

Arabo-Andalusian Zellij Images Retrieval Method Based on the Maximum Common Sub-graph

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Abstract—This paper describes a new indexing and retrieval method that can be used for indexing an Arabo-Andalusian pattern images database where each image is characterized by its zellij Tiles (elementary shape) and their spatial relationship. The proposed method allows representing this spatial relationship by an adjacency graph, itself represented by an adjacency matrix. The search of patterns similar to a given one presented as a query is then realized by calculating a similarity index characterizing the maximum common sub-graph between query image and model images of the database.

The performance of the proposed method is then tested on a database of rosettes (decors which begin at a central point and grow radially outward), built for the purpose of this work.

Keyword-Content-based image Retrieval and indexing, Adjacency graph, Maximum common sub-graph, Arabic-Andalous decors, Geometrical Art, Similarity measure.

I. INTRODUCTION

Mosaic tiles, also referred to Zellij, Moorish, Andalusian, Moresque or Fes tiles, have an antique look. This form of art flourished during the Hispano-Moresque era during the Marinid dynasty around the 14th century; they are still prevalent in Spanish and Portuguese architecture and they can be seen in many Moorish monuments, mosques, churches, houses, and palaces. This art refers to arbo-moresque, arabo-moorish or arabo-andalusian.

Arabo-Andalusian patterns are made of the arabesque, the glazed ceramic mosaic (or zellij in Arabic) and calligraphy [1]. This kind of design is a cultural heritage which has been always a source of inspiration for artists and a tool for socio-economic development.

It is well known that the zellij patterns addressed in this work can be divided in 3 categories (Fig.1):

- *Wallpaper* [2]: patterns that admit translations in two or more directions.
- *Rosette* [3]: patterns which begin at a central point and grow radially outward. The central point is a star with 8, 16, 24, 40, 48, 72 and 96 petals, called order of the rosette. Then, the pattern grows by using zellij tiles generated from this central star.
- *Frieze* [4]: patterns that admit translation in only one direction.

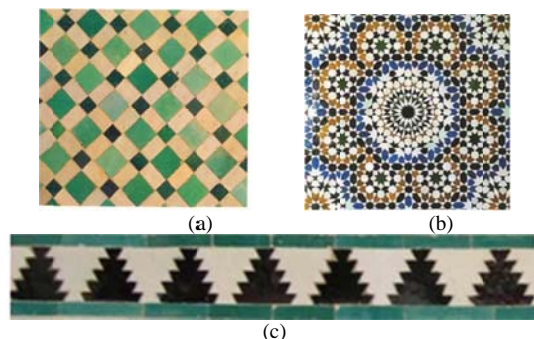


Fig.1. Example of each decor: (a) *Wallpaper*, (b) *Rosette*, (c) *Frieze*

In this paper, we are interested in one of the most beautiful and well known forms of geometric art which is the rosette. The particularity comes from the periodicity and symmetry of tile patterns.

Recently, questions have been raised about the virtual conservation and management of collections and their accessibility to experts such as archeologists, art historians, and even to the large public. Requirements concern facilities:

- For cataloguing collections and distant consultation, and also for intelligent retrieval tools.
- For virtual conservation, reconstruction (e.g., restoration, missing tiles).

To achieve this objective, several works of geometrical patterns modeling as well as automatic methods for indexing and retrieving images of Islamic patterns databases are reported in the literature. Here is a short description of them.

In Abas [5].the author discussed the evolution of classical geometric methods for Islamic geometrical patterns and developed algorithms based on group theory for efficient generation of all crystallographic repeat patterns using modern computer graphics. In Grunbaum and al. [6] the authors decompose periodic Islamic patterns by their symmetry groups, obtaining a fundamental region they use to derive properties of the original pattern. Elsewhere, Grunbaum derived a fundamental region by decomposing the star pattern by their symmetry groups [7].In Kaplan [8].the author presented a procedure for constructing Islamic star patterns based on placing radially-symmetric motifs in a formation dictated by a tiling of the plane, and showed some styles in which they can be rendered. Castera [1].presented a technique based on the construction of networks of eightfold stars and “Safts”.

In Albert and al. [9].the authors propose a method based on the detection of symmetry in a decor, but no measurement of symmetry in the image is calculated.

Several others research works tackled the zellij’s images indexing and retrieval. A. Zarghili and al.([10]-[11]) propose a method to index an Arabo-Moresque decor database which is not based on symmetry. They use a supervised Mosaicking technique to capture the whole principal geometric information (connected set of polygonal shapes, called “spine”) of a pattern. The spine is then described by using Fourier shape descriptors to allow retrieving images even under translation, rotation and scale. But, the drawback of this method is related to the manual extraction of the spine.

In Djibril [12].the authors represent a rosette by its minimal triangle, called fundamental region, by considering the groups of symmetry. Then, the characteristics of the rosette are represented by the color histogram corresponding to the fundamental region. The drawback of this method is related to the fact that tiles included in this zone and their spatial arrangement are not taken into account, but no work has presented the decor of zellij by an adjacency graph, and then used a similarity measurement to compare the decor of zellij.

In this paper, we propose a novel method for which a rosette is represented by its fundamental region as in ([4], [12] - [13]). Then, the fundamental region is characterized by the adjacency graph representing the spatial arrangement of its belonging zellij tiles.

For the retrieval operation we propose to use the similarity measurement between two graphs G_1 and G_2 , which represent respectively the query image and the model image. So the flowchart of the proposed method is as the following:

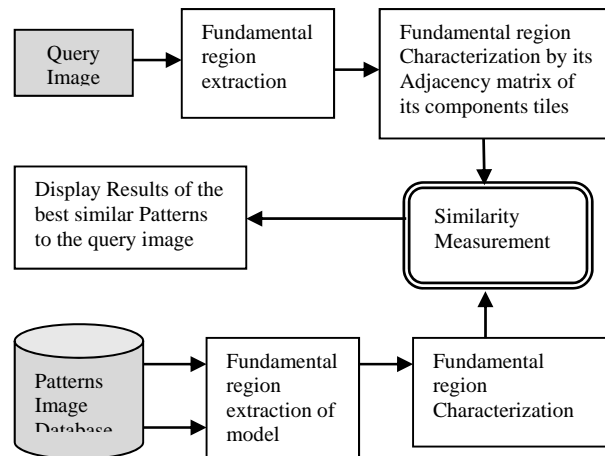


Fig. 2. Content-Based image Indexing and Retrieval Process of the proposed method

The rest of the paper is organized as follows: Section 2 describes the method used for the extraction of the fundamental region. Section 3, demonstrates the representation of a rosette by its corresponding adjacency graph and its adjacency matrix. Section 4, defines the similarity index used to retrieve the best similar zellij pattern to a query one. The results of the proposed method, by using images of the constructed database, are then presented in section 5. The last second is left for the conclusion of this work.

II. FUNDAMENTAL REGION EXTRACTION

A rosette is represented by a set of zellij tiles with a central symmetry. The observation of symmetries can reduce the decor to a minimal symmetrical part, called the fundamental region ([4], [12] - [13]). From this representative region the whole image can be regenerated. The automatic extraction of this region is based on the following steps:

1. Detecting the rotation center of the rosette image,
2. Completing the image structure by using symmetry information.
3. Computing the angle of rotation and number of folds.
4. Extracting the fundamental region

Fig. 3-b illustrates the result obtained by the application of this method to the image of fig. 3-a.

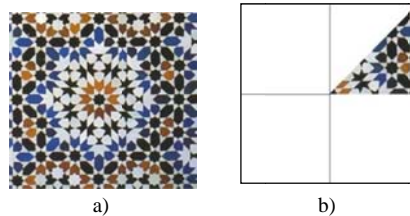


Figure 3. Rosette and its fundamental region

A. Characterization of the Fundamental region by the adjacency matrix

As it is shown by Fig. 3-b, the fundamental region of zellij pattern is constructed by a set of connected tiles with different shapes. Therefore, the characterization of this region can be achieved by identifying all the belonging tiles and then modeling their spatial arrangement.

The identification of each tile is based on a segmentation operation (para.3.1) followed by a pattern recognition operation. The objective is to identify each tile by a label, specified in figure 4 by names used by Moroccan Craftmen.

For the modeling of spatial arrangement of tiles, we propose to use the adjacency graph ([14]-[15]), which is itself represented by an adjacency matrix ([16]-[17])

Saft 	Mrabaa 	Koura
Mthemmen 	Kif Amel 	Khathem
Louza 	Mahhara 	Qfa
Mkinsa 	Thaleth 	Baoula thaltha

Fig.4. Examples of Tiles and their corresponding used names

B. Image segmentation

Generally, image segmentation is the process of isolating objects in the image, i.e., partitioning the image into disjoint regions, such that each region is homogeneous with respect to a certain property, such as grey value, color or texture.

The segmentation approach can be classified into two classes: contour-based segmentation and region-based segmentation.

The watershed algorithm takes a very different approach, compared to the other classical approaches mentioned before.

Watershed segmentation, a very prominent segmentation scheme with many advantages for image segmentation in such a way that it ensures the closed region boundaries and gives solid results.

Many methods and approaches of watershed method have been proposed in the literature ([18], [19], [20], [21]-[22]). The choice of a method generally depends on the processed images. In this work we propose a hierarchical segmentation method using two successive watershed method [23], based on the following Meyer's Algorithm [20].

Algorithm 1. Meyer's Algorithm :

A set of markers, pixels where the flooding shall start, are chosen. Each is given a different label.

1. The neighboring pixels of each marked area are inserted into a priority queue with a priority level corresponding to the gray level of the pixel.
2. The pixel with the highest priority level is extracted from the priority queue. If the neighbors of the extracted pixel that have already been labeled all have the same label, then the pixel is labeled with their label. All non-marked neighbors that are not yet in the priority queue are put into the priority queue.
3. Redo step 3 until the priority queue is empty.

The non-labeled pixels are the watershed lines.

Fig.5 shows examples of results obtained by applying this method to some zellij images.



Fig. 5. Original Images and their segmented version by Meyer's Watershed algorithm

C. Fundamental Region Tiles Recognition

To identify which of the basic shapes of the zellij tiles database corresponds to each tile extracted from the image after segmentation, we applied the pattern recognition method using a dissimilarity measurement between the extracted form and the tiles' image database. This index/measurement represents a set of parameters, invariant to translation, rotation and scale parameter [24]. Fig. 6 shows the flowchart of this method.

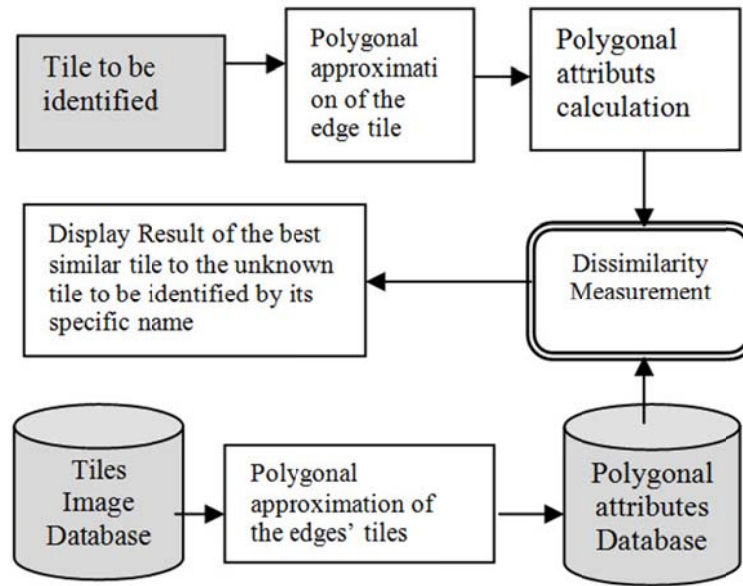


Fig. 6. Flowchart of the proposed method to recognize the specific name of an extracted tile belonging to fundamental region

1) Polygonal approximation of the tile edge and its normalization

The purpose of a polygonal approximation method of an edge is to extract from a string of edge points, successive segments to minimize a global error criterion or respect a local approximation error. To this end, many methods have been developed ([25], [26] - [27])

In this work, we have used the method developed by Huang and Wang [24] This choice is motivated by the simplicity of implementation and the good behavior with noise.

2) Tile characterization by polygonal Attributes

The tile recognition requires the characterization of its polygon representative by a set of attributes invariant to translation, rotation and scale parameter. The following attributes used in this work are those proposed by Huang and Wang [24].

- *Polar distance*: The polar distance r_i is denoted as the distance between vertex point P'_i and its respective centroid.
- *Polar angle*: The polar angle θ_i is denoted as the slope of the line connecting a vertex point i and its centroid.
- *Vertex angle*: The vertex angle a_i is denoted as the angle between the two line segments $[P'_{i-1}, P'_i]$ and $[P'_i, P'_{i+1}]$.
- *Chord length*: Let i denotes the i th chord length of the normalized polygon, which is the distance between the two consecutive vertex points, $[P'_i, P'_{i+1}]$.

The above polygonal attributes are illustrated in Fig 7:

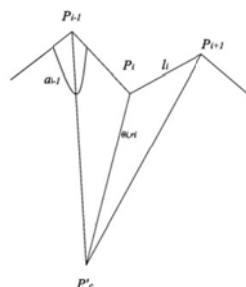


Fig.7. The illustration of the polygonal attributes.

3) Dissimilarity index

The dissimilarity index used in this work is that based on the comparison of normalized polygons, characterized by their attributes [24].

4) Region Adjacency Graphs (RAGs) modeling the adjacency relationships between tiles

After the extraction of all tiles by using segmentation followed by a pattern recognition operation, leading to identify each tile by a label or a specific name, we build the region adjacency graph by using the algorithm proposed in [28].

Algorithm2. Build region adjacency graph:

```

create a new graph  $G$ 
for each region  $r$  do
  add a new vertex to  $G$ 
  set relative size of region
  set average pixel value encompassed by region
end for
for each row  $i$  do
  for each column  $j$  do
    let  $m$  be the region to which pixel  $i,j$  belongs
    let  $n$  be the region to which pixel  $i+1,j$  belongs
    if  $m \neq n$  then
      if there is no arc in  $G$  from  $m$  to  $n$  then
        add an arc in  $G$  from vertex  $m$  to vertex  $n$ 
        set increment in average pixel value
      else if there is no arc in  $G$  from  $n$  to  $m$  then
        add an arc in  $G$  from vertex  $n$  to vertex  $m$ 
        set increment in average pixel value
      end if
    end if
  end for
end for

```

As a result, the obtained adjacency graph will be represented by the corresponding adjacency matrix ([16]-[17]).

III. CONTENT BASED RETRIEVAL PROPOSED METHOD

D. Definition of similarity measure based on the Maximum Common Sub-Graph(MCS)

Given a database of Zellij patterns and a query pattern, the task is to retrieve one or several patterns from the database that are similar to the query. By using the graph as a representation of Zellij pattern, the retrieval operation becomes a graph-matching problem. For this purpose, we propose to use the following similarity measure, based on maximum common sub-graph (MCS) ([29],[30],[31], [32], [33]-[35]) .

$$d_{MCS}(G_1, G_2) = 1 - \frac{|MCS(G_1, G_2)|}{\max\{|G_1|, |G_2|\}}$$

Where $|MCS(G_1, G_2)|$ denotes the number of vertices/ edges from the maximum common sub-graph [39]. $|G_1|$ and $|G_2|$ denote the number of vertices/ edges of G_1, G_2 respectively.

The formal description of the MCS is as follows:

Having two graphs G_1 and G_2 , What is the largest induced sub-graph of G_1 isomorphic to an induced sub-graph of G_2 ?

Many algorithms about MCS have been proposed ([29],[30],[31], [32], [33]-[35]) .

A different strategy for deriving the MCS first obtains the association graph of the two given graphs and then detects the maximum clique (MC) of the latter graph ([31], [35]-[36]).

The use of this computational method reveals two main drawbacks; the first one is its computational complexity. This is an inherent difficulty of the graph-matching problem. A brute-force approach requires a computational cost of $O(n!)$ for a graph with n nodes. The sub-graph isomorphism is proven to be NP-complete [37]. The second drawback is related to the results accuracy (section 5).

To overcome these drawbacks, the proposed method keeps only the relevant relationships between vertices. Being restricted to the search of MCS between two graphs with the same vertices (after the removal of vertices and not common edges) significantly reduces the number of calculations to be performed. Indeed, the aim is to find the most similar patterns to a query one (with the same tiles and the same spatial arrangement of tiles).

E. Practical method for calculation of the MCS

To better understand the proposed method, we provide an illustration of the different steps needed for this purpose.

a) First step aims to delete rows and columns not common between adjacency matrix of query and model images (Fig. 8). This operation leads to common adjacency matrices given by Fig. 9.

- b) The second step aims to extract common sub- graphs (see algorithm 3).
- c) In the last step, the MCS is extracted; it is the graph with the largest number of the vertices, from graphs found in the second step (Fig.10).

Algorithm 3. Extract common sub-graphs:

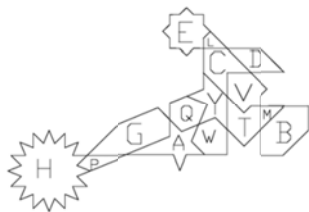
V_q = set the vertices of common adjacency matrix of query image, V_m = set the vertices of common adjacency matrix of model image,

G_c = common sub-graph, V_c = set the vertices of G_c .

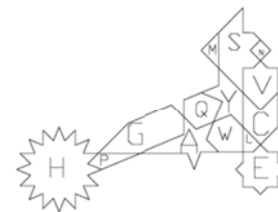
Initialize G_c with the first element of the first column of M_q : ($G_c=V_{q1}$)

```

for all  $V_q$  in  $M_q$  do
  for all  $V_m$  in  $M_m$  do
    if ( $V_q, V_m$ )=1 and ( $V_q, V_c$ )=1 and ( $V_m, V_c$ )=1 then
      add vertices to  $G_c$ 
    end if
  end for
end for
  
```



Query image



Model image

	H	P	G	A	Q	W	Y	T	B	M	V	C	D	L	E
H	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0
P	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0
G	1	1	0	1	1	0	0	0	0	0	0	0	0	0	0
A	0	1	1	0	1	1	1	0	0	0	0	0	0	0	0
Q	0	0	1	1	0	1	1	0	0	0	0	0	0	0	0
W	0	0	0	1	1	0	1	1	0	0	0	0	0	0	0
Y	0	0	0	1	1	1	0	1	0	0	1	1	0	0	0
T	0	0	0	0	0	1	1	0	1	1	1	0	0	0	0
B	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0
M	0	0	0	0	0	0	0	1	1	0	1	0	0	0	0
V	0	0	0	0	0	0	1	1	0	1	0	1	1	0	0
C	0	0	0	0	0	0	1	0	0	0	1	0	1	1	0
D	0	0	0	0	0	0	0	0	0	0	1	1	0	1	0
L	0	0	0	0	0	0	0	0	0	0	0	1	1	0	1
E	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0

Adjacency matrix of query image

	H	P	G	A	Q	Y	W	L	E	C	V	N	S	M
H	0	1	1	0	0	0	0	0	0	0	0	0	0	0
P	1	0	1	1	0	0	0	0	0	0	0	0	0	0
G	1	1	0	1	1	0	0	0	0	1	0	0	0	0
A	0	1	1	0	1	1	1	0	0	0	0	0	0	0
Q	0	0	1	1	0	1	1	0	0	0	0	0	0	0
Y	0	0	0	1	1	0	1	1	0	1	1	0	1	1
W	0	0	0	1	1	1	0	1	1	1	0	0	0	0
L	0	0	0	0	0	1	1	0	1	1	0	0	0	0
E	0	0	0	0	0	0	1	1	0	1	0	0	0	0
C	0	0	0	0	0	1	1	1	1	0	1	0	1	0
V	0	0	0	0	0	1	0	0	0	1	0	1	1	0
N	0	0	0	0	0	0	0	0	0	0	1	0	1	0
S	0	0	0	0	0	1	0	0	0	1	1	1	0	1
M	0	0	0	0	0	1	0	0	0	0	0	0	1	0

Adjacency matrix of model image

Fig.8. Query and model image, with their corresponding matrices

	H	P	G	A	Q	W	Y	M	V	C	L	E
H	0	1	1	0	0	0	0	0	0	0	0	0
P	1	0	1	1	0	0	0	0	0	0	0	0
G	1	1	0	1	1	0	0	0	0	0	0	0
A	0	1	1	0	1	1	1	0	0	0	0	0
Q	0	0	1	1	0	1	1	0	0	0	0	0
W	0	0	0	1	1	0	1	0	0	0	0	0
Y	0	0	0	1	1	1	0	0	1	1	0	0
M	0	0	0	0	0	0	0	0	1	0	0	0
V	0	0	0	0	0	0	1	1	0	1	0	0
C	0	0	0	0	0	0	1	0	1	0	1	1
L	0	0	0	0	0	0	0	0	0	1	0	1
E	0	0	0	0	0	0	0	0	0	1	1	0

Common Adjacency matrix of query image denoted Mq

	H	P	G	A	Q	Y	W	L	E	C	V	M
H	0	1	1	0	0	0	0	0	0	0	0	0
P	1	0	1	1	0	0	0	0	0	0	0	0
G	1	1	0	1	1	0	0	0	0	1	0	0
A	0	1	1	0	1	1	1	0	0	0	0	0
Q	0	0	1	1	0	1	1	0	0	0	0	0
Y	0	0	0	1	1	0	1	1	0	1	1	1
W	0	0	0	1	1	1	0	1	1	1	0	0
L	0	0	0	0	0	1	1	0	1	1	0	0
E	0	0	0	0	0	0	1	1	0	1	0	0
C	0	0	0	0	0	1	1	1	1	0	1	0
V	0	0	0	0	0	1	0	0	0	1	0	0
M	0	0	0	0	0	1	0	0	0	0	0	0

Common Adjacency matrix of model image denoted Mm

Fig.9. Query and Model image with their common adjacency matrices

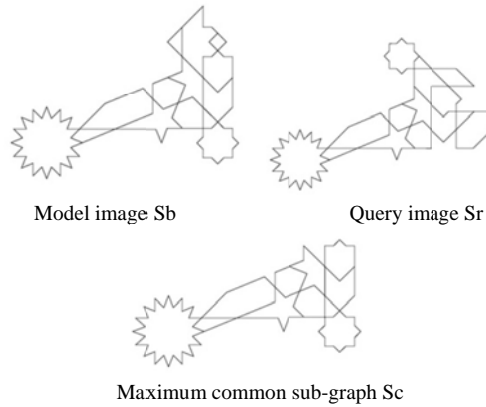


Fig.10. Example for Maximum common sub-graph

IV. EXPERIMENTS

This section is devoted to the experimental evaluation of the proposed method. Efficiency of the proposed method can be measured in terms of:

- Execution time (time complexity): A measure of the amount of time required to execute an algorithm
- The amount of memory required (space complexity)

For most of the algorithms, time complexity comparisons are more interesting than space complexity comparisons.

The following Table I show that the proposed method is faster than the Maximum clique method.

TABLE I. SAMPLE EXPERIMENTAL

Number of Tiles	Simulation Time (s)	
	Maximum clique	Proposed Method
7	6	10 ⁻⁹
9	8	10 ⁻⁸
14	32	1
15	64	1
18	121	1

F. Analysis of simulation results

The performance of the proposed method is shown by the comparison of its obtained results with those obtained by the MC method. For this purpose, a database of rosettes is used.

Fig. 12 gives the results obtained by applying the method based in MC on the association graph, while

Fig. 13 gives the results obtained by applying the proposed method. Table II show the good performance of the proposed method in comparison with the MC method, regarding the number of corresponding tiles in retrieved images.

TABLE II. SIMULATION RESULTS

Retrieved Images	Maximum clique	Proposed Method
	Number of corresponding tiles	Number of corresponding tiles
1st image found	3	19
2nd mage found	2	15
3rd image found	1	8
4th image found	15	8
5th Image found	19	8
6th Image found	5	7

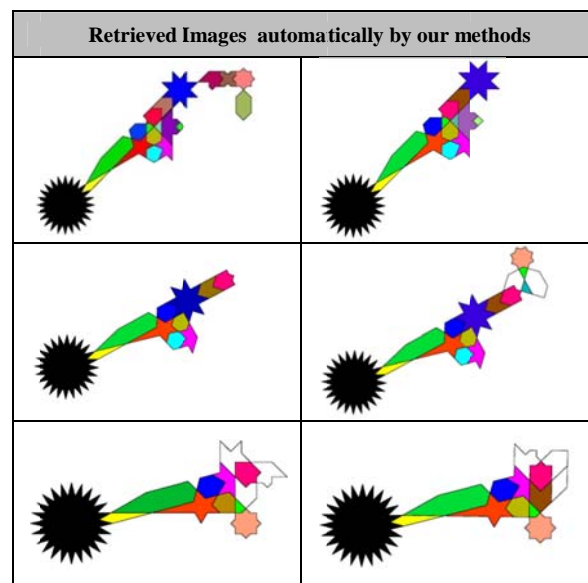
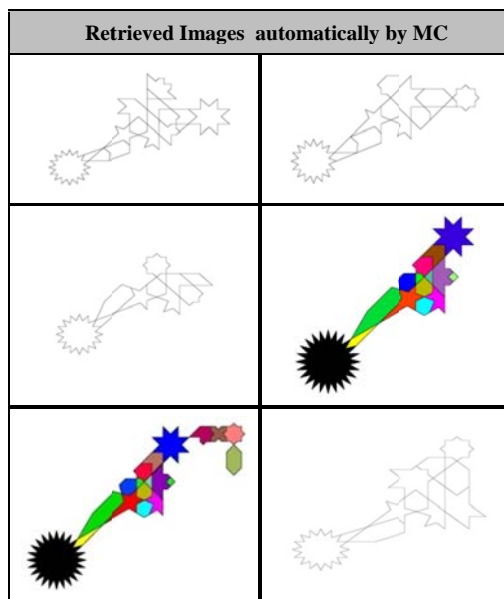
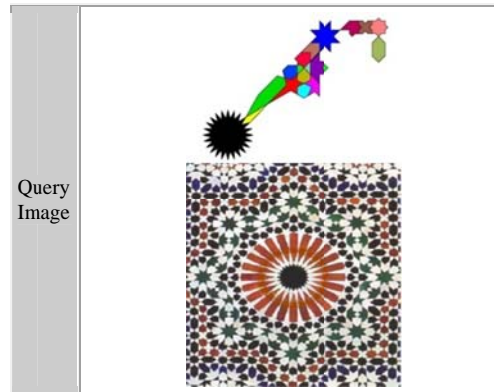


Fig.11. Results obtained by using the method of MC and by using the proposed method

Fig. 12 illustrates results obtained by applying the proposed method to two different query images.

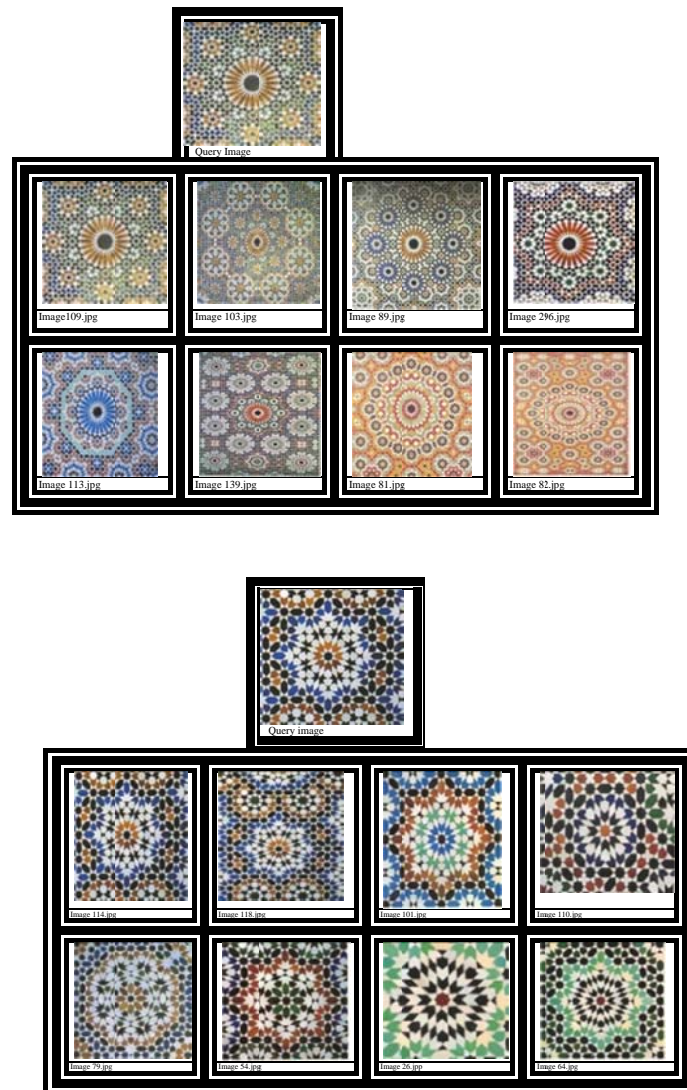


Fig.12. Simulation results

V. CONCLUSION

In this paper, an Arabo-Andalusian pattern images indexing and retrieval method, based on graphs matching, has been proposed. The problem of computing the similarity between two pattern images is transformed to that dealing with computing the measure of similarity between two adjacency graphs.

The adjacency graph which illustrates the spatial relationship of the adjacent tiles forming the pattern are extracted from the zellij pattern, and represented by adjacency matrix. In order to calculate the measure of similarity between two graphs we propose to use a similarity measure computed on the common sub-graph.

The performance of the proposed method is then tested on a database of rosettes, built for the purpose of this work. This performance is measured by the accuracy of the retrieved decors similar to a query one and by the low complexity time in comparison with the maximum clique method.

REFERENCES

- [1] J. Castera, "Arabesques," éditions ACR, 1996. ISBN :2-86770-096-5 (version française 1996).
- [2] D.Schattschneider, "The plane symmetry groups: Their recognition and notation", Amer. Math. Monthly, vol. 85 , 1978.
- [3] S. K. Craig, "Computer Generated Islamic Star Patterns." In Reza Sarhangi, editor, Proceedings of Bridges 2000, Mathematical Connections in Art, Music and Science. Winfield, Kansas, USA, 28-30 July 2000.
- [4] L. Seungkyu, R.Collins, and L., Yanxi, "Rotation Symmetry Group Detection Via Frequency Analysis of Frieze-Expansions," in Dept. of Computer Science and Engineering ,The Pennsylvania State University
- [5] S-J, Abas, Salman, A. S., "Geometric and Group-Theoretic Methods for Computer Graphics Studies of Islamic Symmetric Patterns," in Computer Graphics Forum, Vol. 1. 1992

- [6] B., Grunbaum, and G. C. Shephard, "Interlace patterns in Islamic and Moorish art," *Leonardo*, 25, 1992.
- [7] B. Grunbaum, "Grid Method Classification of Islamic Geometric Patterns", AMS, Vol. 53, 2006.
- [8] C. S. Kaplan, "Computer Graphics and Geometric Ornamental Design," Ph.D., University of Washington, Seattle, 2002.
- [9] F. Albert Gil, J. Gomis, J. M. Valiente, "Reconstruction techniques in the image analysis of Islamic mosaics from the Alhambra," *Dans Computer Graphics International*, pages 618-621, 2004.
- [10] A. Zarghili, N. Gadi, R. Bensliman, and K. Bouatouch, "Arabo-Moresque decor image retrieval system based on mosaic representations," *Journal of Cultural Heritage*, 2001.
- [11] A. Zarghili, J. Kharroubi, R. Benslimane, "Arabo-Moresque decor images retrieval system based on spatial relationships indexing," *Journal of Cultural Heritage*, Volume 9, numéro 3, pages 317-325, 2008.
- [12] M. Djibril, R. Oulad Haj Thami, "Islamic geometrical patterns indexing and classification using discrete symmetry groups," *JOCCH* 1(2), 2008.
- [13] S. K. Craig, D.H. SALESIN, "Islamic Star Patterns in Absolute Geometry," *ACM Transactions on Graphics*, Vol. 23, No. 2, April 2004.
- [14] P. Bodic, P. Héroux, S. Adam, H. Locteau, J. Bilong Mboumba, Y. Lecourtier, "Programmation linéaire en nombres entiers pour la recherche d'isomorphisme de sous-graphe," hal-00492174, version 1 - 15 Jun 2010.
- [15] J. Yves, "Graphes en analyse d'images et reconnaissance des formes," Ramel, Laboratoire Informatique de Tours - FRANCE, JIRC09 - Janvier 2010.
- [16] L. Cerf, T. Bao Nhan Nguyen, J-F. Boulicaut, "Discovering Relevant Cross-Graph Cliques in Dynamic Networks," Université de Lyon, CNRS/INSA-Lyon, LIRIS, UMR5205, F-69621, France.
- [17] I. Zager, and G. Verghese, "Graph similarity and Matching". eecs, MIT, 2005.
- [18] S. Beucher, C. Lantuejoul, "Use of watersheds in contour detection". Proc. Int. Workshop on image processing, real-time estimation, Rennes (France), 1979.
- [19] L. Vincent, P. Soille, "Watersheds in digital space and efficient algorithm based on immersion simulations" *IEEE Transactions on PAMI*, 1991.
- [20] F. Meyer, S. Beucher, "Morphological segmentation," *JVCIR*, 11, No 1:21-46, 1990.
- [21] F. Meyer, "Topographic distance and watershed lines". *Signal Processing*, 38(1):113-125, 1993.
- [22] L. Najman, M. Couprie, "Watershed algorithms and contrast preservation" In: DGCI'03. Volume 2886 of Lecture Notes in Computer Sciences. Springer Verlag, 2003.
- [23] F. Touzani, M. Harti, R. Benslimane, "Watershed for segmentation the Décor of Zellij". International Conference on Software Engineering, Database and Expert System (SEDEXS'12), 2012.
- [24] L.K. Huang, J. Wang, "Efficient Shape Matching Through Model Based Shape Recognition". *Pattern Recognition*, Vol. 29, No. 2, 1996.
- [25] T. Davis, "Fast Decomposition of Digital Curves into Polygons Using the Haar Transform. *IEEE Transactions on PAMI*, vol. 21, no 8, 1999.
- [26] P. L. Rosin, "Techniques for Assessing Polygonal Approximations of Curves". *IEEE Transactions on PAMI*, vol. 19, 1997.
- [27] Yin, P., Tabu, A., 2000. Search Approach to Polygonal Approximation of Digital Curves. *International Journal of Pattern Recognition and Artificial Intelligence*, vol. 14, no 2.
- [28] B. Ricardo, V. Gabriel "An Image Similarity Measure based on Graph Matching String Processing and Information". IEEE, 2000.
- [29] T. Akutsu, "A polynomial time algorithm for finding a largest common subgraph of almost trees of bounded degree. *IEICE Trans, Fundamentals*, E76-A(9), 1993.
- [30] H. Bunke, and K. Shearer, "A graph distance metric based on the maximal common subgraph". *Pattern Recognition Letters*, 19(3-4), 1998.
- [31] H. Bunke, P. Foggia, G. Corrado, C. Sansone and M. Vento, "A comparison of algorithms for maximum common subgraph on randomly connected graphs. in *Syntactical and Structural Pattern Recognition and Statistical Pattern Recognition*, 2002.
- [32] G.C. Garcia, I. L. Ruiz, and M.A. Grmez-Nieto, "Step-by-step calculation of all maximum common substructures through a constraint satisfaction based algorithm" *J. Chem. Inf. Comput. Sci.*, 44:30-41, 2004.
- [33] J.W. Raymond, E.J. Gardiner, and P. Willet, "RASCAL. Calculation of graph similarity using maximum common edge subgraphs. *Comps. J.* 2002.
- [34] C. Tonnelier, P. Jauffret, T. Hanser, and G. Kaufman, "Machine learning of generic reactions: 3. an efficient algorithm for maximal common substructure determination". *Tetrahedron Comput. Methodol.*, 3(6), 1990.
- [35] B. T. Messmer, "Efficient Graph Matching Algorithms for Preprocessed Model Graphs," Ph.D. Thesis, Inst. of Comp. Science and Appl. Mathematics, University of Bern, 1996.
- [36] D. H. Ballard, and C. M. Brown, "Computer Vision". Prentice-Hall, Englewood Cliffs, N.J.,
- [37] K. Mehlhorn, "Graph Algorithms and NP-completeness," Springer-Verlag New York, Inc., New York, NY, USA, 1984.