Production Scheduling for Products on Different Machines with Setup Costs and Times

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***Abstract*- Every company needs production scheduling in order to remain in the competing market and meet the customer needs. In this paper a production scheduling model for a variety of products in different packaging, on separate production lines in a juice factory is presented. There is sequence dependency between products. The production scheduling model is presented based on mixed integer linear programming. It includes setup times and costs. Furthermore a decision support system has been developed for production scheduling in order to help the manager in decision processing. The model is applied to a juice factory.**

Keywords: Production Scheduling , Mixed IntegerLinear Programming Model , Decision Support System

I-INTRODUCTION

Nowadays in order to supply customer needs and respond to market demand, in most factories various products are produced. Due to a competitive market, production scheduling is one of the most important tasks of managers. Several methods have been used to solve production scheduling problems, one of these methods is linear programming. This technique was proposed in 1969 by Wanger. One type of production planning problem is production planning with setup times/costs. Gupta & Kyparisis [3] researched production planning on a single machine with setup cost/time. Wortman [8] emphasized on the importance to consider sequence-dependent setup times for the effective management of manufacturing capacity.

In 1999, allahverdi et al.[1] presented a research on scheduling problems with separate setup times and cost. Reklaitis [7] has performed a comprehensive review of scheduling problems with considering sequence-dependent transitions between products. Lim and Karimi [4] have presented an article about a scheduling problem involving setup times but without consideration to setup cost. Allahverdi, H.M.Soroush [2] presented a research about the importance of reducing setup times and costs and they emphasized on the advantages of considering setup costs and times. Philip Doganis, Haralambos Sarimveis [5] have presented optimal scheduling model based on Mixed Integer Linear Programming (MILP). Their model involved setup times and setup cost. The scheduling has done on a single machine.

In 2007[6], they presented an optimal scheduling problem based on MILP for yogurt packaging lines that consisted of multiple parallel machines, that different products couldn't be produced synchronously by two machines.

The rest of the paper is structured as follow: In the next section, the problem is described. In section III the model formulation is presented. A decision support system is developed in section IV .In the last section a case study is applied to the model. The paper ends with conclusion.

II. MOTIVATING EXAMPLE and PROBLEM DEFINITION

In this paper a production scheduling was designed in the juice production lines of a company. The problem in this paper is to optimally schedule the juice production operations on different machines over a schedule horizon. Different kinds of fruit juice in different packaging are produced in the factory, but all fruit juices are not produced in all types of packaging.

Not only each type of packaging has separate production line, but also some types of packaging have more than one production line. Daily production time can not exceed 21 h, since all the machines should be CIP[1] after last production in a day which it takes about three hours. Also the machines must be cleaned (CIP) between two different products. The inventory levels at the beginning and at the end of the scheduling horizon should be considered. It should be noted that the production on a machine is done based on the sales priorities.

III. MODEL FORMULATION

The model is formulated as a Mixed Integer Linear Programming problem and in order to determine the quantity of products which should be produced on each machine on each day.

A.Notation

i day

j,l products

p packaging

x Number of packaging

m machines

N Scheduling horizon (days)

R Number of products

cstorage storage cost($)

rescost remained demand cost($)

demand(i,j,p) Demand for product j in packaging p in day i(1000 packs)

setup cost(j,l,p,m) Change over cost from product j to l in packaging p on machine m($)

setup time(j,l,p,m) Change over time from product j to l in packaging p on machine m(h)

s(j,p,m) machine speed for product j in packaging p on machine m(1000 packs/h)

openinv(j,p) opening inventory level of product j in packaging p(1000 packs)

Targetinv(j,p) target inventory level of product j in packaging p(1000 packs)

Prod(i,j,p,m) production quantity of product j in packaging p on machine m in day i(1000 packs)

Inv(i,j,p) inventory level of product j in packaging p at the end of day i(1000 packs)

time(i,m) total running time of machine m in day i(h)

bin(i,j,p,m) binary variable for production of product j in packaging p on machine m in day i(1/0)

binsetup(i,j,l,p,m) binary variable for change over of product j to l in packaging p on machine m in day i(1/0)

res (i,j,p) remained demand of product j in packaging p in day i(1000 packs)

B. Parameters

The required values for solving the model are input as parameters.

- Number of products
- Scheduling horizon
- Demand of each product in each packaging on each day
- Setup cost for each transition between products in each packaging on each machine

[1] Cleaning In Place

- Setup time for each transition between products in each packaging on each machine
- Inventory cost
- Machine speed for each product in each packaging
- Inventory level at the beginning of the scheduling horizon for each product in each packaging
- Inventory level at the end of the scheduling horizon for each product in each packaging

C. Decision Variables

After solving the model and getting the solution, these variables get value.Some of these variables are binary.

1) Continuous Variables

- The production quantity of each product in each packaging in each machine for each day.
- The inventory level of each product in each packaging in each machine for each day.
- Total utilization of each machine for each day
- The demand of each product in each packaging that wasn't produced in each day

2) Binary Variables

- Binary Variables for each combination of (day, product, packaging ,machine) that indicate whether the product in the particular packaging and particular machine will be produced in the particular day
- Binary Variables for each combination of (day, product, product, packaging, machine) that

indicate whether the change between products in the particular packaging and particular machine will happen in the particular day.

D. objective Function

The objective function is to minimize the production cost that involves setup cost, inventory cost and the cost for remained demand. The cost of raw materials and labor cost don't include.

$$\begin{aligned} \text{Min} \ & \sum_i \sum_j \sum_l \sum_p \sum_m \text{setup cost}(\mathrm{j,l,p,m}).\,\text{binsetup}(\mathrm{i,j,l,p,m}) \\ + \ & \sum_i \sum_j \sum_p \text{Inv}(\mathrm{i,j,p}).\,\text{cstorage} \\ + \ & \sum_i \sum_j \sum_p \text{rescost}\,.\,\text{res}(\mathrm{i,j,p}) \qquad (1) \end{aligned}$$

E. Constraints

1) Production Capacity

$$\text{Prod}(\mathrm{i,j,p,m}) \leq \text{M}.\,\text{bin}(\mathrm{i,j,p,m}) \quad \forall\, i,j,p,m \qquad (2)$$

$$\text{Prod}(\mathrm{i,j,p,m}) \geq 0 \quad \forall\, \mathrm{i,j,p,m}$$

Constraint (2) shows the relationship between binary variables and continuous variables. Considering the above constraint the product j in the packaging p on machine m in day i will be produced if only the binary variable bin(i,j,p,m) has the value of 1. Otherwise (bin(i,j,p,m) =0) this product won't be produced in packaging p on machine m in day i.

M is a maximum production quantity that is allowed for product j. (production capacity)

Another constraint could express the minimum quantity of production if it is necessary.

2) Relation between packaging and machine

$$\mathrm{Prod}(i,j,p,m) = 0 \qquad \forall\, p = 1\,, \geq 3 \qquad (3)$$

$$\mathrm{Prod}(i,j,p,m) = 0 \qquad \forall\, p = 2\,, m \leq 2\,, m > 3$$

$$\mathrm{Prod}(i,j,p,m) = 0 \qquad \forall\, p = 3\,, m \leq 3\,, m > 4$$

$$\mathrm{Prod}(i,j,p,m) = 0 \qquad \forall\, p = 4\,, m \leq 4\,, m > 5$$

Above constraints indicate that which kinds of packaging could be produced in each machine. For p=1 (200S packaging) the machines 1& 2 are used.

3) Inventory Levels

$$\mathrm{openinv}(j,p) + \sum_m \mathrm{Prod}(1,j,p,m) + \ \mathrm{res}(1,j,p) \ =$$

$$\mathrm{demand}(1,j,p) \ + \ \mathrm{Inv}(1,j,p) \qquad \forall\, j, p \qquad (4)$$

The inventory level of product j in packaging p at the beginning of the scheduling horizon plus the summation of production quantity of product j in packaging p in the first day in each machine, plus the remained demand of product j in packaging p in the first day, must equal the demand of product j in packaging p in the first day, plus the inventory level of product j in packaging p at the end of the first day

$$\mathrm{Inv}(i-1,j,p) + \textstyle\sum_m \mathrm{Prod}(i,j,p,m) + \ \mathrm{res}(i,j,p) \ = \\ \mathrm{demand}(i,j,p) + \ \mathrm{Inv}(i,j,p) + \mathrm{res}(i-1,j,p)$$

$$i > 1\,, \forall\, j, p \qquad (5)$$

$$\mathrm{Inv}(i,j,p) \geq 0 \qquad \forall\, i, j, p$$

For all next days, the inventory level at the end of the previous day plus the total production quantity of product j in packaging p in day i on all machines plus the remained demand of product j in packaging p in day i, must equal the demand of product j in packaging p in day i, plus the inventory level of product j in packaging p at the end of the day i, plus the remained demand of product j in packaging p at the end of the previous day. The inventory levels must be equal or grater than zero.

Considering to constraint (5) the daily demand of product that wasn't satisfied in previous day, adds to this constraint to be produced in this day if it is possible.

Constraints (4), (5) calculate the inventory level and the remained demand of products at the end of each day. The remained demands at the end of the scheduling horizon are the shortage of products for this period.

4) Target inventory

$$\mathrm{Inv}(N,j,p) = \mathrm{Targetinv}(j,p) \qquad \forall\, j, p \qquad (6)$$

Constraint (6) states that the inventory levels for each product in each packaging at the end of the scheduling horizon must be equaled to target inventory

5) Time Constraints

$$\mathrm{time}(i,m) = \textstyle\sum_j \sum_p \sum_m (\,\mathrm{Prod}(i,j,p,m)/s(j,p,m)) \ + \\ \textstyle\sum_i \sum_j \sum_l \sum_p \sum_m \mathrm{setup\ time}(j,l,p,m) \ .\, \mathrm{binsetup}(i,j,l,p,m)$$

$$\forall\, i, m$$

$$\mathrm{time}(i,m) \leq 21 \qquad \forall i, m$$

$$(7)$$

The above constraints indicate the total running time of each machine in each day. The total usage time includes the production times and setup times for transition between products. The total time for each machine must not exceed 21 hours a day, since the machines are cleaned (CIP) at the end of the last production in a day. This operation takes about three hours.

6) Binary Constraints

binsetup(i,j,l,p,m) ≤ 1+ (1− bin(i,j,p,m)) + (1− bin(i,l,p,m)) − (b*$\sum_{j+1}^{l-1}$ bin(i, k, p, m)) $\quad \forall i, j, p, m, \forall l > j \quad$ (8)

binsetup(i,j,l,p,m) ≥ bin(i,j,p,m) + bin(i,l,p,m) −1− $\sum_{j+1}^{l-1}$ bin(i, k, p, m))

$\forall\ i, j, p, m,\ \forall l > j \qquad (9)$

binsetup(i,j,l,p,m) ≤ bin(i, j, p, m)

$\forall\ i, j, p, m,\ \ \forall l > j \qquad (10)$

binsetup(i,j,l,p,m) ≤ bin(i, l, p, m)

$\forall\ i, j, p, m\ , \forall l > j \qquad (11)$

binsetup(i,j,l,p,m) = 0

$\forall\ i, j, p, m,\ \forall l \le j \qquad (12)$

constraints (8-12) state the relationship between bin(i,j,p,m), binsetup (i,j,l,p,m) .The value of binsetup(i,j,l,p,m) is equal to 1, if only the bin(i,j,p,m), bin(i,l,p,m) have the value of 1 and also the summation of bin(i, k, p, m) is equal to 0. In other words the transition between products j,l in packaging p on machine m in day i is being done, if the products j and l are produced in packaging p on machine m in day i and any other products with higher priority than product l is not produced on that machine. If one of these binary variables has the value of 0 the transition is not done and the value of binsetup(i,j,l,p,m) is becoming equal to 0.Constraint (12) indicates the sequence dependency between products.

$$\sum_{j=1}^{R}\sum_{p=1}^{X} \mathrm{bin}(i, j, p, m) \ - \sum_{j=1}^{R}\sum_{l=1}^{R}\sum_{p=1}^{X} \mathrm{binsetup}(i, j, l, p, m) \ \le 1$$

$$\forall\ i, m \qquad (13)$$

The above constraint indicates that in each day for each machine the numbers of products are produced minus the numbers of setups between them should be equal or less than 1.

IV. DECISION SUPPORT SYSTEM

In the previous section the production scheduling model was presented. In order to use the model easily and help the manager to evaluate solutions a decision support system has been developed. Figure 1 shows the structure of the DSS.

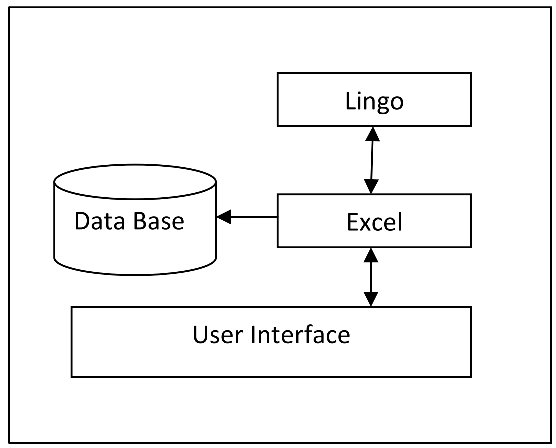

Figure1.The Structure of DSS

The MILP optimization problem that was formulated in section III was solved by Lingo 8. Microsoft Excel is used as an interface in order to link Lingo to user interface. Therefore the Lingo model embedded in Microsoft Excel . Input and output forms were designed in visual basic. User fills the input forms and the data send to excel and then to lingo. The lingo solves the model and the results send to excel to save in data base and will be shown to user in output forms. The data base stores and manages all model data. The data base was designed in Microsoft access. The user could retrieves data from data base. Some of the input and output forms were shown in figures 2 to 4.

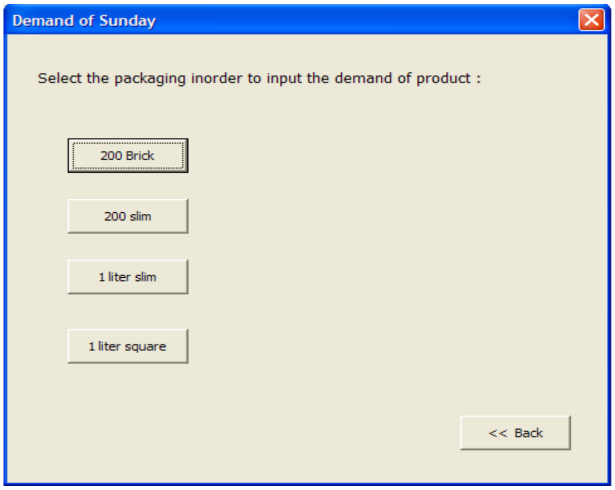

Figure 2.Selecting the packaging for demand

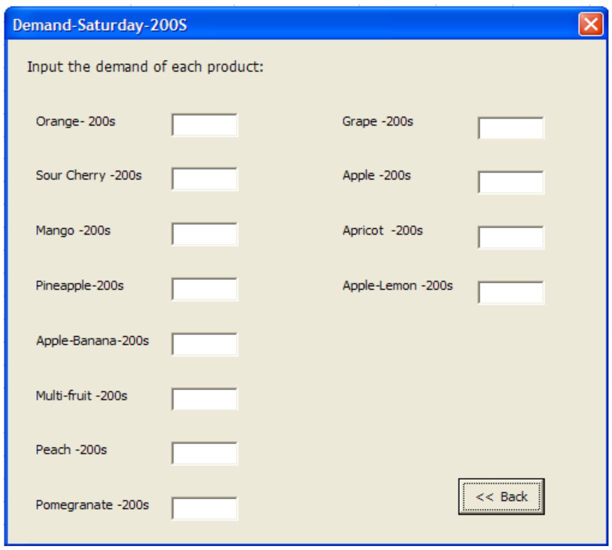

Figure 3.Demand Form for 200s

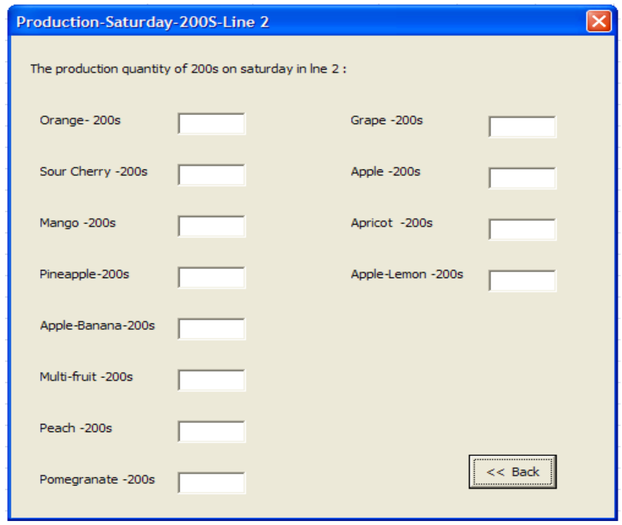

Figure 4.Production Form for 200s in line 2

V.CASE STUDY

The case study presented in this section concerns about a juice factory which 16 different juices are produced in it. Juices have 4 different packaging (200 Brick, 200 slim, 1 liter slim, 1 liter square). The products are produced in 5 machines. Machines could not produce juice in different packaging. The factory has two different lines to produce juice in packaging 200 slim. The products form a sequence according to their sales priority. In order to produce two products in a line, the one with the higher priority is produced before the other. The production lines must be washed before the transition between products, this causes setup times and setup costs. The total machines utilization time is less than or equal to 21, since the machines must be washed after the last production in a day, that takes 3 hours. The scheduling horizon is 6 days. The maximum production quantity is set to 420000.The target inventories are set to 10 for all juices. Some data of the model are presented in below tables. The problem consists of 60316 constraints and 17535 variables. The MILP formulated problem was solved in LINGO 8.0 software. Tables III-VII show production scheduling.

TABLE I

PRODUCTION SEQUENCE

Priority	Product
1	Orange (p1)
2	Sour Cherry (p2)
3	Mango (p3)
4	Pineapple (p4)
5	Apple-Banana (p5)
6	Multi-fruit (p6)
7	Peach (p7)
8	Pomegranate (p8)
9	Grape (p9)
10	Apple (p10)
11	Suger Free Orange (p11)
12	OrangeCarrot (p12)
13	Apricot (p13)
14	Apple-Lemon (p14)
15	Grapefruit (p15)
16	Tomato (p16)

TABLE II

PACKAGING AND MACHINE SPEED (1000 PACKS/H)

Packaging	Machine ID	Machine Speed
1 (200 slim)	1,2	20
2 (1 liter)	3	6
3 (200 B)	4	6
4 (1 liter square)	5	6

TABLE III

PRODUCTION SCHEDULE FOR 200S PACKAGING IN LINE 1

(1000 PACKS)

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
P1	0	0	0	0	0	0
P2	278	100	0	0	0	0
P3	0	0	258	0	120	0
P4	0	0	0	0	0	0
P5	0	0	0	0	0	0
P6	0	0	0	0	0	0
P7	0	0	0	58	0	150
P8	0	0	0	0	0	0
P9	0	0	0	0	0	0
P10	0	0	0	0	0	0
P11	0	0	0	0	0	0
P12	0	0	0	0	0	0
P13	0	0	0	0	0	0
P14	0	0	0	0	0	0
P15	0	0	0	0	0	0
P16	0	0	0	0	0	0

TABLE IV

PRODUCTION SCHEDULE FOR 200S PACKAGING IN LINE 2

(1000 PACKS)

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
P1	160	0	0	0	272	0
P2	0	0	0	0	0	0
P3	0	0	0	0	0	130
P4	0	0	0	0	79	210
P5	0	0	0	0	0	0
P6	0	0	0	162	0	0
P7	0	0	0	0	0	0
P8	0	189	0	0	0	0
P9	0	0	0	0	0	0
P10	0	0	0	0	0	0
P11	0	0	0	0	0	0
P12	0	0	0	0	0	0
P13	0	0	0	0	0	0
P14	0	0	0	0	0	0
P15	0	0	0	0	0	0
P16	0	0	0	0	0	0

TABLE V

PRODUCTION SCHEDULE FOR 1 LITER PACKAGING (1000 PACKS)

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
P1	65	126	0	95	0	40
P2	61	0	55	0	30	0
P3	0	0	0	0	0	29
P4	0	0	49	0	0	0
P5	0	0	0	0	0	0
P6	0	0	22	0	20	0
P7	0	0	0	31	2	0
P8	0	0	0	0	74	20
P9	0	0	0	0	0	0

TABLE VI

PRODUCTION SCHEDULE FOR 200B PACKAGING (1000 PACKS)

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
P1	30	100	0	50	0	10
P2	0	0	0	0	0	0
P3	0	0	0	0	0	0
P4	0	0	0	0	0	0
P5	0	0	0	0	0	0
P6	0	0	0	0	0	0
P7	0	0	0	0	0	0
P8	0	0	0	0	0	0
P9	0	0	0	0	0	0
P10	40	0	54	0	0	10
P11	0	0	0	0	0	0
P12	0	0	0	0	0	0
P13	0	0	0	0	0	0
P14	0	0	0	0	0	0
P15	0	0	0	0	0	0
P16	0	0	0	0	0	0

TABLE VII

PRODUCTION SCHEDULE FOR 1 LITER SQUARE PACKAGING (1000 PACKS)

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
P1	0	0	0	0	0	0
P2	0	0	0	0	0	0
P3	0	0	0	0	0	0
P4	0	0	0	0	0	0
P5	0	0	0	0	0	0
P6	0	0	0	0	0	0
P7	0	0	0	0	0	0
P8	0	0	107	0	13	10
P9	24	116	0	0	0	10
P10	0	0	4	126	0	0
P11	22	0	0	0	108	0
P12	40	0	0	0	0	50
P13	0	0	0	0	0	0
P14	0	0	0	0	0	0
P15	40	10	0	0	5	35
P16	0	0	15	0	0	21

TABLE VIII

REMAINED DEMAND IN 1 LITER PACKAGING (1000 PACKS)

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
P1	126	0	95	0	40	0
P2	55	55	0	30	0	0
P3	0	0	0	0	0	0
P4	49	49	0	0	0	0
P5	0	0	0	0	0	0
P6	0	0	20	20	0	0
P7	0	0	0	2	0	0
P8	0	0	0	0	20	0
P9	0	0	0	0	0	0

TABLE IX
MACHINE UTILITY (H)

	Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
Machine1 (200s)	13.9	5	12.9	2.9	6	7.5
Machine2 (200s)	8	9.45	0	8.1	20.71	20.16
Machine3 (1 liter)	21	21	21	21	21	14.83
Machine4 (200B)	12.5	15	9	21	15	16.66
Machine5 (1liter square)	9.16	8.33	11.66	10.33	13.33	4.83

VI. CONCLUSION

The production scheduling for juice factory was presented in this paper. The factory had 5 separate production lines and the products were produced in 4 different packaging. Setup costs, setup times and the sequence dependency were considered. In order to problem features the model was formulated as mixed integer linear programming. The model was solved by LINGO 8.0 software for a case study. It indicated the products quantity which should be produced in each day on each machine. The results were presented in the tables. A decision support system was developed for effective use of the production scheduling model. The DSS helps the manager in decision making process. The model helps the manger to respond better to customers needs with minimum production cost.

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