\Im(1) = 1-\frac{1}{1+\ell}$$

$$\Im(2)= \left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+4\ell}\right\}+\frac{1}{1+4\ell}\right]$$

$$\Im(3)= \left[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+4\ell}+\frac{1}{1+9\ell}\right\}+\left\{\frac{1}{1+5\ell}+\frac{1}{1+13\ell}+\frac{1}{1+10\ell}\right\}-\frac{1}{1+14\ell}\right]$$

$$\Im(4)= \Bigg[1-\left\{\frac{1}{1+\ell}+\frac{1}{1+4\ell}+\frac{1}{1+9\ell}+\frac{1}{1+16\ell}\right\}$$

$$+\left\{\frac{1}{1+5\ell}+\frac{1}{1+13\ell}+\frac{1}{1+25\ell}+\frac{1}{1+10\ell}+\frac{1}{1+17\ell}++\frac{1}{1+20\ell}\right\}$$

$$-\left\{\frac{1}{1+14\ell}+\frac{1}{1+21\ell}+\frac{1}{1+26\ell}+\frac{1}{1+29\ell}\right\}+\frac{1}{1+30\ell}\Bigg]$$

Case 8:- When $\ell_i = i\,\ell$ and $k_i^* = \frac{1}{i}$

$$\Im(1) = 1-\frac{1}{1+\ell}$$

$$\Im(2)= \left[1-\frac{2}{1+\ell}+\frac{1}{1+2\ell}\right]$$

$$\Im(3)= \left[1-\frac{2}{1+\ell}+\frac{3}{1+2\ell}-\frac{1}{1+3\ell}\right]$$

$$S\Im(4)= \left[1-\frac{4}{1+\ell}+\frac{6}{1+2\ell}-\frac{4}{1+3\ell}+\frac{1}{1+4\ell}\right]$$

TABLE I: When $\ell_i = \ell$ and $k_i^* = i!$

ρ	ℑ(1)	ℑ(2)	ℑ(3)	Sℑ(1)	Sℑ(2)	Sℑ(3)	Sℑ(4)
0.40	0.2857	0.1847	0.1744	0.0334	0.4704	0.6449	0.6783
0.41	0.2908	0.1898	0.1796	0.0417	0.4805	0.6601	0.7018
0.42	0.2958	0.1948	0.1846	0.0499	0.4905	0.6752	0.7251
0.43	0.3007	0.1997	0.1897	0.0578	0.5004	0.6901	0.7479
0.44	0.3056	0.2047	0.1947	0.0656	0.5102	0.7049	0.7705
0.45	0.3103	0.2096	0.1996	0.0733	0.5199	0.7195	0.7928
0.46	0.3151	0.2144	0.2045	0.0807	0.5295	0.7340	0.8147
0.47	0.3197	0.2192	0.2094	0.0881	0.5389	0.7483	0.8364
0.48	0.3243	0.2240	0.2142	0.0953	0.5483	0.7625	0.8578
0.49	0.3289	0.2287	0.2190	0.1023	0.5575	0.7765	0.8789
0.50	0.3333	0.2333	0.2237	0.1093	0.5667	0.7904	0.8997
0.51	0.3377	0.2380	0.2284	0.1161	0.5757	0.8041	0.9202
0.52	0.3421	0.2425	0.2331	0.1228	0.5846	0.8177	0.9405
0.53	0.3464	0.2471	0.2377	0.1293	0.5935	0.8312	0.9605
0.54	0.3506	0.2516	0.2422	0.1358	0.6022	0.8444	0.9802
0.55	0.3548	0.2560	0.2468	0.1422	0.6108	0.8576	0.9998
0.56	0.3590	0.2604	0.2512	0.1484	0.6194	0.8706	1.0190

TABLE II: When $\ell_i = \ell$ and $k_i^* = \frac{1}{i!}$

ρ	ℑ(1)	ℑ(2)	ℑ(3)	Sℑ(1)	Sℑ(2)	Sℑ(3)	Sℑ(4)
1.00	0.5000	0.2333	0.0627	-0.7321	0.7333	0.7961	0.0640
1.10	0.5238	0.2560	0.0729	-0.6868	0.7798	0.8527	0.1659
1.20	0.5455	0.2776	0.0832	-0.6462	0.8231	0.9062	0.2600
1.30	0.5652	0.2981	0.0935	-0.6096	0.8634	0.9569	0.3473
1.40	0.5833	0.3177	0.1039	-0.5764	0.9010	1.0049	0.4285
1.50	0.6000	0.3363	0.1142	-0.5461	0.9363	1.0504	0.5043
1.60	0.6154	0.3539	0.1245	-0.5183	0.9693	1.0938	0.5755
1.70	0.6296	0.3708	0.1346	-0.4927	1.0004	1.1350	0.6424
1.80	0.6429	0.3868	0.1447	-0.4689	1.0297	1.1744	0.7055
1.90	0.6552	0.4021	0.1547	-0.4468	1.0573	1.2119	0.7651
2.00	0.6667	0.4167	0.1645	-0.4262	1.0833	1.2478	0.8216
2.05	0.6721	0.4237	0.1693	-0.4164	1.0958	1.2652	0.8488
2.10	0.6774	0.4306	0.1741	-0.4069	1.1080	1.2821	0.8753
2.15	0.6825	0.4373	0.1789	-0.3976	1.1198	1.2988	0.9011
2.20	0.6875	0.4439	0.1837	-0.3887	1.1314	1.3150	0.9263
2.25	0.6923	0.4503	0.1884	-0.3800	1.1426	1.3310	0.9510
2.30	0.6970	0.4566	0.1930	-0.3715	1.1535	1.3466	0.9750
2.35	0.7015	0.4627	0.1976	-0.3633	1.1642	1.3618	0.9985
2.36	0.7024	0.4639	0.1986	-0.3617	1.1663	1.3649	1.0032

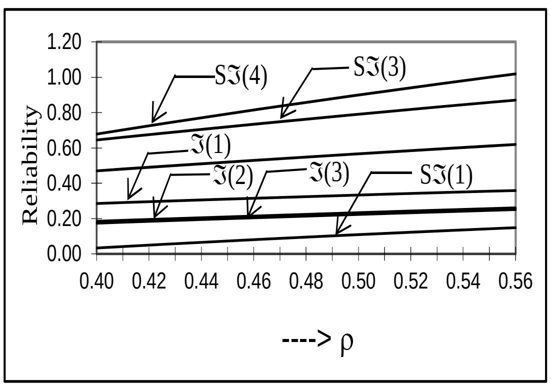

Figure 1: When $\ell_i = \ell$ and $k_i^* = i!$

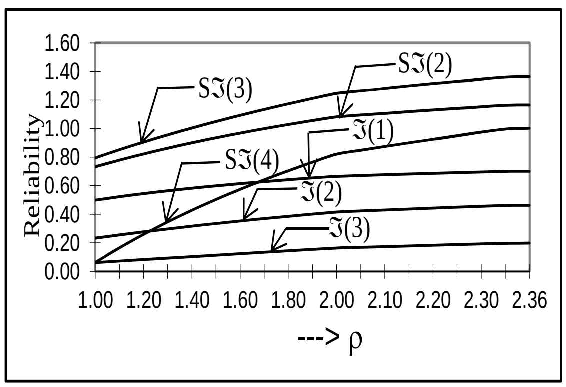

Figure 2: When $\ell_i = \ell$ and $k_i^* = \frac{1}{i!}$

TABLE III: When $\ell_i = i\,\ell$ and $k_i^* = i!$

ρ	ℑ(1)	ℑ(2)	ℑ(3)	Sℑ(1)	Sℑ(2)	Sℑ(3)	Sℑ(4)
0.18	0.1525	0.0975	0.0949	0.0057	0.2500	0.3449	0.3506
0.19	0.1597	0.1043	0.1018	0.0171	0.2640	0.3658	0.3829
0.20	0.1667	0.1111	0.1087	0.0280	0.2778	0.3864	0.4145
0.21	0.1736	0.1179	0.1155	0.0385	0.2914	0.4069	0.4454
0.22	0.1803	0.1246	0.1223	0.0487	0.3049	0.4272	0.4758
0.23	0.1870	0.1313	0.1290	0.0585	0.3183	0.4472	0.5057
0.24	0.1935	0.1379	0.1356	0.0680	0.3314	0.4671	0.5350
0.25	0.2000	0.1444	0.1422	0.0772	0.3444	0.4867	0.5639
0.26	0.2063	0.1509	0.1488	0.0861	0.3573	0.5061	0.5922
0.28	0.2188	0.1637	0.1616	0.1034	0.3825	0.5441	0.6475
0.30	0.2308	0.1762	0.1742	0.1197	0.4070	0.5812	0.7009
0.32	0.2424	0.1884	0.1865	0.1353	0.4309	0.6174	0.7527
0.34	0.2537	0.2004	0.1985	0.1503	0.4541	0.6526	0.8029
0.36	0.2647	0.2120	0.2102	0.1646	0.4767	0.6869	0.8515
0.38	0.2754	0.2234	0.2216	0.1784	0.4987	0.7204	0.8987
0.39	0.2806	0.2289	0.2272	0.1850	0.5095	0.7367	0.9218
0.40	0.2857	0.2344	0.2328	0.1916	0.5201	0.7529	0.9445
0.41	0.2908	0.2399	0.2382	0.1980	0.5306	0.7689	0.9669
0.42	0.2958	0.2452	0.2436	0.2044	0.5410	0.7846	0.9890
0.43	0.3007	0.2505	0.2489	0.2106	0.5512	0.8001	1.0107

TABLE IV: When $\ell_i = i\,\ell$ and $k_i^* = \frac{1}{i!}$

ρ	ℑ(1)	ℑ(2)	ℑ(3)	Sℑ(1)	Sℑ(2)	Sℑ(3)	Sℑ(4)
0.90	0.4737	-0.4098	0.1582	0.0444	0.0639	0.2221	0.2665
0.94	0.4845	-0.3782	0.1674	0.0481	0.1064	0.2738	0.3219
0.98	0.4949	-0.3479	0.1765	0.0519	0.1470	0.3235	0.3754
1.02	0.5050	-0.3190	0.1854	0.0557	0.1859	0.3713	0.4271
1.06	0.5146	-0.2914	0.1943	0.0596	0.2232	0.4174	0.4771
1.20	0.5455	-0.2032	0.2243	0.0735	0.3422	0.5666	0.6401
1.24	0.5536	-0.1802	0.2326	0.0775	0.3734	0.6060	0.6835
1.28	0.5614	-0.1581	0.2408	0.0816	0.4033	0.6441	0.7257
1.32	0.5690	-0.1368	0.2488	0.0856	0.4322	0.6810	0.7666
1.36	0.5763	-0.1163	0.2567	0.0897	0.4600	0.7167	0.8065
1.40	0.5833	-0.0965	0.2645	0.0938	0.4868	0.7514	0.8452
1.44	0.5902	-0.0774	0.2722	0.0979	0.5128	0.7849	0.8828
1.48	0.5968	-0.0590	0.2797	0.1019	0.5378	0.8175	0.9195
1.50	0.6000	-0.0500	0.2834	0.1040	0.5500	0.8334	0.9374
1.52	0.6032	-0.0412	0.2871	0.1060	0.5620	0.8491	0.9551
1.54	0.6063	-0.0325	0.2908	0.1081	0.5738	0.8646	0.9726
1.55	0.6078	-0.0282	0.2926	0.1091	0.5796	0.8722	0.9813
1.56	0.6094	-0.0240	0.2944	0.1101	0.5854	0.8798	0.9899
1.57	0.6109	-0.0198	0.2962	0.1111	0.5911	0.8873	0.9984
1.58	0.6124	-0.0156	0.2980	0.1121	0.5968	0.8948	1.0069

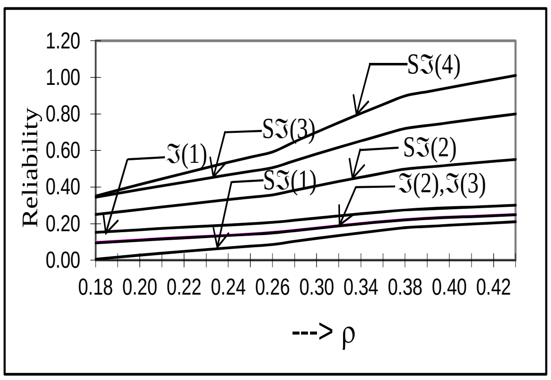

Figure 3: When $\ell_i = i\,\ell$ and $k_i^* = i!$

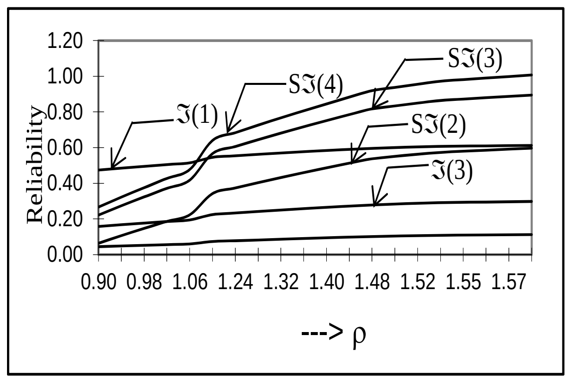

Figure 4: When $\ell_i = i\,\ell$ and $k_i^* = \frac{1}{i!}$

TABLE V : When $\ell_i = \ell$ and $k_i^* = i$

ρ	ℑ(1)	ℑ(2)	ℑ(3)	Sℑ(1)	Sℑ(2)	Sℑ(3)	Sℑ(4)
0.03	0.0291	0.0032	0.0007	0.0002	0.0323	0.0330	0.0332
0.06	0.0566	0.0112	0.0041	0.0022	0.0678	0.0719	0.0741
0.09	0.0826	0.0225	0.0107	0.0069	0.1051	0.1158	0.1226
0.12	0.1071	0.0360	0.0200	0.0143	0.1431	0.1631	0.1774
0.15	0.1304	0.0509	0.0313	0.0240	0.1813	0.2126	0.2366
0.18	0.1525	0.0666	0.0442	0.0355	0.2191	0.2633	0.2988
0.21	0.1736	0.0828	0.0581	0.0484	0.2564	0.3145	0.3629
0.24	0.1935	0.0993	0.0728	0.0622	0.2928	0.3656	0.4278
0.27	0.2151	0.1179	0.0899	0.0785	0.3330	0.4229	0.5014
0.30	0.2308	0.1321	0.1032	0.0914	0.3629	0.4660	0.5574
0.33	0.2481	0.1482	0.1185	0.1064	0.3963	0.5149	0.6212
0.36	0.2647	0.1641	0.1338	0.1214	0.4288	0.5626	0.6840
0.39	0.2806	0.1796	0.1490	0.1364	0.4602	0.6092	0.7455
0.41	0.2908	0.1898	0.1590	0.1463	0.4805	0.6395	0.7858
0.44	0.3056	0.2047	0.1737	0.1610	0.5102	0.6840	0.8449
0.47	0.3197	0.2192	0.1882	0.1754	0.5389	0.7271	0.9026
0.50	0.3333	0.2333	0.2024	0.1896	0.5667	0.7690	0.9587
0.51	0.3377	0.2380	0.2070	0.1943	0.5757	0.7827	0.9770
0.52	0.3421	0.2425	0.2116	0.1989	0.5846	0.7963	0.9952
0.53	0.3464	0.2471	0.2162	0.2035	0.5935	0.8097	1.0132

TABLE VI: When $\ell_i = \ell$ and $k_i^* = \frac{1}{i}$

ρ	ℑ(1)	ℑ(2)	ℑ(3)	Sℑ(1)	Sℑ(2)	Sℑ(3)	Sℑ(4)
1.00	0.5000	0.2333	0.1590	-0.6032	0.7333	0.8923	0.2891
1.05	0.5122	0.2448	0.1671	-0.5774	0.7570	0.9241	0.3467
1.10	0.5238	0.2560	0.1751	-0.5529	0.7798	0.9550	0.4020
1.15	0.5349	0.2669	0.1831	-0.5295	0.8018	0.9849	0.4554
1.20	0.5455	0.2776	0.1910	-0.5072	0.8231	1.0140	0.5069
1.25	0.5556	0.2880	0.1988	-0.4857	0.8436	1.0423	0.5566
1.30	0.5652	0.2981	0.2065	-0.4652	0.8634	1.0698	0.6047
1.35	0.5745	0.3080	0.2141	-0.4454	0.8825	1.0966	0.6512
1.40	0.5833	0.3177	0.2216	-0.4264	0.9010	1.1226	0.6962
1.45	0.5918	0.3271	0.2290	-0.4080	0.9189	1.1479	0.7398
1.50	0.6000	0.3363	0.2363	-0.3904	0.9363	1.1725	0.7822
1.55	0.6078	0.3452	0.2435	-0.3733	0.9531	1.1965	0.8233
1.60	0.6154	0.3539	0.2506	-0.3567	0.9693	1.2199	0.8632
1.65	0.6226	0.3625	0.2575	-0.3407	0.9851	1.2427	0.9020
1.70	0.6296	0.3708	0.2644	-0.3252	1.0004	1.2648	0.9397
1.75	0.6364	0.3789	0.2712	-0.3101	1.0153	1.2865	0.9764
1.76	0.6377	0.3805	0.2726	-0.3071	1.0182	1.2907	0.9836
1.77	0.6390	0.3821	0.2739	-0.3042	1.0211	1.2950	0.9908
1.78	0.6403	0.3837	0.2753	-0.3013	1.0240	1.2992	0.9979
1.79	0.6416	0.3852	0.2766	-0.2984	1.0268	1.3034	1.0050

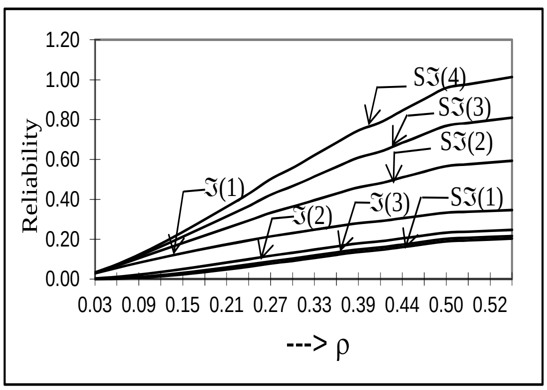

Figure 5: When $\ell_i = \ell$ and $k_i^* = i$

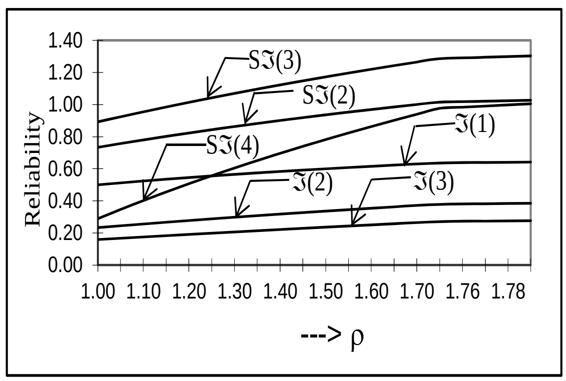

Figure 6: When $\ell_i = \ell$ and $k_i^* = \frac{1}{i}$

TABLE VII: When $\ell_i = i\,\ell$ and $k_i^* = i$

ρ	ℑ(1)	ℑ(2)	ℑ(3)	Sℑ(1)	Sℑ(2)	Sℑ(3)	Sℑ(4)
0.03	0.0291	0.0291	0.0029	0.0021	0.0583	0.0611	0.0632
0.06	0.0566	0.0566	0.0134	0.0118	0.1132	0.1266	0.1384
0.09	0.0826	0.0826	0.0291	0.0272	0.1651	0.1942	0.2215
0.12	0.1071	0.1071	0.0477	0.0457	0.2143	0.2620	0.3077
0.15	0.1304	0.1304	0.0677	0.0657	0.2609	0.3286	0.3943
0.18	0.1525	0.1525	0.0882	0.0862	0.3051	0.3933	0.4796
0.20	0.1667	0.1667	0.1019	0.1000	0.3333	0.4353	0.5352
0.22	0.1803	0.1803	0.1155	0.1136	0.3607	0.4762	0.5898
0.24	0.1935	0.1935	0.1289	0.1271	0.3871	0.5160	0.6431
0.26	0.2063	0.2063	0.1421	0.1403	0.4127	0.5548	0.6951
0.28	0.2188	0.2188	0.1551	0.1533	0.4375	0.5926	0.7459
0.30	0.2308	0.2308	0.1677	0.1660	0.4615	0.6293	0.7953
0.32	0.2424	0.2424	0.1801	0.1785	0.4848	0.6650	0.8435
0.34	0.2537	0.2537	0.1922	0.1906	0.5075	0.6997	0.8903
0.36	0.2647	0.2647	0.2040	0.2025	0.5294	0.7335	0.9360
0.38	0.2754	0.2754	0.2156	0.2141	0.5507	0.7663	0.9804
0.40	0.2857	0.2857	0.2268	0.2254	0.5714	0.7982	1.0236

TABLE VIII: When $\ell_i = i\,\ell$ and $k_i^* = \frac{1}{i}$

ρ	ℑ(1)	ℑ(2)	ℑ(3)	Sℑ(1)	Sℑ(2)	Sℑ(3)	Sℑ(4)
0.05	0.0476	0.0043	0.0006	0.0001	0.0519	0.0525	0.0526
0.10	0.0909	0.0152	0.0035	0.0010	0.1061	0.1096	0.1106
0.15	0.1304	0.0301	0.0093	0.0035	0.1605	0.1699	0.1734
0.20	0.1667	0.0476	0.0179	0.0079	0.2143	0.2321	0.2401
0.25	0.2000	0.0667	0.0286	0.0143	0.2667	0.2952	0.3095
0.30	0.2308	0.0865	0.0410	0.0224	0.3173	0.3583	0.3807
0.35	0.2593	0.1068	0.0547	0.0319	0.3660	0.4207	0.4526
0.40	0.2857	0.1270	0.0693	0.0426	0.4127	0.4820	0.5246
0.45	0.3103	0.1470	0.0844	0.0543	0.4574	0.5418	0.5961
0.50	0.3333	0.1667	0.1000	0.0667	0.5000	0.6000	0.6667
0.55	0.3548	0.1859	0.1157	0.0796	0.5407	0.6564	0.7360
0.60	0.3750	0.2045	0.1315	0.0928	0.5795	0.7110	0.8039
0.62	0.3827	0.2119	0.1378	0.0982	0.5946	0.7324	0.8306
0.64	0.3902	0.2191	0.1441	0.1036	0.6093	0.7534	0.8570
0.66	0.3976	0.2262	0.1503	0.1090	0.6238	0.7741	0.8831
0.68	0.4048	0.2333	0.1565	0.1144	0.6380	0.7945	0.9090
0.70	0.4118	0.2402	0.1627	0.1199	0.6520	0.8147	0.9346
0.72	0.4186	0.2470	0.1689	0.1253	0.6657	0.8345	0.9599
0.74	0.4253	0.2538	0.1750	0.1308	0.6791	0.8541	0.9849
0.76	0.4318	0.2605	0.1811	0.1362	0.6923	0.8733	1.0096

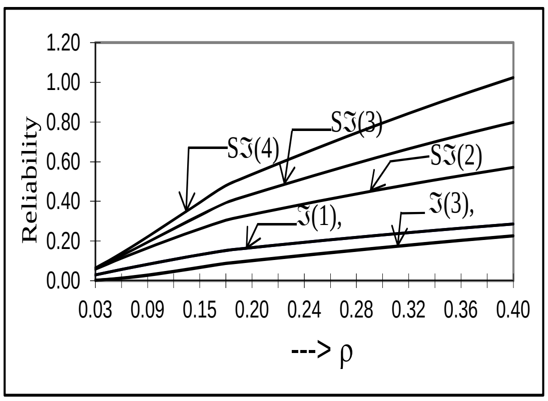

Figure 7: When $\ell_i = i\,\ell$ and $k_i^* = i$

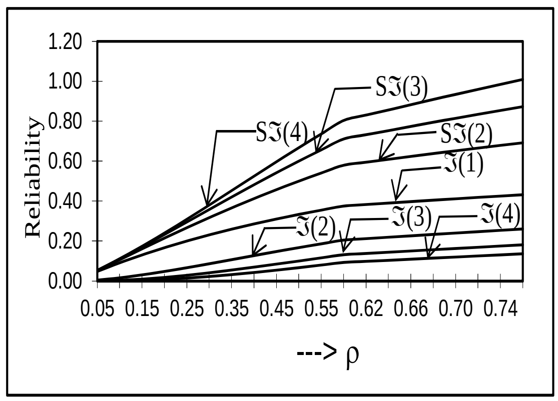

Figure 8: When $\ell_i = i\,\ell$ and $k_i^* = \frac{1}{i}$

V. CONCLUSION

<u>Case 1</u>:- When $\ell_i = \ell$ and $k_i^* = i!$

The results in this case are shown in Table-I and Fig.1, for $\ell = 0.4$ to 0.59, we observed that ℑ(1) > ℑ(4) and ℑ(2), ℑ(4) are coincided and Sℑ(4) > Sℑ(3) > Sℑ(1).

<u>Case 2</u>:- When $\ell_i = \ell$ and $k_i^* = \frac{1}{i!}$

The results in this case are shown in Table-II and Fig.2, for $\ell = 1$ to 2.90, we observed that ℑ(n) is increases for all specified values of ℓ except ℑ(4) is negative and Sℑ(n) decreases with n .

<u>Case 3</u>:- When $\ell_i = i\,\ell$ and $k_i^* = i!$

The results in this case are shown in Table-III and Fig.3, for ℓ = 0.1 to .48, we observed that $\Im(2)$ and $\Im(3)$ are coincided, and $\Im(1) > \Im(4)$. $S\Im(n)$ increases with n and tends to 1.

Case 4:- When $\ell_i = i\ell$ and $k_i^* = \frac{1}{i!}$

The results in this case are shown in Table-IV and Fig.4, for ℓ = 0.9 to 1.76, we observed that $\Im(1)$ and $\Im(3)$ are parallel to X-axis and $\Im(2)$ is negative. $S\Im(n)$ increases with n and tends to 1.

Case 5:- When $\ell_i = \ell$ and $k_i^* = i$

The results in this case are shown in Table-5 and Fig.V, for ℓ = 0.03 to 0.6, we observed that $\Im(3)$ and $\Im(4)$ are equal and $\Im(1) > \Im(2)$. $S\Im(n)$ increases with n and tends to 1.

Case 6:- When $\ell_i = \ell$ and $k_i^* = \frac{1}{i}$

The results in this case are shown in Table-6 and Fig.VI, for ℓ = 1 to 2, we observed that $\Im(1) > \Im(2) > \Im(3)$ and $\Im(4)$ is negative. $S\Im(n)$ increases with n and tends to 1.

Case 7:- When $\ell_i = i\ell$ and $k_i^* = i$

The results in this case are shown in Table-7 and Fig.VII, for ℓ = 0.03 to 0.53, we observed that $\Im(3)$ and $\Im(4)$ are coincided and also $\Im(1)$ and $\Im(2)$ coincided. $S\Im(n)$ increases with n and tends to 1.

Case 8:- When $\ell_i = i\ell$ and $k_i^* = \frac{1}{i}$

The results in this case are shown in Table-8 and Fig.VIII, for ℓ = 1 to 2, we observed that $\Im(n)$ decreases $S\Im(n)$ increases with n and tends to 1.

VI. ACKNOWLEDGEMENT

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