# Machine Repair Problem with K-Type Warm Spares, Multiple Vacations for Repairmen and Reneging

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***Abstract-*** **In this paper, a multi-repairmen machine repair problem comprising of M operating units together with k type of warm spares has been investigated. The failed units are immediately attended by repairmen if available. When there are no failed units in the queue for repair, the repairmen leave for a vacation of random length. When the vacation period is terminated, the repairmen return and check for any failed unit in the queue. If the queue is non-empty, the repairmen start repairing of failed units until the queue becomes empty, and again they leave for another vacation. The repair rate of the repairmen depends upon the workload. The caretaker of the failed machines may renege on finding all repairmen busy. The steady state queue size distribution and other important performance measures have been derived using matrix recursive approach. Furthermore, a numerical example provides a profound understanding of the sensitivity of parameters with respect to various performance measures.**

***Keywords:*** **Machine repair, Multi-repairmen, Warm spares, Reneging, Multiple vacations, Matrix recursive approach, Queue size.**

## I. INTRODUCTION

The machining systems have pervaded every field of our lives in different activities as such ensuring our almost total dependence on them. As the time progresses, a machine becomes prone to failure. The failure of machines may result in loss of production, money, goodwill etc.. This situation can be handled by providing spare part support and corrective maintenance provided by the repairman.

In modern context, the failure and repair are coupled events in a typical machining system. Many researchers have suggested the provision of repair crew and spare part support to ensure the desired efficiency of the machining system. The important contributions in the areas of machine repair problem are due to [1], [12], [15] and many others. Cost benefit analysis of series system with warm standby components was done by [16] and [17]. [5] analyzed an M/M/C interdependent machining system with mixed spares and controllable failure and repair rates. [6] examined a manufacturing system consisting of M operating units and S spares under the supervision of a group of repairmen.

As soon as a machine is failed, it is sent to repair facility. In case, when all repairmen are busy and there is a long queue of failed units, a newly failed unit may not like to join the queue (i.e. balk) or may leave the system after spending some time in queue (i.e. renege) due to impatience or otherwise. Hence, while developing a realistic queueing model for machine repair problem, it becomes mandatory to include the discouragement behavior of the failed units too. Some notable works in this regard are as follows. [2] investigated machine repair problem with balking, reneging and warm spares. M/M/R machine repair problems with balking and reneging were studied by [7] and [11] for cost analysis purpose. The probabilistic analysis of a repairable machining system using warm spares with balking and reneging concepts was done by [14]. Machine repair system with standby components, balking and reneging was investigated by [4] using diffusion process.

Queueing systems with vacations have many applications in machining systems working in industrial environment such as manufacturing and production systems. When there is no failed unit present in the system; what should a repairmen do? The answer is, instead of remaining idle during this period, the repairmen may go for a vacation and can utilize this time to do some ancillary work such as preventive maintenance, proper arrangement of tools etc.. Over last three decades, a substantial amount of work has been done to examine queueing systems with vacations.

Machine interference problem with warm spares, server vacations and exhaustive service was tackled by [3]. [18] analyzed a multi-server queueing system in which d ($\leq$c) out of total c servers take synchronous vacations at a service completion instant. The stationary distributions of queue length and waiting time were obtained with the help of matrix geometric method. Later, [13] incorporated a two-threshold vacation policy in multi-server queueing systems and by using matrix analytic method, obtained stationary queue size distribution. A Markovian queue with two heterogeneous servers and multiple vacations was analyzed by [9]. They used matrix geometric approach to calculate queue length distribution and mean system size. [10] used matrix geometric approach to obtain steady state solution of GI/Geo/1 queue with working vacations and vacation interruption.

The scarcity of the works related to multi-repairmen problem with spares and vacations motivated us to develop multi-repairmen machine repair model with multiple vacations. By incorporating k- type of warm spares along with the concepts of common cause failure and reneging factor, the model becomes more robust and realistic. Recently, a comparable M/M/R model was developed by [8], who used two types of warm spares to assist the system, along with single and multiple vacation policies. The investigation has been organized in different sections as follows. In section II, the model has been described by stating the requisite assumptions and notations for mathematical formulation purpose. The state dependent failure and repair rates along with the system states notations are given in section III and the balance equations governing the model have been constructed. The matrix method to evaluate the probabilities has been presented in section IV. For various performance measures explicit expressions have been established in section V. Some special cases have been discussed in section VI. Numerical results and sensitivity analysis have been provided in section VII. Finally, the concluding remarks have been made in section VIII.

## II. MODEL DESCRIPTION

Consider a multi component machining system having operating units as well as k-type of warm spares and a repair facility consisting of C repairmen. For the mathematical formulation of the model, we assume that

There is provision of $S_i$ (i=1, 2,…, k) spare units of ith type along with M operating units.

- The operating units and ith type spare units fail in Poisson fashion with rate $\lambda$ and $\alpha_i$ , i = 1,2,3,…,k , respectively. Also $\alpha_1>\alpha_2>\alpha_3>....>\alpha_k$.
- The machining system can also fail due to some common cause in Poisson fashion with rate $\lambda_c$.
- The failed unit is immediately replaced by an available spare of higher failure rate and sent to repair facility.
- The switch over time from standby state to operating state is considered negligible.
- After being repaired, the unit joins the operating group if the system works with less than M units otherwise put with the standby group.
- The repair time of failed units is assumed to be exponentially distributed.
- The repairmen repair the failed units with rate $\mu$ till there are n<C failed units in the system and with faster rate $\mu_1$ when the number of failed units becomes equal or exceeds the number of repairmen i.e. $n \geq C$.
- When the repairmen becomes idle, he leaves for a vacation; the vacation time is exponentially distributed with rate $\theta$.
- The repairmen after returning from vacation check for any failed unit in the queue; if any, it is repaired, otherwise repairmen leave for another vacation. This trend continues, until the repairmen find any failed unit in the queue, on returning from vacation.

The failure and repair rates are given by

$$\lambda_n = \begin{cases} M\lambda + \lambda_c + (S_1 - n)\alpha_1 + \sum_{i=2}^{k} S_i\alpha_i, & 0 \le n < S_1 \\ M\lambda + \lambda_c + (\sum_{i=1}^{j} S_i - n)\alpha_i + \sum_{i=j+1}^{k} S_i\alpha_i, & \sum_{i=1}^{j-1} S_i \le n < \sum_{i=1}^{j} S_i; \\ & j = 2,3,...,k \\ (K-n)\lambda + \lambda_c, & \sum_{i=1}^{k} S_i \le n < M + \sum_{i=1}^{k} S_i = K \end{cases} \quad \text{…(1)}$$

and

$$\mu_n = \begin{cases} n\mu, & 0 < n < C \\ C\mu_1 + (n-C)\nu, & C \le n \le K \end{cases} \quad \text{…(2)}$$

The system states are represented in the form (i,j) where, i denotes the number of busy repairmen and j is used to denote the number of failed machines.

We denote the probabilities of state (i,j) by $X_{i,j}$

## III. THE GOVERNING EQUATIONS

The steady state equations governing the model for different states of the servers are constructed as follows:

### A. *When all repairmen are on vacation i.e. i=0.*

$$-\lambda_0 X_{0,0} + \mu X_{1,1} = 0 \quad \text{…(3)}$$

$$-(\lambda_n + C\theta)X_{0,n} + \lambda_{n-1}X_{0,n-1} = 0, \quad 1 \le n \le K-1 \quad \text{…(4)}$$

$$-\lambda_{K-1}X_{0,K-1} + C\theta X_{0,K} = 0 \quad \text{…(5)}$$

### B. *When some repairmen are on vacation i.e. $1 \le i \le C-1$*

$$-(\lambda_i + i\mu)X_{i,i} + i\mu X_{i,i+1} + [(C - \overline{i-1})\theta]X_{i-1,i} + (i+1)\mu X_{i+1,i+1} = 0, \quad i = n \le C-1 \quad \text{…(6)}$$

$$-[\lambda_n + i\mu + (C-i)\theta]X_{i,n} + i\mu X_{i,n+1} + \lambda_{n-1}X_{i,n-1} + [(C - \overline{i-1})\theta]X_{i-1,n} = 0, \quad 1 < n < C-1 \quad \text{…(7)}$$

$$-[\lambda_{C-1} + i\mu + (C-i)\theta]X_{i,C-1} + [(C - \overline{i-1})\theta]X_{i-1,C-1} + [i\mu_1 + (C-i)\nu]X_{i,C} + \lambda_{C-2}X_{i,C-2} = 0, \quad 1 < n = C-1 \quad \text{…(8)}$$

$$-[\lambda_n + (i\mu_1 + (n-i)\nu) + (C-i)\theta]X_{i,n} + [(C - \overline{i-1})\theta]X_{i-1,n} + [i\mu_1 + (n+1-i)\nu]X_{i,n+1} + \lambda_{n-1}X_{i,n-1} = 0, \quad i < n, \quad C \le n \le K-1 \quad \text{…(9)}$$

$$-[(i\mu_1 + (K-i)\nu) + (C-i)\theta]X_{i,K} + [(C - \overline{i-1})\theta]X_{i-1,K} + \lambda_{K-1}X_{i,K-1} = 0, \quad \text{…(10)}$$

### C. *When all repairmen are busy in providing repair, i.e.*

*i=C.*

$$-(\lambda_C + C\mu_1)X_{C,C} + \theta X_{C-1,C} + (C\mu_1 + \nu)X_{C,C+1} = 0 \quad \text{…(11)}$$

$$-[\lambda_n + (C\mu_1 + (n-C)\nu)]X_{C,n} + \theta X_{C-1,n} + [C\mu_1 + (n+1-C)\nu]X_{C,n+1} + \lambda_{n-1}X_{C,n-1} = 0,$$
$$C+1 \le n \le K-1 \quad \text{…(12)}$$

$$-[C\mu_1 + (K-C)\nu)]X_{C,K} + \theta X_{C-1,K} + \lambda_{K-1}X_{C,K-1} = 0 \quad \text{…(13)}$$

## IV. MATRIX RECURSIVE METHOD

Consider an irreducible Markov chain with transition probability matrix Q which can be represented in the following block tri-diagonal structure.

$$Q = \begin{bmatrix} D_0 & U_0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ L_1 & D_1 & U_1 & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & L_2 & D_2 & U_2 & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ \vdots & \vdots & L_3 & D_3 & U_3 & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & & \ddots & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & & \vdots & \vdots & L_{C-2} & D_{C-2} & U_{C-2} & \vdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & \vdots & L_{C-1} & D_{C-1} & U_{C-1} \\ 0 & \cdots & & \cdots & & \cdots & \cdots & 0 & L_C & D_C \end{bmatrix}$$

The matrix Q is a square matrix and of order (K+1) (C+1)-C(C+1)/2 and is composed of following sub-matrices

$$D_0 = \begin{bmatrix} -\lambda_i & \lambda_i & 0 & 0 & 0 & \cdots & \cdots & 0 \\ 0 & -(\lambda_{i+1}+C\theta) & \lambda_{i+1} & 0 & 0 & \vdots & \vdots & \vdots \\ 0 & 0 & -(\lambda_{i+2}+C\theta) & \lambda_{i+2} & 0 & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & -(\lambda_{K-1}+C\theta) & \lambda_{K-1} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -C\theta \end{bmatrix}$$

$$D_i = \begin{bmatrix} v_{i,i} & w_{i,i} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ u_{i+1,i} & v_{i+1,i} & w_{i+1,i} & 0 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & u_{i+2,i} & v_{i+2,i} & w_{i+2,i} & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & u_{i+3,i} & v_{i+3,i} & w_{i+3,i} & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \vdots & v_{K-3,i} & w_{K-3,i} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & u_{K-2,i} & v_{K-2,i} & w_{K-2,i} & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & u_{K-1,i} & v_{K-1,i} & w_{K-1,i} \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & u_{K,i} & v_{K,i} \end{bmatrix}, \quad 1 \le i \le C$$

where

$$v_{j,i} = \begin{cases} \lambda_j + (C-i)\theta + i\mu + (j-i)\nu, & \text{if } i+1 \le j \le C-1 \\ \lambda_j + (C-i)\theta + i\mu_1, & \text{if } C \le j \le K-1 \\ (C-i)\theta + i\mu_1, & \text{if } j = K \\ \lambda_j + i\mu, & \text{if } 0 < i = j \le C-1 \\ \lambda_j + i\mu_1, & \text{if } i = j = C \end{cases}$$

$$w_{j,i} = \{\lambda_i, \quad \text{if } i \le j \le K-1$$

and

$$u_{j,i} = \begin{cases} i\mu + (j-i)\nu, & i+1 \le j \le C-1 \\ i\mu_1 + (j-i)\nu, & C \le j \le K \end{cases}$$

For $0 \le i \le C\text{-}1$

$$U_i = \begin{bmatrix} 0 & 0 & \cdots & \cdots & \cdots & 0 & 0 \\ (C-i)\theta & 0 & \cdots & \cdots & \cdots & 0 & 0 \\ 0 & (C-i)\theta & \cdots & \cdots & \cdots & 0 & 0 \\ \vdots & 0 & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & 0 & \vdots \\ 0 & 0 & 0 & 0 & 0 & (C-i)\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & (C-i)\theta \end{bmatrix}$$

and

$$L_i = \begin{bmatrix} z_i & 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \\ \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 \end{bmatrix}, \quad 0 \le i \le C$$

where,

$$z_i = \begin{cases} i\mu & 1 \le i \le C-1 \\ i\mu_1 & i = C \end{cases}$$

The dimensions of $D_0$, $D_i$, $U_i$ and $L_i$ are (K+1)×(K+1), (K+1-i)×(K+1-i), (K+1-i)×(K-i) and (K+1-i)×(K+2-i), respectively. The stationary probability vector X = $[X_0, X_1, X_2, \ldots, X_C]$, where, $X_i = \{X_{i,i}, X_{i,i+1}, X_{i,i+2}, \ldots, X_{i,K}\}$ for $0 \le i \le C$ is a 1×(K+1-i) vector and is evaluated using XQ=0 along with the normalizing condition

$$\sum_{i=0}^{C} X_i e = 1 \quad \text{…(14)}$$

Here e is column vector with all elements equal to one.

Computation of stationary probabilities

The balance equations (3)-(13) can be written in matrix form as follows:

$$X_0D_0 + X_1L_1 = 0 \quad \text{…(15)}$$
$$X_{i-1}U_{i-1} + X_i D_i + X_{i+1}L_{i+1} = 0, \quad 1 \le i \le C-1 \quad \text{…(16)}$$
$$X_{C-1}U_{C-1} + X_CD_C = 0 \quad \text{…(17)}$$

$X_i$ can be obtained by solving the above equations in terms of $X_0$ as

$$X_i = X_{i-1}\eta_i, \quad 1 \le i \le C-1 \quad \text{…(18)}$$

where the stationary probability vector $X_i$ is a function of transition rates between $i^{th}$ state and its preceding state

Equation (17) gives

$$X_C = -X_{C-1}U_{C-1}D_C^{-1} \quad \text{…(19)}$$

Putting i=1 in equation (18) and using equation (15), we have

$$X_0 (D_0+L_1\eta_1) = 0 \quad \ldots(20)$$

where

$$\eta_i = -U_{i-1}(D_i + \eta_{i+1} + L_{i+1})^{-1} \text{ ; for i= 1,2,..,C-1} \quad \ldots(21)$$

Also we have $\eta_C = -U_{C-1}D_C^{-1}$ …(22)

$X_0$ can be obtained by using equation (14). With the help of eqs (18) and (19), we evaluate the probability vector $X_i$ (i=1,2,…,C).

## V. PERFORMANCE MEASURES

Various performance measures of machining system in terms of probabilities can be obtained. Now we establish some results as follows:

- The expected number of failed units in the system is

$$E(N) = \sum_{i=0}^{C}\sum_{n=i}^{K} nX_{i,n} \quad \ldots(23)$$

- The expected number of failed units in the queue is

$$E(N_q) = \sum_{i=0}^{C}\sum_{n=i}^{K}(n-i)X_{i,n} \quad \ldots(24)$$

- The expected number of operating machines in the system is

$$E(O) = M - \sum_{i=0}^{C}\sum_{\sum_{j=1}^{k}S_j+1}^{K}(n - \sum_{j=1}^{k}S_j)X_{i,n} \quad \ldots(25)$$

- The expected number of spare units of type 1 acting as standby is

$$E(S_1) = \sum_{i=0}^{C}\sum_{n=i}^{S_1}(S_1 - n)X_{i,n} \quad \ldots(26)$$

- The expected number of j (j = 1,2,…,k-1) type spare units in the system acting as standby is given by

$$E(S_j) = S_j\sum_{i=0}^{C}\sum_{n=1}^{\sum_{q=1}^{j-1}S_q}X_{i,n} + \sum_{i=0}^{C}\sum_{n=\sum_{q=1}^{j-1}S_q+1}^{\sum_{q=1}^{j}S_q}\left(\sum_{q=1}^{j}S_q - n\right)X_{i,n}\,,$$

j =2,3,…,k …(27)

- The expected number of spare units acting as standby is

$$E(S) = \sum_{i=1}^{k}E(S_i) \quad \ldots(28)$$

- The expected number of busy repairmen in the system

$$E(B) = \sum_{i=1}^{C}\sum_{n=i}^{K} i\,X_{i,n} \quad \ldots(29)$$

- The expected number of vacationing repairmen in the system is obtained as

$$E(V) = \sum_{i=0}^{C}\sum_{n=i}^{K}(C-i)\,X_{i,n} \quad \ldots(30)$$

- Machine availability is evaluated as

$$M.A. = 1 - \frac{E(N)}{K} \quad \ldots(31)$$

- The effective failure rate is given by

$$\lambda_{eff} = \sum_{i=0}^{C}\sum_{n=i}^{K-1}\lambda_n\,X_{i,n} \quad \ldots(32)$$

## VI. SPECIAL CASES

Here we present some earlier existing results as special cases, which have been deduced by setting appropriate parameters in our model.

**Case 1:** By setting $\theta = 0$, $\alpha_1, \alpha_2, \ldots, \alpha_k=\alpha$ and $\mu=\mu_1$, our model matches with the M/M/R machine repair model with warm spares, which was developed by ***Sivazlian and Wang (1989).***

**Case 2:** When C=1, k=2, $\alpha_1 = \alpha_2 = \alpha$, $\lambda_c=0$ and $\mu=\mu_1$, our model reduces to M/M/1 machine repair model with spare and server multiple vacations considered by ***Gupta (1997).***

**Case 3:** Putting $\theta = 0$, $\alpha_1, \alpha_2, \ldots, \alpha_k=0$, $\mu=\mu_1$, K=L and S=K-1, we obtain classical M/M/R/M/M queueing model ***(cf. Gross and Harris, 1985).***

**Case 4:** If k=2, $\lambda_c=0$, $\nu=0$ then our model corresponds to machine repair model with two type spares and multiple vacation policy which was recently developed by ***Ke and Wang (2007).***

## VII. NUMERICAL EXPERIMENT

In this section, the effects of varying parameters on various performance measures such as E(O), E(S), E(B), E(V), E(N) and M.A. have been displayed through tables and graphs. For numerical calculations we set default parameters as M=10, C=4, $S_1$=1, $S_2$=2, $S_3$=3, $\lambda_c$=0.05, $\alpha_1$= 0.3, $\alpha_2$= 0.2, $\alpha_3$= 0.1. Tables I and II display the effect of $\lambda$ and $\mu$ on E(O), E(S), E(B) and E(V) by varying $\theta$. It is noticed that E(O), E(S) and E(V) decrease (increase) while E(B) increases (decreases) as $\lambda$ ($\mu$) increases. When the effect is observed for increasing values of $\theta$, table I shows that E(O), E(S) and E(B) increase but E(V) decreases whereas it is clear from table II that E(O), E(S) and E(V) increase while E(B) decreases.

Figs 1 and 2 depict, the effect of $\lambda$ on E(N) and M.A. by varying values of $\alpha$, $\theta$ and $\nu$ respectively. For the increasing values of $\lambda$, E(L) shows a sharp increasing trend while M.A. displays decreasing trend. Fig. 1(a) shows a reasonable increment in E(N) with the increasing server's failure rate ($\alpha$). As the vacation ($\theta$) starts increasing, a notable downfall in queue length can be observed in fig 1(b). In fig. 1(c), we note that the queue size is not much affected by the increasing values of reneging parameter and it comes down slightly. Figs 2(a)-(c) exhibit the trend of M.A. for increasing values of $\alpha$, $\theta$ and $\nu$, respectively. From fig. 2(a), it is noted that M.A. becomes lessen for increasing values of $\alpha$. When the repairmen start returning from vacation frequently i.e. $\theta$ increases, M.A. also increases remarkably, as can be observed from fig. 2(b). In fig. 2(c), the impact of varying reneging parameter is very slight which was also observed in case of its effect on E(N) in fig. 1(c). Overall, we conclude that

- As we expect, with the increasing failure rate, the queue of failed units build up with fast pace whereas the availability of operating units and spares ceases as such more repairmen become busy.
- As the rate of returning of vacationing repairmen increases, the repair process speeds up which helps in the reduction of queue length; as a result, the operating units and spares start accumulating.

## VIII. CONCLUDING REMARKS

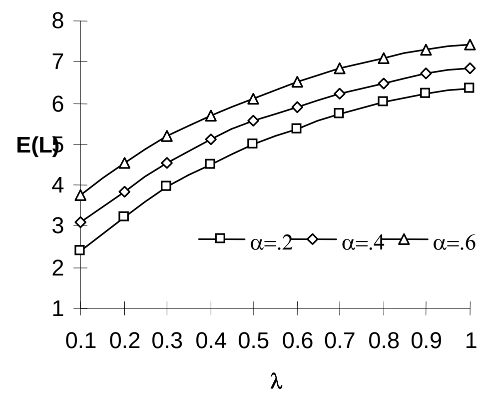
In this investigation, we have developed a performance model for a multi-component machining system with multiple repairmen and common cause failure based on queue theoretic approach. The provision of k- types of warm spares has been included by keeping in mind the pragmatic economical and physical constraints so that the machining system can work efficiently and smoothly. The concepts of common cause failure and reneging incorporated in our model fit in real time frame. Multiple vacations for repairmen considered have a significant role in reducing the cost involved as vacationing repairmen can do some ancillary work during vacation period. Various measures, which play a vital role in deciding the performance of the system, have been established. Numerical experiment performed may be helpful to explore the effects of parameters on performance measures. The model developed may be applied by the production managers, system engineers, management personal etc. in industrial organizations, who face difficulty in the maintenance of the system while making decision regarding the installation of the number of machines and the repairmen to continue the operation.

| θ | λ | E(O) | E(S) | E(B) | E(V) |
|---|---|---|---|---|---|
| 0.2 | 0.1 | 9.75 | 4.94 | 1.54 | 2.45 |
| | 0.3 | 9.49 | 3.98 | 1.84 | 2.15 |
| | 0.5 | 9.40 | 3.29 | 1.99 | 2.00 |
| | 0.7 | 9.25 | 2.90 | 2.04 | 1.95 |
| | 0.9 | 9.23 | 2.82 | 2.12 | 1.87 |
| 0.3 | 0.1 | 9.91 | 5.01 | 1.59 | 2.40 |
| | 0.3 | 9.69 | 3.77 | 1.86 | 2.13 |
| | 0.5 | 9.65 | 3.18 | 2.01 | 1.98 |
| | 0.7 | 9.63 | 2.75 | 2.09 | 1.90 |
| | 0.9 | 9.62 | 2.60 | 2.12 | 1.87 |
| 0.4 | 0.1 | 9.99 | 5.30 | 1.80 | 2.19 |
| | 0.3 | 9.82 | 3.40 | 1.90 | 2.09 |
| | 0.5 | 9.79 | 2.88 | 2.07 | 1.92 |
| | 0.7 | 9.78 | 2.54 | 2.10 | 1.89 |
| | 0.9 | 9.78 | 2.33 | 2.18 | 1.81 |

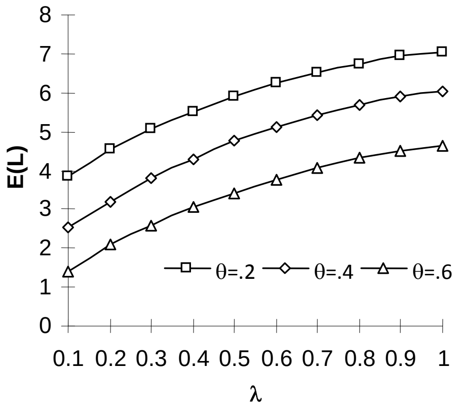
Table I: Performance measures by varying λ and θ

| θ | μ | E(O) | E(S) | E(B) | E(V) |
|---|---|---|---|---|---|
| 0.2 | 1 | 9.45 | 2.68 | 2.50 | 1.49 |
| | 2 | 9.56 | 2.76 | 2.33 | 1.66 |
| | 3 | 9.72 | 3.01 | 2.16 | 1.83 |
| | 4 | 9.74 | 3.29 | 1.85 | 2.14 |
| | 5 | 9.82 | 3.34 | 0.06 | 3.93 |
| 0.3 | 1 | 9.80 | 2.70 | 2.37 | 1.62 |
| | 2 | 9.84 | 3.14 | 2.31 | 1.68 |
| | 3 | 9.85 | 3.52 | 2.13 | 1.86 |
| | 4 | 9.86 | 3.77 | 1.45 | 2.54 |
| | 5 | 9.89 | 3.86 | 0.03 | 3.96 |
| 0.4 | 1 | 9.85 | 2.71 | 2.33 | 1.66 |
| | 2 | 9.91 | 3.34 | 2.27 | 1.72 |
| | 3 | 9.93 | 3.65 | 2.07 | 1.92 |
| | 4 | 9.94 | 4.10 | 1.23 | 2.76 |
| | 5 | 9.94 | 4.16 | 0.02 | 3.97 |

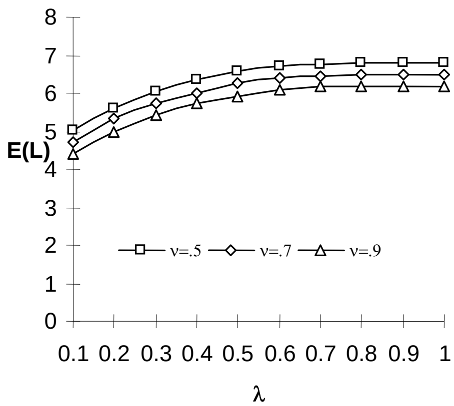
Table II: Performance measures by varying μ and θ




(a)

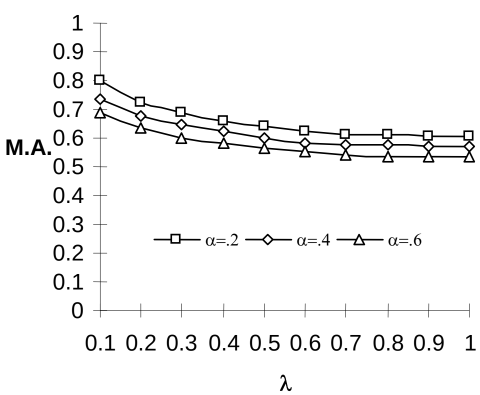



(b)

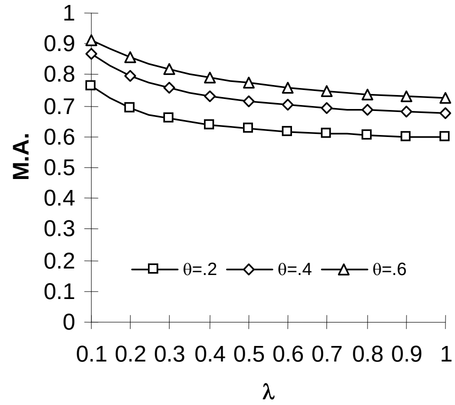



(c)

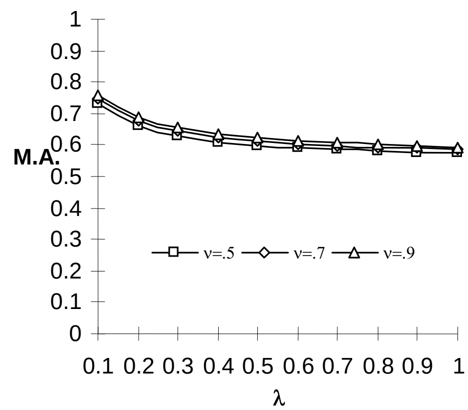
Fig. 1: E(N) vs λ for varying (a) α (b) θ (c)ν




(a)




(b)




(c)

Fig. 2: M.A. vs λ for varying (a) α (b) θ (c)ν

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