Transient analysis of lossy multiconductor transmission lines model based by the characteristics method

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Abstract— **In this paper, we will present a low losses multiconductor transmission lines model valid both in the time and the frequency domains. This model is an extension of the characteristics method (Branin method) valid for lossless lines. Our model permits to describe the line as a quadripole whose advantage is to avoid the presupposed applied charges conditions in its extremes. This permits it to be easily introduced in the circuit simulators such as Spice, Esacap and Saber, and it is valid both in time and frequency domains. Diverse examples of applications products by the literature are presented to validate these method and to show their interests.**

Keywords— *Characteristic method, Time domain, Transmission line, Multiconductor line, Loss, Modeling.*

I. INTRODUCTION

Due to the important increase in equipments and industrial applications, the problems related to the frequency mounting and the effects of interconnections are various (distortion, attenuation, crosstalk ...). Moreover the losses can play a very important role in the degradation and attenuation of signals traveling through the line. Taking into consideration the framework of the inter-equipment electromagnetic compatibility (EMC), skin and proximity effects complicates further those problems in high frequency. This necessitates, therefore, a physical model adapted to the transmission lines.

The cascade model [1][2] allows to model the transmission line in the form of a RLCG circuit. It can also be applied in case of MTL line, which becomes complicated in use beyond two conductors. And it yields to the undesirable oscillations in time domain (phenomenon of Gibbs). And it needs an important calculating time, which makes it inefficient.

The finite differences method in time domain (FDTD) [2][3][4] is an analytic method consisting in dividing the time and the space domains where the solution is seek into a regular network (a mesh). This requires a transformation in order to pass to the frequency domain using the "Time Domain to Frequency Domain" method (TDFD) [4].

The major difficulty is, then, to have an MTL line model distributed valid both in time and frequency domains, with and without losses.

This paper presents a time domain low loss multiconductor transmission lines model. Our model is an extension of the characteristics method (Branin method) [2][5][6] which is valid for lossless lines. Our model is permitting a modelling of the line in the form of a quadripole whose advantage is to avoid the presupposed applied charges conditions in its extreme. This permits to the model to be easily introduced in the circuit simulators as Spice, Esacap [7] and Saber, and it is valid both in the time and the frequency domains [8].

Several applications examples, in the literature, are presented to validate the model and to show its interest and the results are compared and confirmed with results of the FDTD method.

II. MTL LINES MODELLING

To model the transmission lines, it is necessary to solve the Telegraphists equations. The time domain equations representation is:

$$\begin{cases} \dfrac{\delta V(z,t)}{\delta z} + RI(z,t) + L\dfrac{\delta I(z,t)}{\delta t} = 0 \\[2mm] \dfrac{\delta I(z,t)}{\delta z} + GV(z,t) + C\dfrac{\delta V(z,t)}{\delta t} = 0 \end{cases} \quad (1)$$

[V] and [I] represent, respectively, the tensions and currents matrix.

[R], [L], [C] and [G] represent, respectively, the resistances, inductances, conductances and capacities matrix. And they include implicitly all the information concerning the transverse section, which permits to characterize a muticonductor structure.

The coefficients of these different matrix are obtained

either by engineering practical techniques [9], or by numerical methods [10][11][12]. The modelling of a lossless and losses MTL lines in time domain by the characteristics method will be presented respectively in the follow paragraphs.

2.1. Lossless line

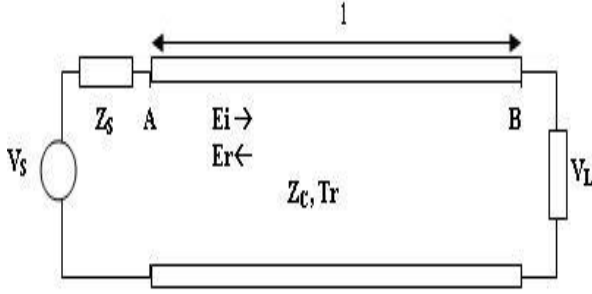The model of a two-conductors line is figured bellow:

Figure 1. Two conductor transmission line.

Z_c is the impedance characteristic, and T_r is the delay of the line.

Branin [6] is the first to propose a numerical model transmission line which permitted to give a simple equivalent circuit of the ideal line. This schema includes two dipoles. In the input dipole (z=0, t) the tension is determined from the reflected tension in extremes of the line (z=l) at the precedent time t- T_r. The same interpretation is applied to the output dipole.

The equivalent schema of the lossless line (R=G=0) is figured bellow [2] [6]:
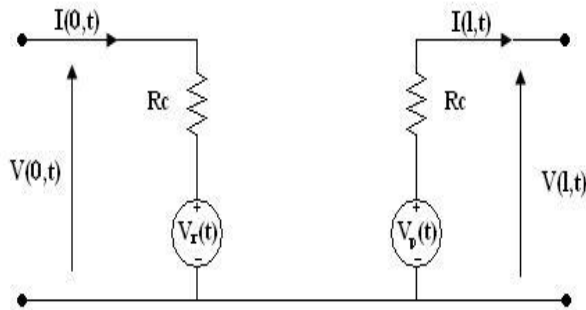

Figure 2. Quadripole representation of the ideal line.

Indeed, it is easy to show that electrical parameters in extremities are linked by means of the following relations:

$$V(0,t) = R_C I(0,t) + V(l,t-T_r) + R_C I(l,t-T_r) \quad (2.a)$$

$$V(l,t) = R_C I(l,t) + V(0,t-T_r) + R_C I(0,t-T_r) \quad (2.b)$$

The equations (2.a) and (2.b) depend only on the secondary parameters (the characteristic impedance and the delay of the line). Hence, the equivalent schema of the figure 2 is established by putting:

$$V_r(l,t) = V(l,t-T_r) + Z_C I(l,t-T_r) \quad (3.a)$$

$$V_i(0,t) = V(0,t-T_r) + Z_C I(0,t-T_r) \quad (3.b)$$

$V_r(t)$ and $V_i(t)$ are, then, calculated using observable parameters at a known time $t-T_r$.

The characteristic method can perhaps be expanded to the multiconductor lines case [2][5][10]. This requires, on the other hand, uncoupled propagation modes on each line. This separation of modes is implemented using the modal method [2][5][10].
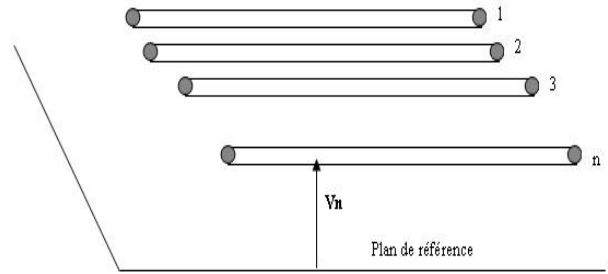

Figure 3. Multiconductor transmission line above a ground plan.

Note that electromagnetic couplings between the lines, the impedances matrix are not diagonal. The benefit of the modal method is then to uncouple the equations in order to be able to diagonalise the matrix.

As we deal with the lossless lines case (G=R =0), the MTL lines equations in the time domain (1) become:

$$\begin{cases} \dfrac{\delta^2 V(z,t)}{\delta z^2} = -LC \dfrac{\delta^2 I(z,t)}{\delta t^2} \\ \dfrac{\delta^2 I(z,t)}{\delta z^2} = -CL \dfrac{\delta^2 V(z,t)}{\delta t^2} \end{cases} \quad (4)$$

The modal method consists of introducing fictitious parameters V_m and I_m, by means of the following linear transformations:

$$\begin{cases} V(z,t) = T_V . V_m(z,t) \\ I(z,t) = T_I . I_m(z,t) \end{cases} \quad (5)$$

The modal matrix T_V and T_I, are to be determined to ensure the uncoupling of propagation modes of the MTL line [2][5][10]. If we apply the transformation to the system of equations (4), we then obtain:

$$\begin{cases} \dfrac{\delta^2 V_m(z,t)}{\delta z^2} = -T_V^{-1} LCT_I \dfrac{\delta^2 I_m(z,t)}{\delta t^2} \\ \dfrac{\delta^2 I_m(z,t)}{\delta z^2} = -T_I^{-1} CLT_V \dfrac{\delta^2 V_m(z,t)}{\delta t^2} \end{cases} \quad (6)$$

We can choose both matrix T_V and T_I such as the system (6) could be uncoupled. In our case of a lossless line placed in a homogeneous and isotropic medium, we can always find the matrix of transformation which diagonalise

simultaneously the matrix L and C.

The two transformation matrix T_V and T_I are linked by the following relation [2]:

$$T_I^t = T_V^{-1} \tag{7}$$

The equations system of a lossless line written in the modal base:

$$\begin{cases} \dfrac{\delta V_m(z,t)}{\delta z} = -L_m \dfrac{\delta I_m(z,t)}{\delta t^2} \\ \dfrac{\delta I_m(z,t)}{\delta z} = -C_m \dfrac{\delta V_m(z,t)}{\delta t} \end{cases} \tag{8}$$

With L_m and C_m are diagonal matrix of dimension NxN :

$$\begin{cases} L_m = T_V^{-1}.L.T_I \\ C_m = T_I^{-1}.C.T_V \end{cases} \tag{9}$$

The system of equations (8) also represents an uncoupled MTL line, which has as characteristic impedance: R_{Cm} and as delay T_{rm}. Where:

$$R_{Cm_{ii}} = \sqrt{\dfrac{L_{m_{ii}}}{C_{m_{ii}}}}, \qquad T_{rm_i} = \dfrac{l}{\sqrt{L_{m_{ii}}C_{m_{ii}}}} \tag{10}$$
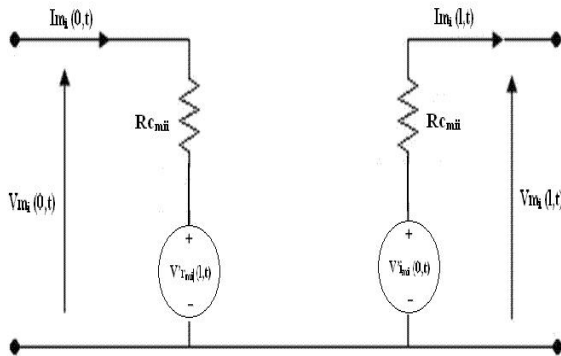

Figure 4. Quadripole representation of the lossless MTL line.

2.2. Lossy line

The losses can play a very important role in the degradation and attenuation of signals transported by line. The losses are caused by a non-null conductivity and the loss of the polarisation medium or imperfect conductors. The losses presented by imperfect conductors are habitually more significant than the losses due to the transmission line. For this reason, we often suppose that the surrounding medium is lossless (G = 0) in the MTL line equations. The resistance due to imperfect conductors is represented in the resistance matrix by a length unity [R].

The equations system (1) becomes:

$$\begin{cases} \dfrac{\delta V(z,t)}{\delta z} + RI(z,t) + L\dfrac{\delta I(z,t)}{\delta t} = 0 \\ \dfrac{\delta I(z,t)}{\delta z} + C\dfrac{\delta V(z,t)}{\delta t} = 0 \end{cases} \tag{11}$$

The equations (11) expressed in the frequency domain by Laplace operator p gives us:

$$\begin{cases} -\dfrac{\delta V}{\delta z} = [R][I] + p[L][I] = [Z][I] \\ -\dfrac{\delta I}{\delta z} = p[C][V] = [Y][V] \end{cases} \tag{12}$$

In general, a line can be modelled by its serial impedance $Z = R + jwL$, and parallel admittance $Y = jwC$, (The dielectrics losses are supposed to be negligible G=0).

For the working frequencies larger than the characteristic frequency of the line ($R/2\pi L$) (low losses hypothesis) [2][5], and using a first development order, we obtain:

$$Z_C = \sqrt{\dfrac{R + jwL}{iwC}} \approx R_C + \dfrac{RR_C}{2jwL} \tag{13}$$

With $R_C = \sqrt{\dfrac{L}{C}}$

The characteristic impedance in that case is equivalent to a characteristic impedance RC serially mounted with a capacity $C_{pf} = \dfrac{2L}{RR_c}$, when the frequency increases ZC becomes equal to RC.

With the same approximation, the constant of propagation becomes:

$$\gamma = \alpha + j\beta = \dfrac{R}{2R_C} + jw\sqrt{LC}$$

To take into account the attenuation of the wave referred to by the term $e^{-\alpha l}$, it is enough to modify Branin's generators of the quadripole representation of the ideal line (cf Fig 2). They become [2][5] :

$$\begin{cases} V_r^{'}(t) = V_r(t).e^{-\alpha l} \\ V_i^{'}(t) = V_i(t).e^{-\alpha l} \end{cases} \tag{14}$$
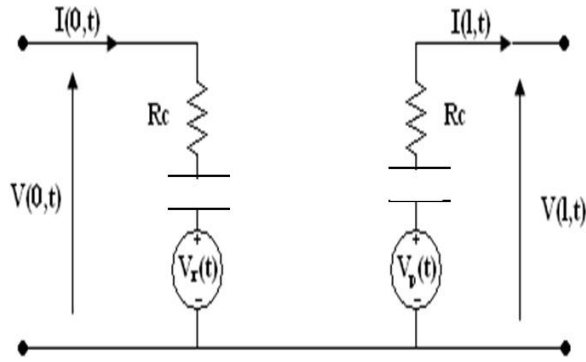
Figure 5. Quadripole representation of the low lossy line.

For a losses multiconductor line, we do the same as in the case of a lossless MTL line. Therefore, in the case of coupled low loss lines, the impedance characteristic and the delay are given by:

$$Z_{Cm} \approx R_{Cm} + \frac{RR_{Cm}}{2jwLm} \qquad , R_{Cm} = \sqrt{\frac{L_m}{C_m}} \qquad (15)$$

This supposes that the losses' resistance is referred by the R matrix whose non-diagonal terms were null. Also, it is enough to bring the following modifications to the generators of the ideal lines, given by relations (3.a) (3.b), which become:

$$V_{rm}(l,t) = T_V^{'} \left\{ V_m(l,t-T_{rm}) + Z_{Cm}I_m(l,t-T_r) \right\} \qquad (16.b)$$

$$V_{im}(0,t)' = T_V^{'} \left\{ V_m(0,t-T_{rm}) + Z_{Cm}I_m(0,t-T_r) \right\} \qquad (16.b)$$

With $T_V^{'} = T_V e^{-\alpha l}$ is the attenuated modal matrix.

The Electric parameters at the extremes of the MTL line are linked by the following relations:

$$V_m(l,t) = Z_{Cm}I_m(l,t) + V_{im}^{'}(0,t) \qquad (17.b)$$

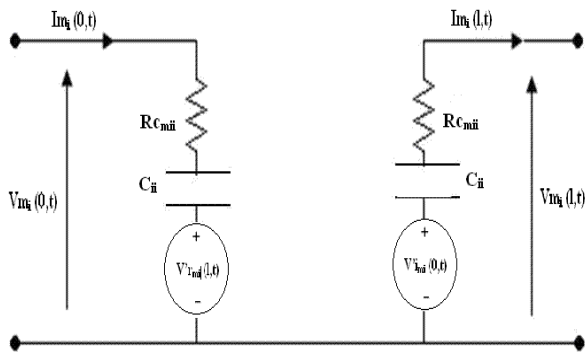$$V_m(0,t) = Z_{Cm}I_m(0,t) + V_{rm}^{'}(l,t) \qquad (17.b)$$

Figure 6. Quadripole representation of the low lossy MTL line.

III. LOW LOSS TRANSMISSION LINE MODEL

3.1. Equivalent circuit

If this were possible, we could simulate the modes with uncoupled, two-conductor lossy modal lines using the previous results for two-conductor lines from Figure 6. The transformations from mode to actual line currents and voltages via equations 5 and 9 could be implemented in the same manner as for lossless lines using controlled sources as illustrated in Figure 7
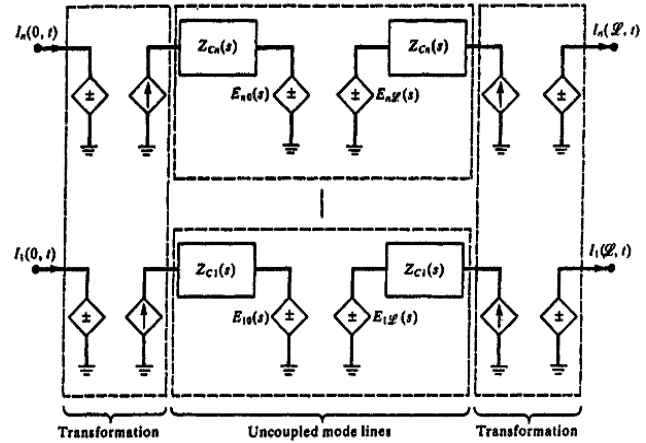

Figure 7. Representation lossy two-conductor mode lines using characteristics method in terms of uncoupled modes.

3.2. ESACAP program

To verify the validity of our model, we simulate two types of multiconductor transmission lines, the "Ribbon Cable" and "2Wire Xtalck in ESACAP circuit simulator [7]. The following program is the principal program under ESACAP Line Ribbon Type Cable.

```
-------------- Programme Start -----------------------------------
$$DES
# Models
$MOD:
Transmission line model Description
END;
-------------- Principal circuit equivalent description --------
$NET:
# Netlist
Principal circuit equivalent description
Extreme line tension as a fonction of modal tension
Modale courent  as a fontion of extreme line current
Call the Line 1 model
Call the Line 2 model
END;
-------------- Time domain Analysis ---------------------------
$$TRA
$PARAM:
Analysis parameters
Time sweep with increments for output tables.
```

Max truncation error in numerical integration
Max   integration
time steps
END;
-------------- Output file --
$DUMP:
Output file
Plot output tension
END;
END; program end
$$STOP program execution stop
-------------- Programme end ----------------------------------

IV. SIMULATION & RESULTS

In this paragraph, we make a comparison between experimental results in the literature for example 1[2] and example 2 [9] and those obtained using a FDTD method and our simulation results using the ESACAP circuit simulator in time domain. This model is valid in time and also in the frequency domain [8].

4.1. Example 1: Ribbon cable

We consider a transmission line of three conductors with 2m length and ligneous parameters, fed by a tensional generator. The in and out charges are equal to 50 Ohms, as indicated on the figure 6.

$$L=\begin{bmatrix} 750.37 & 508.69 \\ 508.69 & 1017.39 \end{bmatrix}(nH/m) \quad R=\begin{bmatrix} 0.19 & 0 \\ 0 & 0.19 \end{bmatrix}(Ohm/m)$$

$$C=\begin{bmatrix} 37.21 & -18.60 \\ -18.60 & 24.85 \end{bmatrix}(pF/m)$$
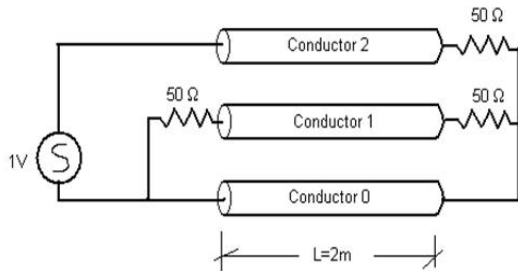

Figure 8a. Electrical representation of the ribbon Cables transmission line to 3 conductors
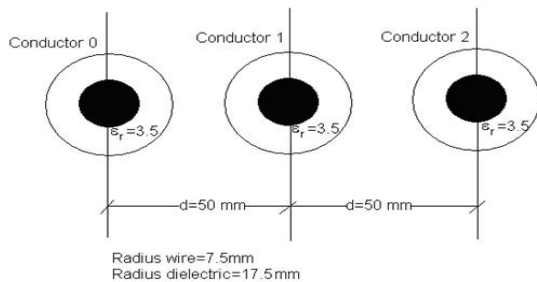

Figure 8b. Dimensions of the ribbon Cables transmission line to 3 conductors.

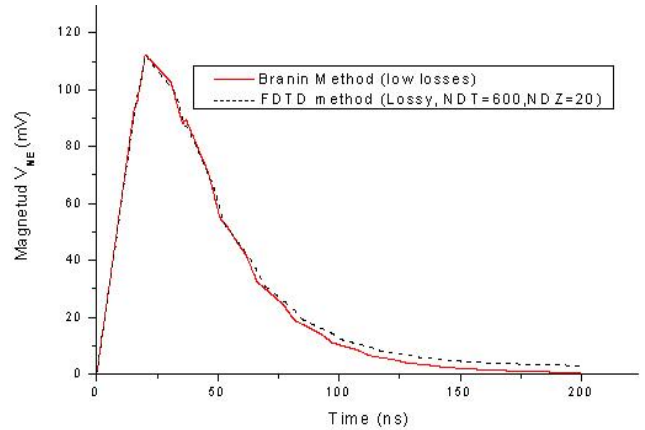In the time domain, the line is fed by an equal amplitude scale 1V, and a rise time tr=20ns.

Figure 9. Result of simulation VNE tension.

The Near End cross-talk predictions (Branin model and FDTD method) are compared to the experimental results [2] in figure 9 and figure 10. The signal rise time is T-rise = 20 ns, is slow compared to return-propagation-time ΔT~2.(3.5ns x 2m) = 14ns. Therefore, the predicted Near-end X-talk on victim line (figure 9) correctly reproduces the actual source signal shape. The multiple reflections on measured waveform, at end of queue, have a single step duration of ΔT~14ns. The same oscillations can be identified on predicted waveform, less evident because the parasitic inductance of termination leads has not been included in the model.
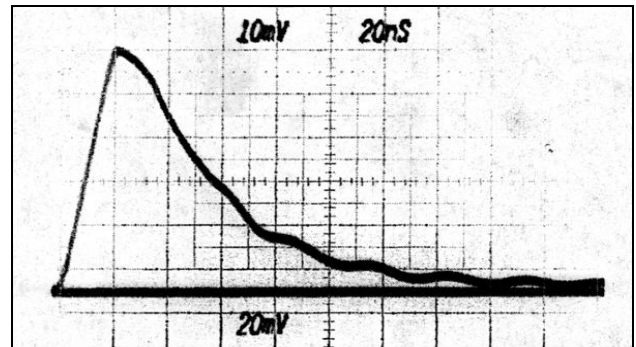

Figure 10. Experimental result for the magnitude of the near-End [2].

4.2. Example 2 : '2Wire Xtalk'

We consider a '2wire-xtalk' transmission line with length 4.674m and of dielectric constant ε =1, fed by a scale with a rise time tr=12ns, the flat top w=20ns, fall time td=12ns, and period T=1us.
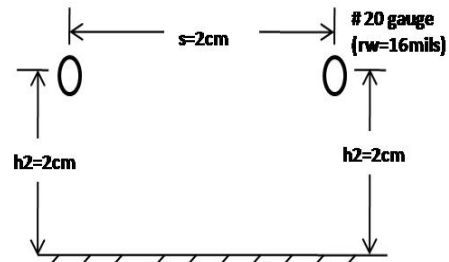

Figure 11a. Geometric configuration of the transmission line '2Wire Xtalk'.
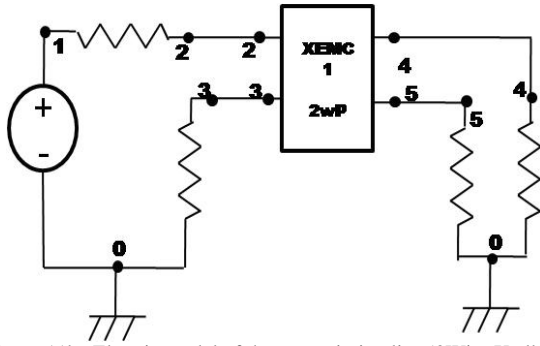
Figure 11b. Electric model of the transmission line '2Wire Xtalk'.

Figure 11.a presents the geometric configuration and electric model of the line '2Wire Xtalk' whose parameters are:

$$L = \begin{bmatrix} 918 & 161 \\ 161 & 918 \end{bmatrix} (nH/m) \quad R = \begin{bmatrix} 0.78 & 0 \\ 0 & 0.78 \end{bmatrix} (Ohm/m)$$

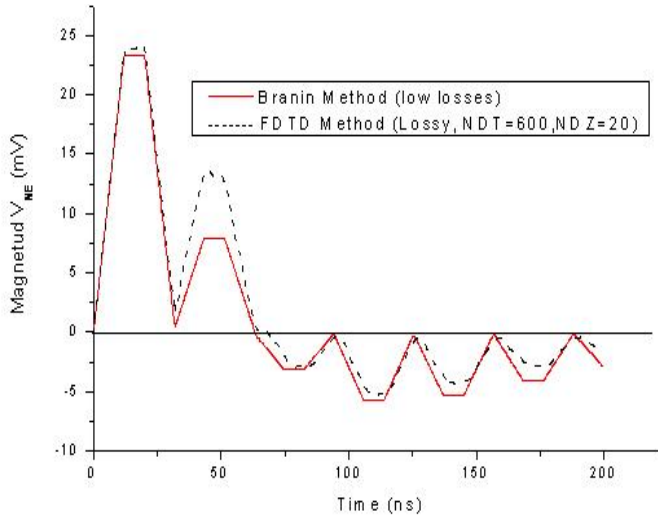$$C = \begin{bmatrix} 12.50 & -2.19 \\ -2.19 & 12.50 \end{bmatrix} (pF/m)$$

Figure 12. The VNE tension obtained for a '2Wire Xtalk' transmission line.

As in Figure 12, the full pulse is now reflected from line terminations, because the total pulse width including rise and fall time is 32ns, and the return propagation time of the bundle is $\Delta T = 2 (3.3ns/m \times 4.64 \ m) = 30.6ns$. That means, on the source side, that almost at the end of input signal the backward end reflection is ready to alter the signal itself. As can be checked [10] the Z-common-mode = 162Ω and Z-differential-mode = 45Ω, that means that the two-wire bundle is totally unmatched for both the propagation modes. Therefore, the first pulse of Near-End X-talk reproduces the input signal shape up to T= 32ns, (as coherent with a loosely coupling configuration) and almost at the end, at T=30.6ns, the backward positive reflection is added to X-talk signal itself.

That creates a sharp corner and to correctly predict its value, a very high time discretisation analysis is required. Then the multiple reflections follow.

The Branin and FDTD method prediction is coherent with experimental result (Figure 13), but some discrepancies can be observed with experimental results. That is again related to parasitic effect of termination's leads, not simulated in the models.
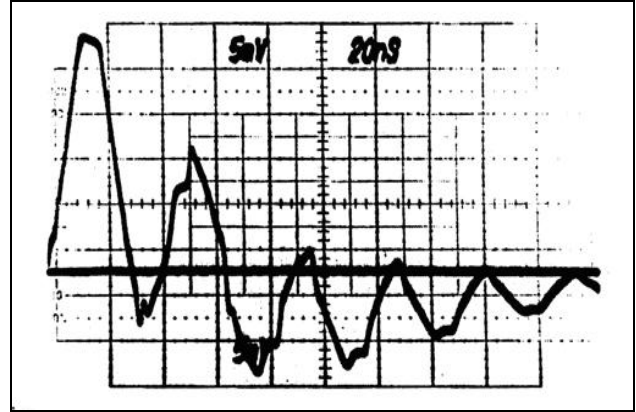

Figure 13. Experimental result for the magnitude of the near-End [10].

V. CONCLUSION

In this paper we proposed a low loss multiconductor transmission line model in time domain. Our model is based on the characteristics method (Branin method). This model is implemented using the ESACAP circuit simulator and it's valid in both the time and the frequency domains. It can be introduced easily in circuit simulators as Spice, Esacap and Saber, but it is only valid for nonlinear loads.

The simulation results are compared to the experimental result and FDTD result, these simulations are presented in the end of this paper to validate the model and show their interest. Our model represents the phenomena related to electromagnetic compatibility in leads mode (crosstalk, ...).

The future work will be devoted to modelling the coupling of an EM wave with a multiconductor transmission line (radiated mode). And to develop the model of both real losses (skin and proximity effect), using a model based on the Pade approximation.

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