Modeling the wait time of a hierarchical system: Case study of an Internal Medicine unit

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***Abstract* –All, such as industrial production (Manufacturing companies), health units are hierarchical at the level of organization. This arrangement permits them to response efficiently to an event in occurrence, such as a patient whom appropriate care has to be dispensed. The quick reaction of the medical unit when faced with a problem is not only an indicator of its reactivity, but also an indicator of its performance.**

In this article, we propose to model the reaction of a medical unit in relationship to parameters, notably the reference periods of the different levels at which decisions are made. We define and express subsequently, the wait times which is a source of delay in the treatment process of a patient from when he arrives. Then, we propose an algorithm which minimizes the wait time.

***Index Terms*—Modeling, wait time, lateness, reactivity, minimization, event.**

I. INTRODUCTION

In Sub-Saharan Africa, one of the most important causes for deaths is the response time which is very important when considering a patient who just gets to a medical unit. This time is an indicator of the performance of the unit. It is therefore important to quantify it precisely, and this takes us to the model. We are considering the case of systems' control and performance.

So much work has been carried out on control and performance of production systems. Just to site a few Chan et al. [1], Folan et al. [2], Hernandez-Silva et al. [3], Gruat La Forme [4]. Different in their methodologies, they all had same objectives, ameliorating system performance. Regnier [5] in his works just like the others elsewhere [6,7], showed that only a hierarchical and multi-level structure (see Fig. 1) is able to respond efficiently to an unforeseen event. She then combines the advantages related to the phenomenon of aggregation [8] and the quality of refinement.

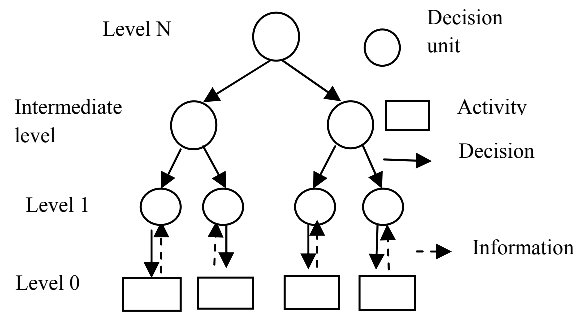

Fig. 1. Example of a hierarchical structure

Therefore a patient who gets to the medical unit needs treatment. This process involves successive passage through all the levels of making decisions. In this light, it's important to determine the rapidity with which the unit defines its reaction as a measure of reaction and wait time for each level in combination with the process to be considered.
In this first part we present the method of our model by making explicit the reactivity and the hypotheses on which it will be based. In the second part, we express in a periodic manner the delay in reaction and the wait time of a hierarchical production system following the event of a disturber. At the end we propose an algorithm which would permit us to reduce the wait times, synonymous. Finally we present a numerical application in the case of an Internal Medicine unit.

II. METHODOLOGY

A. Hypotheses of the model

This model which is based on the GRAI model [9,10] is established on a certain number of hypotheses. The arrival of a patient is considered as an even with pathology to be treated:

- The propagation of the event: it appears at a level which does not treat it, it has repercussion at a higher level. This repercussion moves from one level to the next until it gets to the level at which it is treated.

- The functioning is periodic: the repercussion from one level to the next is made of two phases, an upstream phase which is the ascending phase (from lower level to higher level) and the downstream phase which corresponds to the repercussion of the reaction from the level at which it is elaborated to an inferior level which has to apply it. In the two phases, the repercussion from one level to the next is done at the end of the period. The manner is said to be periodic.
- The transmission from one level to the other of the event or reaction is not instantaneous. There exists a non-zero delay in transmission upstream and downstream between two consecutive levels.
- On each level, there exists a shift (which could be zero) between the reference date, time origins (t_o) and the start date of the reference period for the level k considered $x^k(0)$. These shifts are not necessarily equal for all the levels.
- We consider the most unfavorable case of an event which appears at the level 0, and which is not treated and has repercussions right up to the level N where it is finally treated. This particular case presents the longest reaction delay.

B. Treatment process of an event

The objective presented in the Fig. 2 is to express the reaction delay of the system as a function of the occurrence date of an unwanted even and the system parameters, notably the start dates of the reference period of the different levels involves in the treatment.

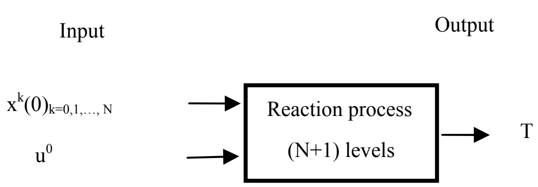

Fig. 2. Objective of the model

$x^k(0)$:Initialization date of the reference period.
u^0: Occurrence date of the event.
T : Reaction time of the medical unit.

$$T=f\left[u^0,\left\{x^k(0)\right\}_{k=0,1,..N}\right]$$

We designate a sub process to every passage of an event in a level. Therefore, every level k, except the highest level (k=N), has two sub processes sp_k and sp_{2N-k} which treats the upstream and downstream events respectively (see Fig. 3). The level N which treats the event has only a single sub process: sp_N

The process therefore has in total 2N+1 sub processes (0,1,……,2N). In every sub process i, sp_i, except the last (i=2N), the event, in the upstream phase passes through four successive states, the reaction in the downstream phase equally passes through four successive states presented in the Table 1. The sub process 2N, sp_{2N}, the last has just the three first stages.

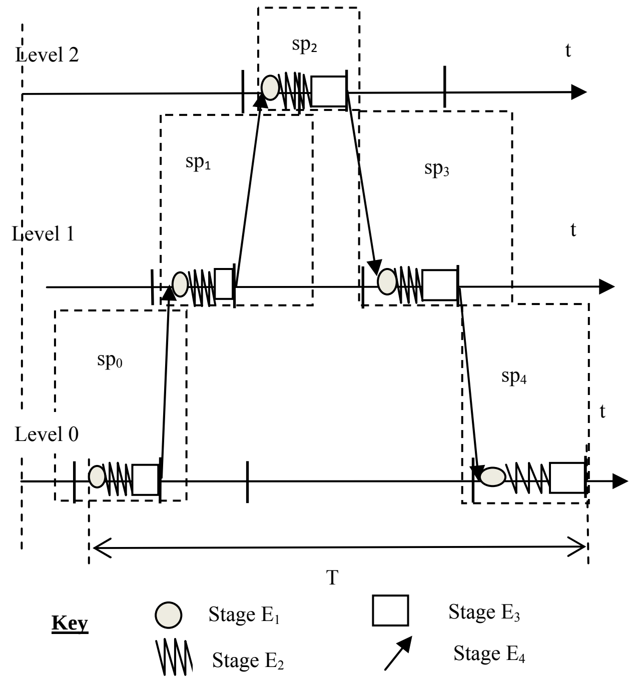

Fig. 3. Example of a circuit on two levels

TABLE 1
DIFFERENTS STATES OF TREATMENT

State	Designation		Duration
	Upstream phase	Downstream phase	
E_1	Evaluation of the gravity	Verification for coherence	$T_{i,1}$
E_2	Preliminary treatment	Elaboration of the decision framework	$T_{i,2}$
E_3	Waiting for the end of the period	Waiting for the end of the period	$T_{i,3}$
E_4	Transfer to a higher level	Transfer to a lower level	$T_{i,4}$

Without any much detail about what is happening in the different states, we simply say that it's the state E_1 in the upstream state which determines the mode of periodic or factual treatment as a function of the appreciation of the gravity.

C. *Evaluation of the reaction time*

We define below the parameters of the model:

t_0 : reference date
k : level considered
i : index of the sub process considered
l : index of the state of the event
j : number of the period order
N : level at which the event is treated
sp_i : sub process i of the system
E_l : state l of the treatment of the event
P_k : duration of a period of the level k
j_i : synchronization period for which the event is treated in sp_i;
$x^k(0)$: start date of the reference period for the level k
x^i_0 : arrival date of the event in the sub process sp_i
$T_{i,l}$: duration of the state l of sp_i
S : execution date of the reaction;
T : reaction delay of the system to the event
u^i : entrance date into sp_i, of the event
$x^k(j)$: finish date of the period j of the level k
x^i_l : finish date of the state E_l for the event sp_i ;
s^i : exit date of the event (end of the last stage) of sp_i ;

For a sub process, the treatment sequence is the same. Fig. 4 presents the dates for which the perturbation in the sub process changes state.

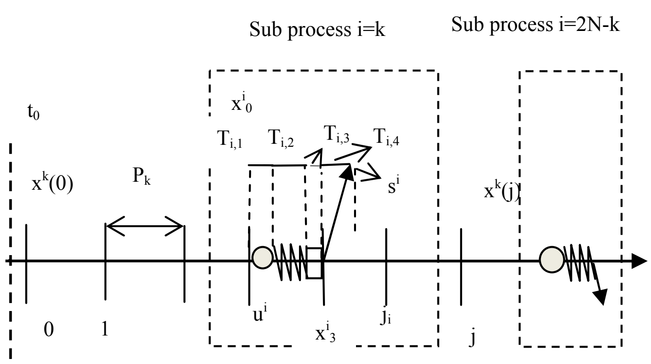

Fig. 4. Duration and change of state in a sub process sp_i

There exists two distinct dynamics in the treatment process. One part is the dynamic of the event (its change of states) which is made at irregular instances and as a function of the duration of the different states which are intrinsic characteristics of the system in relation to a given event. The other is the dynamics of decision making which is regular, because it is periodic at each level.

However, the two dynamics have to be synchronized so that the event can pass from the state E_3 to the state E_4 (see Fig. 4) before a decision relative to its treatment is finally taken. One of the two dynamics has to adapt itself to the other. This is what makes the difference between the periodic conduct and factual conduct.

In factual conduct, it is the dynamic of decision making that adapts itself to that of the event, given that it's irregular; the factual conduct is forced to be irregular.
On the contrary, in periodic conduct, it's the dynamic of the event which adapts to that of decision making. This is what will involve the wait times before the treatment of the event.

In reality, the two modes coexist in the designation of mixed conduct. There it operates on a periodic conduct, but for critical events, decision is taken without waiting for the end of the period.

The passage from a period j to the nest j+1, on a given level k, effects itself at finish date of the period k, $x^k(j)$, which is given by:

$$x^k(j)=P_k+x^k(j-1)$$

or :

$$x^k(j)=jP_k+x^k(0)$$

In periodic conduct, the event is treated in a sub process sp_i, at a period j_i, of the level k (where the sub process appears), which we determine as follows:

$$\begin{cases} j_i = \lambda \quad \text{if } \exists\ \lambda \in \text{IN such that } u^i + T_{i,1} + T_{i,2} - x^k(0) = \lambda P_k \\ j_i = E\left(\dfrac{u^i + T_{i,1} + T_{i,2} - x^k(0)}{P_k}\right) + 1 \quad \text{if not} \end{cases}$$

E represents the real part of x.

The dates for change of states of the event (passage from the state E_l to the state E_{l+1}), for each of the four states in the sub process sp_i, x_l^i, are given by:

$$\begin{cases} x^i_l = x^i_{l-1} + T_{i,l} \quad \forall l \in \{1,2,4\} \\ x^i_3 = x^k(j_i) \quad l = 3 \end{cases}$$

For l=3, the equation which we have, shows clearly the synchronization between the two dynamics. It permits us to determine the date for which transfer decision for the event is taken. This date coincide with the end of the synchronization period j_i, of the sub process sp_i.

The entrance u^i and the exit s^i of sp_i in the upstream phase of the process are such that:

$$\begin{cases} u^i = x^i_0 \\ s^i = x^i_4 \end{cases}$$

We thus obtain:

$$\begin{cases} x_1^i = x_0^i + T_{i,1} \\ x_2^i = x_1^i + T_{i,2} \\ x_3^i = x^k(0) + j_i.P_k \\ x_4^i = x_3^i + T_{i,4} \end{cases}$$

What proceeds the exit date is therefore:

$S^i = x^k(0) + j_i P_k + T_{i,4}$

This result is true for all the sub processes i, except for the last, i=2N, for which reason the state E_4 does not exist, consequently $T_{2N,4}=0$. We have:

$S^{2N} = x^{2N}(0) + j_{2N} P_0$

The entrance date of an event in a sub process is equal to its exit date from the proceeding sub process.

Parameters at entrance:

$$\begin{cases} u^0 \\ x^k(0) \quad \forall k = 0,1,\ldots,N \end{cases}$$

Parameters:

$$\begin{cases} P_k \quad \forall k = 0,1,\ldots, N \\ T_{i,l} \quad \forall l \in \{1,2,4\} \text{ et } i = 0,1,\ldots 2N \end{cases}$$

Except $T_{2N,4}$ which does not exit.

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Calculation:

For i=0,1,…,2N-1

$$\begin{cases} s^i = x^k(0) + j_i P_k + T_{i,4} \\ u^{i+1} = s^i \end{cases}$$

For i =2N

$S^{2N} = x^0(0) + j_{2N} P_0$

Reaction delay represents the time elapses between the occurrence and execution of the response. With reference to our model difference have to be made between the exit date of the process event (exit date of the last sub process sp_{2N}) and the occurrence date of the event at the first level 0. This is then written as:

$$T = S^{2N} - u^0$$

Or:

$$T = \left(x^0(0) + j_{2N} P_0\right) - u^0$$

We therefore have an expression for the reaction delay as a function of the system parameters.

D. Calculating the wait time.

At each level K of decision making the state E_3 in the upstream phase and downstream phase represents the wait for the end of the period.

For this reason we are going to establish another expression for the delay in the previous reaction. It's gotten by uniquely expressing as a sum, on the entire process, the duration of the events in all the different states of every sub process:

$$T = \left(\sum_{i=0}^{2N} T_{i,3}\right) + \left(\sum_{i=0}^{2N}\sum_{l=1}^{2} T_{i,l} + \sum_{i=0}^{2N-1} T_{i,4}\right)$$

Which is of the form: (1)+(2)

Where:

$$(1) = \left(\sum_{i=0}^{2N} T_{i,3}\right)$$

And

$$(2) = \left(\sum_{i=0}^{2N}\sum_{l=1}^{2} T_{i,l} + \sum_{l=0}^{2N-1} T_{i,4}\right)$$

This expression illustrates that reaction delay is made of:

One part (1), constitutes the wait time

The other part (2), constitutes the actual time for the process, therefore has an incompressible priority.

Approaching this expression for the reaction time using that which has been obtained previously, the wait time (1) is written:

$$\sum_{i=0}^{2N} T_{i,3} = T - \left(\sum_{i=0}^{2N}\sum_{l=1}^{2} T_{i,l} + \sum_{i=0}^{2N-1} T_{i,4}\right)$$

Or:

$$\sum_{i=0}^{2N} T_{i,3} = \left(j_{2N} P_0\right) - \left(u^0 - x^0(0) + \sum_{i=0}^{2N}\sum_{l=1}^{2} T_{i,l} + \sum_{i=0}^{2N-1} T_{i,4}\right)$$

In this equation, for a given system and event, only j_{2N} varies as a function of the start dates of the reference period for the levels. All the other terms are constants.

In order to reduce the reactivity delay, it is imperative to reduce the wait times, $T_{k,3}$ and $T_{2N-k,3}$ (duration of the stage E_3), of the two sub processes upstream and downstream, appearing at the level k by adjusting the start date $x^k(0)$, of the reference period of the level, in a manner to cancel one of the two wait times. The adjustment on a level is carried out in the following manner:

if $\min(T_{k,3}, T_{2N-k,3}) \leq x^k(0)$, then
$x^k(0)=x^k(0) - \min(T_{k,3}, T_{2N-k,3})$
if not
$x^k(0)=P_k + [x^k(0) - \min(T_{k,3}, T_{2N-k,3})]$

The result is the elimination of the shorter of the two wait times. We obtain a new start date for the reference period and a new wait time which is smaller.

For the entire treatment process, we successively apply the same principle to all levels of the process starting with the lowest preference. The algorithm below permits us to effect this calculation:

$x^k(0)=0 \quad \forall\ k=0,1 ;\ldots,N$
for k ranging from 0 to N, Do :
if $\min(T_{k,3}, T_{2N-k,3})=0$, then
$k=k+1$
If not, if $\min(T_{k,3}, T_{2N-k,3}) \leq x^k(0)$
$x^k(0)= x^k(0) - \min(T_{k,3}, T_{2N-k,3})$
If not
$x^k(0)=P_k + [x^k(0) - \min(T_{k,3}, T_{2N-k,3})]$
End if
$k=k+1$
End if
End

III. APPLICATION

We conducted our study based on the services of an Internal Medicine Unit of a hospital in Cameroon. It is an ideal milieu for the application. There work is usually carried out in a tight flux with the numerous pathologies requesting expertise and the availability of different members of the different members of the unit therefore the many equipments that are used to carry out diagnosis. This indicates that in the procedures to handle patients, the decision making trend should be clearly identified and function perfectly in order to react immediately.

A. *Identification of the different levels of decision making and the stages*

The information are contained in the tables 2,3 and 4 below:

TABLE 2
DIFFERENT LEVEL OF DECISION MAKING

Level k	Intervener
0	Nurse
1	Senior Nurse
2	Doctor, Head of Service

At every level of decision making, though the chain of the stages is the same what ever the level k of decision making, the description of the task executed is not the same. As an illustration we consider the example of the levels 0 and 2

Level 0: Nurse

TABLE 3
STAGES OF THE LEVEL 0

State	Upstream Phase		Downstream Phase	
	Description	Duration	Description	Duration
E_1	Administrative Reception of the patient, talk with the patient	$T_{1,0}$	Reception of instructions	$T_{1,4}$
E_2	Examination of the blood pressure and weight.	$T_{2,0}$	Administration of treatment, control and follow up	$T_{2,4}$
E_3	Wait time for the end of the period	$T_{3,0}$	Wait end of the period	$T_{3,4}$
E_4	Transfer to the next level	$T_{4,0}$		

Level 2: Doctor Head of Service

TABLEAU 4
STAGES OF LEVEL 2

State	Last level	
	Description	Duration
E_1	Reception of the patient's card, talk with patient.	$T_{1,2}$
E_2	Carry out final diagnosis	$T_{2,2}$
E_3	Wait end of period	$T_{3,k}$
E_4	Transfer of card to level 1	$T_{4,2}$

In the process and procedures of taking charge of a patient at a level k, the presence of interveners from the previous level is not necessary for the stages E_1 or E_2 seen for the two.

B. *Application*

The data for the example are as follows:

Time unit is the minute
The reference date is any minute considered to be the time origin.
The occurrence date of the event after the reference minute is u^0=2mn.
The periods of the levels are: P_0=3mn, P_1=3mn and P_2=7mn.
We initialize the reference period of all the levels to the reference date $t_0=0$. That's to say: $x^1(0)=x^2(0)=x^3(0)=0$.

The duration of the dates of the different stages of each sub process are given in the TABLE 5 below:

TABLE 5
DURATION OF THE STAGES

Sub process i	Duration $T_{i,1}$	Duration $T_{i,2}$	Duration $T_{i,3}$

0	3	2	2
1	2	2	2
2	2	3	2
3	2	2	2
4	1	2	

The simulation we realized on Excel gives us the results which we've regrouped in the table below:

Calculated data			
u^i	$T_{i,3}$	j_i	s^i
2	2	3	11
11	0	5	17
20	6	4	30
30	2	12	38
48	1	14	42

The exit date of the event is s^i=42mn
The reaction delay is T=40mn
The total wait time is 11mn

Next we apply the algorithm to reduce the wait times at the different levels.

We obtain the following results per level:

For the level 0, sub processes sp_0 and sp_4

None of the wait times is zero, we proceed to the adjustment. The smallest wait time is $T_{4,3}$=1mn in sp_0. It is superior to $x^0(0)=0$. The new value of $x^0(0)$ is:
$x^0(0)=P_0+[x^0(0)-\min(T_{0,3},T_{4,3})]=3+(0-1)=2$

We obtain the following results:

Parameters			Results						
$X^0(0)$	$X^1(0)$	$X^2(0)$	$T_{0,3}$	$T_{1,3}$	$T_{2,3}$	$T_{3,3}$	$T_{4,3}$	s	T
2	0	0	1	1	6	2	0	41	39

The new wait times are 10min

For the level 1, sub process sp_1 and sp_3

None of the wait times are zero, we proceed to the adjustment. The smallest wait time is $T_{1,3}$=1mn in sp_1, it is greater than $x^1(0)=0$. The new value of $x^1(0)$ is:
$x^1(0)=P_1+[x^1(0)-\min(T_{1,3},T_{3,3})]=3+(0-1)=2$

We have the following results

Parameters			Results						
$X^0(0)$	$X^1(0)$	$X^2(0)$	$T_{0,3}$	$T_{1,3}$	$T_{2,3}$	$T_{3,3}$	$T_{4,3}$	s	T
2	2	0	1	0	0	2	1	35	33

The new wait times are 4mn

For the level 2, sub process sp_2

The wait times $T_{2,3}$ is zero. We do not adjust the start date of the reference period for this level. We conserve $x^2(0)=0$. The result is the same to that obtained previously.

Parameters			Results						
$X^0(0)$	$X^1(0)$	$X^2(0)$	$T_{0,3}$	$T_{1,3}$	$T_{2,3}$	$T_{3,3}$	$T_{4,3}$	s	T
2	1	0	1	0	0	2	1	35	33

At the exit of the level 2, we obtain a total wait time of 4mn instead of 11mn that was at first. Bringing back the time unit of the previous example which was the minute, the reduction of 7mn obtained on the reaction delay which brings it back to 33mn is important for the life of a patient.

We think on the other part that the wait time of 4mn to the end of the process are incompressible in the measure or, after the principle of the method, one of the two wait times at a level is zero.

IV. CONCLUSION AND PERSPECTIVES

We have in this article established a relation which gives us the delay in reaction of a hierarchical system, using a periodic conduct following an event that occurs in the system. This delay is expressed as the time that runs between the occurrence of an event and the execution of the response (reactivity), and it's a function of system characteristics only. We have seen that the reaction delay is composed of two parts, one part which is incompressible and the other part which is the wait time.

We then proposed an algorithm which will permit us to determine the start dates of the reference periods, which minimizes the wait times to the end of period for each level and minimizes as well the reaction delay.

We have realized an application which shows that the model proposed and the algorithm to reduce the wait times gives us results that are globally satisfactory.

We wish to continue this work by evaluating the impact on the delay in reacting to a diagnostic error at any given level in the upstream phase. The expression of reaction delay is sensibly modified according to whether the error is treated in a periodic or factual conduct.

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