Fast Digital Measurement of Low Frequencies in a Narrow Band

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***Abstract*— Conventional digital method for low frequency measurement based on counting the pulses over a fixed amount of time requires abnormally large time. On the other hand, if the reciprocal of the time period is measured, the time is reduced to one period only. Based on this principle, a method is presented for low-frequency measurement with reasonable accuracy over a narrow band of frequencies. The method is verified experimentally. This will find many applications such as measurement of power frequency changes**.

Keywords- Low frequency measurement, fast frequency measurement, inverse period measurement, power frequency measurement

I. INTRODUCTION

For low frequency measurement, *analog* method [1] can be used. The digital value of the time period is sensed and digitally latched and applied to a multiplying digital to analog converter (DAC). The DAC is reconfigured as a divider by connecting the DAC in the feedback loop of an operational amplifier [2] [3]. The other input (Reference input) is –1 V dc. Thus, the output voltage is proportional to the reference voltage multiplied by the reciprocal of digital input, and, hence, proportional to the instantaneous rate of the signal. To achieve good accuracy, the analog circuit needs a careful adjustment, and with very low frequency signals the sample and hold droop is a serious source of error.

The *conventional digital method of cycle counting* requires a long time for low frequency measurement. For example, a signal of 1 Hz frequency requires 100 s for an accuracy of $\pm$1%. One possible alternative is to multiply the signal frequency by a known factor 10^k (k is a positive integer) and then measure this high frequency. Frequency multiplication can be accomplished with a PLL or a suitable frequency multiplier.

The circuit proposed by Smallwood [4] uses digital techniques, and the accuracy is, therefore, independent of the input signal frequency, but there is a limit on the ratio of the upper to the lower frequency for a given accuracy.

Commercial digital instruments which produce a digital output once per cycle, measure the period of the signal and then find its reciprocal using an IC calculator chip.

In this paper, we propose a simple, accurate and fast low-frequency measurement for narrow band around a nominal frequency. It is based on measuring the reciprocal of period measurement. A practical example for need of such a measurement is the power frequency, 50 Hz with $\pm$ 5% variation. Many sensors give output as frequency. Hence small variations in the non-electrical quantities such as pressure, temperature, can be measured with this device.

II. MEASUREMENT OF LOW FREQUENCY

Up counting method

Let f_o be the nominal frequency and $T_o = 1/f_0$ be the corresponding time period. If the clock pulses of frequency f_C are counted in the period T_o such that the display is exactly f_o, then the number of pulses counted will be

$$T_o f_C = \frac{f_C}{f_0} = f_o. \tag{1}$$

This requires the clock frequency

$$f_C = f_o^{\,2}. \tag{2}$$

At any other frequency *f*, the counter reading follows the straight line passing through origin and (T_0, f_0)., i.e., the relation

$$f = f_o^{\,2} T. \tag{3}$$

The plots of actual and displayed readings against period *T* are shown as curves (a) and (b), respectively, in Fig. 1. Note that, as *f* increases above f_o, reading decreases and vice-versa. Hence, the error goes on increasing on either side of f_o. Two methods to circumvent this problem are given below.

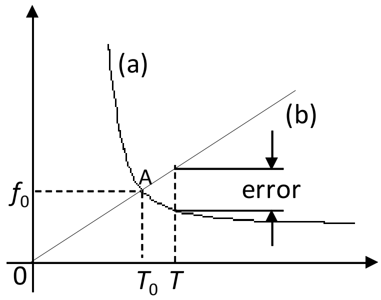

Figure 1. Plots of actual reciprocating and practical curves

Method 1: Complementary counting

Instead of displaying the actual counter reading, its complement is displayed. This can be accomplished by initially setting the counter at its full value, i.e., 10^n - 1, where *n* is the number of digits displayed, and by using the counter in the down count mode. Thus, the actual curve has the equation (a straight line passing through the points (f_0, T_0) and (10^n-1,0)

$$f = (10^n - 1) - \left[(10^n - 1) f_0 - f_0^{\,2}\right] T.$$

$$f = (10^n - 1) - f_C T. \tag{4}$$

Also

Comparing the two equations, we get

$$f_C = (10^n - 1) f_0 - f_0^{\,2}. \tag{5}$$

Note the following:

1. In the vicinity of f_o, reading is close to *f*. Thus, the method is reasonably accurate only over a narrow band around f_0.
2. It can be shown from eqn (5) that the two curves (a) and (b) cross at two frequencies, as shown in Fig. 2, given by the roots of the equation

$$f^2 - (10^n - 1)f + f_C = 0.$$

As we know that one of the roots is f_0, the other has to be $(10^n - 1) - f_o$. Thus the meter can be used for the nominal frequency f_0 or its complement. The displayed reading is less for lower frequencies and more for higher frequencies for nominal frequency f_0 and vice versa for the complementary frequency.

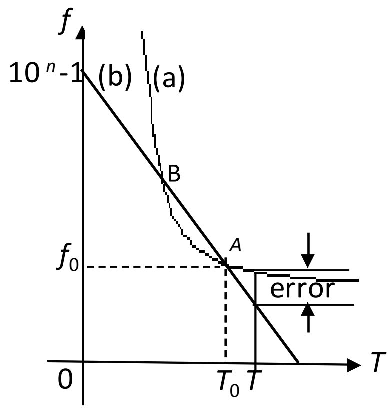

Figure 2. Plots of actual reciprocating and practical curves

Method 2: Down counting

A more accurate solution is to simulate the actual period reciprocating curve $f = 1/T$ by a straight line approximation such that the value f_o and the slope $-1/T_o^2$ at the point (T_o, f_o) of the actual reciprocating curve are the same for the straight line. Thus, the straight line will have the equation

$$f = 2f_0 - \frac{1}{T_0^2}T = 2f_0 - f_0^2 T.$$

Also
$$f = 2f_0 - f_C T. \tag{6}$$

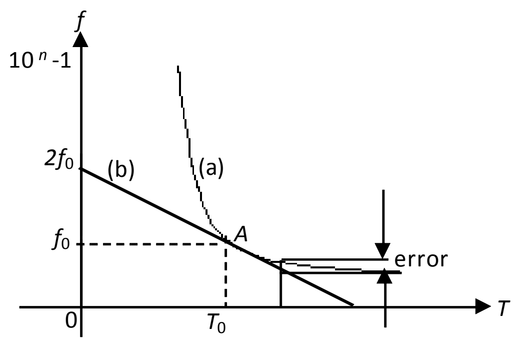

Figure 3. Plots of actual reciprocating and practical curves when $2f_o < (10^n - 1)$

Comparing the above two equations, we get

$$f_C = f_0^2. \tag{7}$$

Now, counting down takes place from $2f_0$ instead of 10^n-1 as in method 1. The actual curve f and its straight line approximation are shown as curves (a) and (b), respectively, in Fig. 3. It is interesting to note from Eqns (4) and (6) that the two straight lines overlap when $2f_0 = (10^n - 1)$, i.e., when the full counter value equals double the nominal frequency. Thus the performance of the two methods will be the same.

Table 1 gives the error at frequencies ±5% of nominal frequencies of 30, 50, 60 by the above two methods. Also for the method 1, we have also given the readings for complement of nominal frequency 30 Hz, i.e., 69 Hz. The case when the nominal frequency is half the full counter reading, i.e, 49.5 Hz is also included.

TABLE I. : ERRORS AT DIFFERENT FREQUENCIES

Nominal frequency f_0	**f_c from Eqn (6)**	**f_c from Eqn (8)**	**Actual Frequency f Hz**	**f from Eqn (4) Hz**	**f from Eqn (5) Hz**	**% Error in method 1**	**% Error in method 2**
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
30	2070	900	28.5	26.37	28.42	-7.47	-3.75
30	2070	900	31.5	33.29	31.43	5.68	-3.39
49.5	2450.25	2450.25	47.05	46.92	46.92	-2.76	-2.76
50	2450	2500	47.5	47.42	47.37	-0.16	-0.27
50	2450	2500	52.5	52.33	52.39	-0.32	-0.20
60	2340	3600	57.0	57.95	56.84	1.66	-0.28
60	2340	3600	63.0	61.86	62.86	-1.80	-0.22
69	2070	-	65.55	67.42	-	2.85	-
69	2070	-	72.45	70.43	-	-2.79	-

From Table 1, we note the following.

1. Unlike in method 1, the errors in method 2 are all negative. This is because the reading is always less than the actual value of the frequency (see Fig. 3). It is, therefore, possible to reduce the spread in error by adjusting the intercept of the straight line (b) in Fig. 3 slightly higher than $2f_o$.

2. In method 1, the sign of the errors is governed by whether the nominal frequency is at point A or at B and higher or lower than the nominal frequencies. For the nominal frequency of 30 Hz the point is A and for 50, 60, 69 Hz the point is at B. For 49.5 Hz both points A and B coincide.

3. As expected, the error in method 2 is less than that in method 1, except in the case of nominal frequency 50 Hz when the frequency measured is lower (47.5) than the nominal frequency. This is because the slope of the straight line (b) in method 2 is greater than that of the line (b) in method 1 and the actual frequency measured is below point A.

4. For the nominal frequency equal to half the counter reading = 49.5 Hz, the two errors are the same.

5. We have calculated the errors at a very large variation in frequencies (5%). Better accuracies will be obtained if the variations are within 2% as in the case of power supply.

To increase the accuracy in display, the clock frequency can be increased. For example for a resolution of 0.01, the clock frequency can be increased by 100 times.

In method 2, if $2f_o > (10^n - 1)$, one additional most significant digit will be required to store the initial value of $2f_o$. Alternatively, the counting can be carried out from $2[(10^n - 1) - f_o]$ (as e as shown in Fig. 4) in the up mode

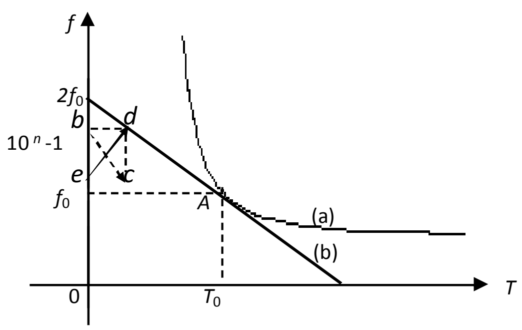

Figure 4. Two modes of operation when $2f_o > (10^n - 1)$

with a frequency f_C. When the counter becomes full, the counting should be switched to the down mode with the same frequency. Else, counting may be in the down mode from initially set value of $10^n - 1$ (as b as shown in Fig. 4) with a frequency f_C. When the count reaches the value $2[(10^n - 1) - f_o$, the counter is again set the full value $10^n - 1$ and allowed to continue in the down count mode with the same frequency f_C. The two modes of operation are shown as edA and bcdA in Fig. 4. One more alternative is that the counter is initially reset to 00…00 and start counting upward with a clock frequency *f'* till the counter is full and then switched over to the down counting with frequency f_C. However, this will require an additional clock of frequency $f' > f_C$. Thus one has to trade off between the increase in cost due to additional components for one more bit or this arrangement.

Note that we cannot measure the frequencies above half of the full range of the counter $(10^n-1)/2$.

Accuracy can be increased by including further *k*-bit counter and increasing the clock frequency by a factor of 10^k. This in effect makes the meter an $(n + k)$ bit device, with the *k* least significant bits discarded.

III. Practical Verification

Frequency meters are designed for the nominal frequencies of 30, 50 and 60 Hz and fabricated for a resolution of 0.01. For 60 Hz, we have used the method 2 with the path edA shown in Fig. 4. The result obtained were within ±8% accuracy for method 1 and -4% for method 2, for a variation of ±5% in the nominal frequency. The range of errors in method 2 can be reduced by slightly increasing the intercept of the curve (b) so that it would become positive for some values and negative for other values.

IV. Conclusion

A method for low-frequency measurement with reasonable accuracy over a narrow band of frequencies around the nominal one in one cycle period has been presented. The method has been verified experimentally. This will find applications such as in the measurement of power frequency variations. This device, coupled with sensors whose output is a low frequency, can measure the changes faster and with better accuracy.

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T S Rathore was born in Jhabhua (M P, India) on Oct. 29, 1943. He received the B Sc (Electrical Engineering), M E (Applied Electronics & Servomechanisms), and Ph D (by research on Passive and Active Circuits) degrees in Electrical Engineering from Indore University, Indore, India in 1965, 1970 and 1975, respectively.

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