

# A Computerized Method for Determining Driving Point Resistance

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**Abstract**—A new computerized method for determination of driving point resistance is proposed. It is suitable for manual calculation as well. It proceeds step by step in a logical way. The network is reduced to a single resistance using parallel, series, star-delta and delta-star reduction techniques. The method is successfully tested manually as well as by simulation on the computer.

**Keyword** - Driving point resistance, Ladder network, Matrix method, Resistance, Star delta transformation

## I. INTRODUCTION

Methods for evaluating the driving point resistance (DPR) of a network are available [1]-[7]. The most simple and straight forward method is reducing the network to a one loop two node circuit using basic rules of series and parallel reduction, and star delta transformation. In this paper, the method is mechanized such that it can be computerized. The main difficulty is to make the computer to understand whether the elements are connected in series, parallel, delta or star. For this a special matrix representation of the network is developed which identifies the type of connection. Once this is achieved, then the task left for the computer is simply to calculate.

## II. MATRIX METHOD

### A. Numbering of junctions

A junction is defined as a meeting point of two or more elements. A junction will be indicated by a 'o'. Consider any two terminal resistive network whose driving point resistance (DPR)  $r_{1,1'}$  at the input terminals 1,1' is to be determined. Mark in serial order starting from the input terminal 1 as 1, all the junctions as 2 to N-1 and the terminal 1' as N, such that, while traversing from junction 1 to N, all the junctions are encountered but once. There can be many possible ways of numbering the junctions.

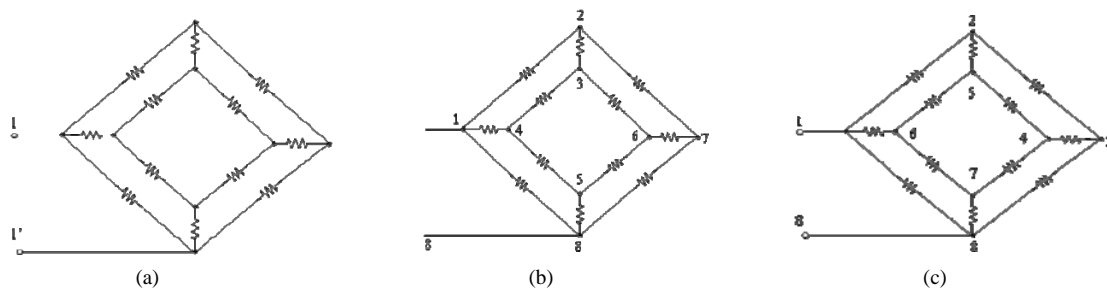


Fig. 1: (a) Circuit for example 1, (b) and (c) two possible numbering of junctions

**Example 1:** Consider the circuit shown in Fig. 1(a). There are 8 junctions. Two possible numbering of junctions are shown in Figs. 1(b) and 1(c).

### B. Formation of matrix

An  $[n \times n]$  matrix is formed as follows. Enter  $R_{ij}$  corresponding to the resistor connected between the junctions  $i$  and  $j$  as an element in the  $i$ th row and  $j$ th column. If there is no resistor, enter  $\infty$ . If there is  $k$  number of resistors connected in parallel between these junctions, all these will appear in the same location. Such a matrix will appear as shown in Table I.

TABLE I  
[n × n] matrix

Junction	1	2	...	N-2	N-1	N
1	0	$R_{1,2}$	...	$R_{1,N-2}$	$R_{1,N-1}$	$R_{1,N}$
2	$R_{2,1}$	0	...	$R_{2,N-2}$	$R_{2,N-1}$	$R_{2,N}$
⋮	⋮	⋮	0	⋮	⋮	⋮
N-2	$R_{N-2,1}$	$R_{N-2,2}$	...	0	$R_{N-2,N-1}$	$R_{N-2,N}$
N-1	$R_{N-1,1}$	$R_{N-1,2}$	...	$R_{N-1,N-2}$	0	$R_{N-1,N}$
N	$R_{N,1}$	$R_{N,2}$	...	$R_{N,N-2}$	$R_{N,N-1}$	0

Note that the *forward diagonal* has all 0 elements. Since  $R_{ij} = R_{ji}$ , the matrix is symmetrical across the forward diagonal. The diagonal immediately next and above the forward diagonal, will be called as *first diagonal (FD)*, next to FD the *second diagonal*, and so on. We shall refer the part of the matrix consisting of the diagonals 1 through N-1 as *upper triangular matrix (UTM)*, and designate it by  $[UTM]_{N-1}$ .

**Example 2:** Consider the network shown in Fig. 2 where the junctions are already numbered. The UTM for the network is shown in Table II. Note that three parallel resistors  $R_2$ ,  $R_3$  and  $R_4$  appear together in the second row and second column.

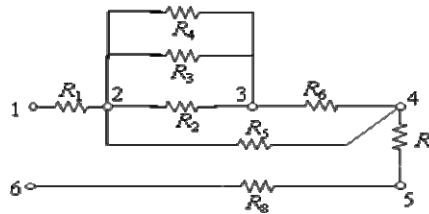


Fig. 2: Network for Example 2

TABLE II  
UTM for circuit shown in Fig. 2

Junction	2	3	4	5	6
1	$R_1$	$\infty$	$\infty$	$\infty$	$\infty$
2	$R_2$ $R_3$ $R_4$	$\infty$	$R_5$	$\infty$	$\infty$
3			$R_6$	$\infty$	$\infty$
4				$R_7$	$\infty$
5					$R_8$

C. Matrix simplifications

Series parallel reduction

Consider the ladder network shown in Fig. 3 with junctions numbered. The corresponding UTM is given in Table III.

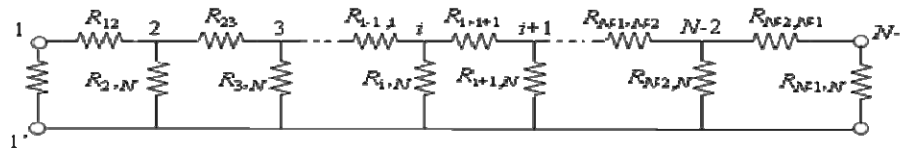


Fig. 3: Ladder network

TABLE III  
UTM for the ladder network of Fig. 2

Junction	2	3	...	<i>I</i>	<i>i+1</i>	...	<i>N-2</i>	<i>N-1</i>	<i>N</i>
1	$R_{1,2}$	$\infty$	...	$\infty$	$\infty$	...	$\infty$	$\infty$	$R_{1,N}$
2		$R_{2,3}$	...	$\infty$	$\infty$	...	$\infty$	$\infty$	$R_{2,N}$
⋮			⋱	⋮	⋮	⋱	⋮	⋮	⋮
<i>i-1</i>				$R_{i-1,i}$	$\infty$	...	$\infty$	$\infty$	$R_{i-1,N}$
<i>i</i>					$R_{i,i+1}$	...	$\infty$	$\infty$	$R_{i,N}$
⋮						⋱	⋮	⋮	⋮
<i>N-2</i>							$R_{N-3,N-2}$	$\infty$	$R_{N-3,N}$
<i>N-1</i>								$R_{N-2,N-1}$	$R_{N-2,N}$
<i>N</i>									$R_{N-1,N}$

Note from eqn (4), the following.

1. All the elements  $R_{i,i+1}$ ,  $i = 1, 2, \dots, N-1$ , appearing in the FD are the series branch resistors of the ladder network.
2. All the elements  $R_{i,N}$ ,  $i = 1, 2, \dots, N-1$ , appearing in the last column are the shunt branch resistors of the ladder 3.
3. All other elements  $R_{i,N} = \infty, j \neq N, j \neq i+1, i, j = 1, 2, \dots, N-1$ . These are the elements located in the area occupied by the diagonals 2 through  $N-1$ . This area will be referred as an *infinite zone (IZ)*. IZ with all elements as  $\infty$  characterizes the ladder network.

The DPR of the network is given by the recursive relation

$$r_{i+1,N} = [r_{i+2,N} + R_{i+1,i+2}] / R_{i+1,N} \quad i = N-3, N-4, \dots, 0$$

$$r_{N-1,N} = R_{N-1,N} \tag{1}$$

where  $x/y = xy/(x + y)$ .

**General networks**

We have seen above that the UTM of a ladder network has all the elements in IZ  $\infty$ . However, a general network may produce finite values also in the IZ. For example, the UTM of the network shown in Fig. 2 given in Table II has  $R_5$  in the IZ. Note that the resistor  $R_5$  is the bridging element between the junctions 2 and 4 in the network. Hence the elements that appear in the IZ are the bridging elements.

A general 2-terminal network may, therefore, consist of series resistors which appear as elements in the SD, parallel resistors that appear as elements in the last column and the bridging resistors that appear as elements in the IZ. We may, therefore, call any 2-terminal network as a *bridged-ladder network* in line with the name bridged-T network.

Any 2-terminal network redrawn in the following way will look as a bridged ladder. After numbering the junctions, place the junctions from 1 to  $N-1$  on an imaginary horizontal line, and the junction  $N$  in another horizontal line below the first line as shown in Fig. 4. Then insert all the resistances between various junctions as in the original network.

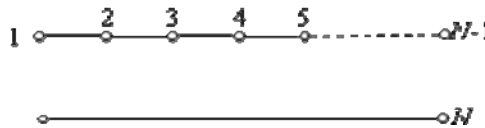


Fig. 4: Arrangement of the junctions

**Example 3:** The network shown in Fig. 5(a) is redrawn following the above procedure as shown in Fig. 5(b).

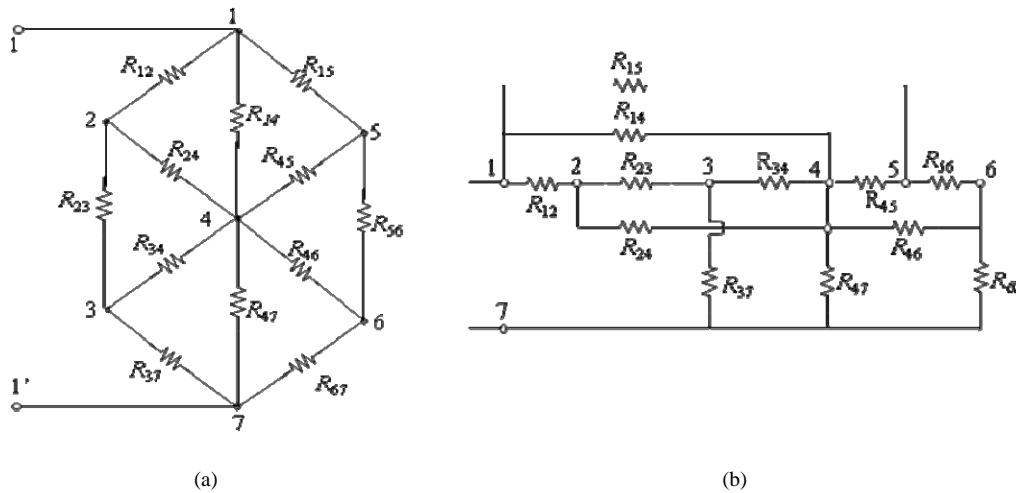


Fig. 5: (a) Network and (b) redrawn to look like a bridged ladder

Some obvious properties of UTM of a ladder network are as follows.

1. All the junctions belong to the basic ladder.
2. If each junction is connected to all other junctions, then IZ will be full with all elements of finite value. This will be referred as *house full IZ*.
3. Number of elements connected at a junction  $k$  is the sum of non-infinite number of elements both in the row and column corresponding to the junction  $k$ .

**Parallel reduction**

Let there be  $k$  number of parallel resistances connected across the junctions  $i$  and  $j$ . These resistances will appear as a cluster at the  $i$ th row and  $j$ th column in UTM. They can be replaced by a single resistance  $R_p$  in the same row and same column given by

$$R_p = 1/[\text{sum of the reciprocals of all the parallel resistances}]. \tag{2}$$

After replacing the string of  $k$  parallel resistors by the equivalent resistance  $R_p$  in the network, the UTM corresponding to the reduced network will be the same UTM except that the entire parallel matrix will be replaced by a single element  $R_p$ . This process will be referred as *parallel reduction rule*.

**Example 4:** Consider the network of Fig. 2 and the corresponding UTM of Table II. Three resistors  $R_2, R_3, R_4$  connected in parallel between the junctions 2 and 3 manifests as a cluster in the row and column corresponding to the junction 2 as shown in the UTM in Table II. Applying the parallel reduction rule, the new UTM obtained is shown in Table IV.

TABLE IV  
New UTM

Junction	2	3	4	5	6
1	$R_1$	$\infty$	$\infty$	$\infty$	$\infty$
2		$R_p$	$R_5$	$\infty$	$\infty$
3			$R_6$	$\infty$	$\infty$
4				$R_7$	$\infty$
5					$R_8$

**Series reduction**

If a string of  $k$  number of resistors  $R_{i+1}, R_{i+2}, \dots, R_{i+k}$  connected in series, they all can be replaced by a single resistance

$$R_s = R_{i+1} + R_{i+2} + \dots + R_{i+k}. \tag{3}$$

Consider the UTM of Table IV. Resistors  $R_7$  and  $R_8$  connected in series in the circuit between the junctions 4 and 5 manifests in the columns corresponding to these junctions. Such a matrix will be referred as *series matrix*. The following are the characteristics of a series matrix.

1. Each series resistor  $R_{ij}$  will appear as the element in the FD and no other element will appear in the  $i$ th row and in  $j$ th column.
2. All other elements in this matrix will be  $\infty$ .

After replacing the string of  $k$  series resistors by the equivalent resistance  $R_s$  in the network, the UTM corresponding to the reduced network will be the same as shown in Table IV by inserting an element  $R_s$  in the  $k$ th column and deleting first  $k-1$  columns and bottom  $k-1$  rows of the series matrix. This will reduce the order of the matrix by  $k$ . This process will be referred as *series reduction rule*.

**Example 5:** After applying the series reduction rule to the Table IV, UTM reduces to

Junction	2	4	6
1	$R_1$	$\infty$	$\infty$
2		$R_5$ $R_{S1}$	$\infty$
4			$R_{S2}$

where  $R_{S1} = R_5 + R_6$ ,  $R_{S2} = R_7 + R_8$ . Further  $R_5$  and  $R_{S1}$  form a parallel combination. After removing them by applying parallel rule the following UTM results.

$$[UTM]_3 =$$

Junction	2	4	6
1	$R_1$	$\infty$	$\infty$
2		$R_{p1}$	$\infty$
4			$R_{S2}$

where  $R_{p1} = \frac{R_{S1}R_5}{R_{S1} + R_5}$ .

Again applying the series reduction rule,

$$[UTM]_1 =$$

Junction	6
1	$R_1 + R_{p1} + R_{S2}$

Thus the input resistance is  $r_{1,6} = R_1 + R_{p1} + R_{S2}$ .

**Bridged-T reduction using delta to star transformation**

A star network shown in Fig. 6(a) can be converted into a bridged-T network by introducing a bridging branch in three possible ways as shown in Figs. (b), (c) and (d). These will be referred as *BT-T*, *BT-R* and *BT-L* respectively, based on the position of the bridging branch on top, right or left, respectively. Note that (1) at junction  $i+1$  the number of elements connected is due to three star branches and is three, (ii) the position of bridging branch is different for BT-T, BT-R and BT-L respectively. The portion of the UTM when  $p < i$  is shown in Table V. Note that three branches  $R_{p,i+1}$ ,  $R_{i,i+1}$  and  $R_{i+1,i+2}$  shown in black color are the same while the location of the bridging element is shown in red color by  $R_L(R_{p,i})$  corresponding to the BT-T, BT-R and BT-L respectively.

After delta resistances are converted into equivalent star resistances and then the two resistances connected in series are replaced by their series equivalent, the UTM of the new network will be as given in Table VI. Note that the red color elements become infinity while the black color elements changed  $R_{Ak}$ ,  $R_{Bk}$  and  $R_{Ck}$  ( $k = T, R, L$ ) as per equation (4). Other elements in the row and column corresponding to  $i+1$  junction will remain infinity, so that the sum of the number of finite elements is still 3.

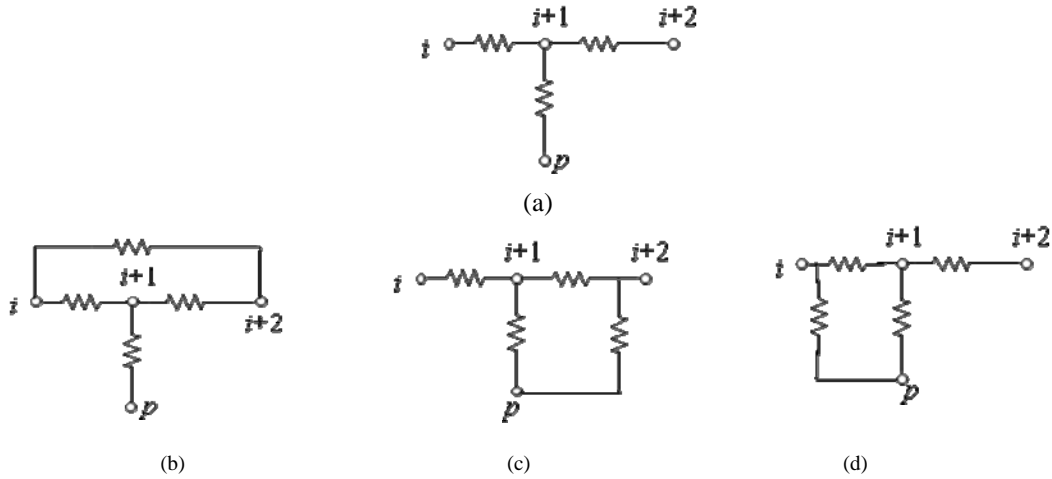


Fig. 6: (a) Star connection, (b) BT-T, (c) BT-R and (d) BT-L

TABLE V  
Portion of the UTM corresponding to bridged-T when  $p < i$

Junction	2	$p$	...	$i-1$	$I$	$i+1$	$i+2$	...	$N$
1						$\infty$			
$\vdots$						$\infty$			
$p$					$\infty, \infty, R_L$	$R_{p,i+1}$	$\infty, R_R, \infty$		
$\vdots$						$\infty$			
$i$						$R_{i,i+1}$	$R_L, \infty, \infty$		
$i+1$							$R_{i+1,i+2}$	$\infty$	$\infty$
$i+2$									
$\vdots$									
$N$									

TABLE VI  
UTM after the bridged-T is converted into an equivalent T

Junction	2	...	$p$	...	$i$	$i+1$	$i+2$	...	$N$
1						$\infty$			
$\vdots$						$\infty$			
$p$					$\infty, \infty, \infty$	$R_{CT}+R_{p,i+1}$ $R_{BR}$ $R_{AL}$	$\infty, \infty, \infty$		
$\vdots$						$\infty$			
$i$						$R_{AT}$ $R_{CR}+R_{p,i+1}$ $R_{BL}$	$\infty, \infty, \infty$		
$i+1$							$R_{BT}$ $R_{AR}$ $R_{CL}+R_{i+1,i+2}$	$\infty$	$\infty$
$i+2$									
$\vdots$									
$N-1$									

where

$$R_{AT} = \frac{R_{i,i+1} \times R_{i,i+2}}{R_{i,i+1} + R_{i,i+2} + R_{i+1,i+2}},$$

$$R_{BT} = \frac{R_{i,i+2} \times R_{i+1,i+2}}{R_{i,i+1} + R_{i,i+2} + R_{i+1,i+2}},$$

$$R_{CT} = \frac{R_{i+1,i+2} \times R_{i,i+1}}{R_{i,i+1} + R_{i,i+2} + R_{i+1,i+2}} \tag{4}$$

$$R_{AR} = \frac{R_{p,i+2} \times R_{p,i+1}}{R_{p,i+1} + R_{p,i+2} + R_{i+1,i+2}},$$

$$R_{BR} = \frac{R_{p,i+2} \times R_{i+1,i+2}}{R_{p,i+1} + R_{p,i+2} + R_{i+1,i+2}},$$

$$R_{CR} = \frac{R_{i+1,i+2} \times R_{p,i+1}}{R_{p,i+1} + R_{p,i+2} + R_{i+1,i+2}} \tag{5}$$

$$R_{AL} = \frac{R_{i,i+1} \times R_{p,i}}{R_{i,i+1} + R_{p,i} + R_{p,i+1}},$$

$$R_{BL} = \frac{R_{p,i} \times R_{p,i+1}}{R_{i,i+1} + R_{i,i+2} + R_{i+1,i+2}},$$

$$R_{CL} = \frac{R_{i+1,i+2} \times R_{p,i+1}}{R_{i,i+1} + R_{i,i+2} + R_{i+1,i+2}} \tag{6}$$

TABLE VII  
UTM of bridged-T network when  $p > i+2$

junction	2	...	$i$	$i + 1$	$i + 2$	...	$p > i + 2$	...	$N$
1				$\infty$					
$\vdots$				$\infty$					
$i$				$R_{i,i+1}$	$R_T$		$R_L$		
$i+1$					$R_{i+1,i+2}$	$\infty$	$R_{i+1,p}$	$\infty$	$\infty$
$i+2$							$R_R$		
$\vdots$									
$N-1$									

TABLE VIII  
UTM after the bridged-T is converted into an equivalent star

Junction	2	...	$i$	$i + 1$	$i + 2$	...	$p > i + 2$	...	$N$
1				$\infty$					
$\vdots$				$\infty, \infty, \infty$					
$i$				$R_{aR}$ $R_{cR} + R_{p,i+1}$ $R_{bL}$	$\infty, \infty, \infty$		$\infty, \infty, \infty$		
$i+1$					$R_{bT}$ $R_{aR}$ $R_{cL} + R_{i+1,i+2}$	$\infty$	$R_{cT} + R_{i+1,p}$ $R_{bR}$ $R_{aL}$	$\infty$	$\infty$
$i+2$							$\infty, \infty, \infty$		
$\vdots$									
$N-1$									

where

$$\begin{aligned}
 R_{aT} &= \frac{R_{i,i+1} \times R_{i,i+2}}{R_{i,i+1} + R_{i,i+2} + R_{i+1,i+2}}, \\
 R_{bT} &= \frac{R_{i,i+2} \times R_{i+1,i+2}}{R_{i,i+1} + R_{i,i+2} + R_{i+1,i+2}}, \\
 R_{cT} &= \frac{R_{i+1,i+2} \times R_{i,i+1}}{R_{i,i+1} + R_{i,i+2} + R_{i+1,i+2}}
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 R_{aR} &= \frac{R_{i+2,p} \times R_{i+1,i+2}}{R_{i+1,p} + R_{i+2,p} + R_{i+1,i+2}}, \\
 R_{bR} &= \frac{R_{i+1,p} \times R_{i+2,p}}{R_{i+1,p} + R_{i+2,p} + R_{i+1,i+2}}, \\
 R_{cR} &= \frac{R_{i+1,i+2} \times R_{i+1,p}}{R_{i+1,p} + R_{i+2,p} + R_{i+1,i+2}}
 \end{aligned} \tag{8}$$

$$\begin{aligned}
 R_{aL} &= \frac{R_{i+1,p} \times R_{i,p}}{R_{i,i+1} + R_{i+1,p} + R_{i,p}}, \\
 R_{bL} &= \frac{R_{i,p} \times R_{i,i+1}}{R_{i,i+1} + R_{i+1,p} + R_{i,p}}, \\
 R_{cL} &= \frac{R_{i,i+1} \times R_{i+1,p}}{R_{i,i+1} + R_{i+1,p} + R_{i,p}}.
 \end{aligned} \tag{9}$$

Similarly the original and the reduced UTMs when  $p > i+2$  will, respectively, be as given in Tables VII and VIII. Alternatively, the new locations of elements can be obtained from  $R_{p,y} = R_{y,p} \Big|_{i+2 \rightarrow i}$ . Note that these changes do not reduce the number of junctions, however, eliminate the bridging resistance. This will be referred to bridged-T to T reduction rule.

**Star to delta reduction**

Star to delta conversion can also be used for reducing the number of junctions. Let there be a star of three resistors  $R_{i,i+1}$ ,  $R_{i+1,i+2}$  and  $R_{i+1,p}$  between the junctions  $i$  and  $i+1$ ,  $i+1$  and  $i+2$ , and  $i+1$  and  $p$ . The equivalent delta resistances are

$$\begin{aligned}
 R_X &= R_{i,i+1} + R_{i+1,i+2} + \frac{R_{i,i+1} \times R_{i+1,i+2}}{R_{i+1,p}}, \\
 R_Y &= R_{i+1,i+2} + R_{i+1,p} + \frac{R_{i+1,i+2} \times R_{i+1,p}}{R_{i,i+1}}, \\
 R_Z &= R_{i+1,p} + R_{i,i+1} + \frac{R_{i+1,p} \times R_{i,i+1}}{R_{i+1,i+2}}
 \end{aligned} \tag{10}$$

An  $n$ -branch star requires an equivalent mesh with  $(1/2)n(n-1)$  sides [5]. Thus, the number of sides in a mesh is minimum when  $n = 3$ . Therefore, converting a star with  $n > 3$  will increase the complexity of circuit rather than reducing it.

**Bridged-T reduction using star to delta transformation**

Consider the bridged-T network depicted in the UTM of Table V. If the star portion of it is replaced by its equivalent delta resistances  $R_X$ ,  $R_Y$  and  $R_Z$ , the new UTM will be as shown in Table IX.

Note that the junction  $i+1$  is eliminated,  $R_X$  got mixed with  $R_{i,i+2}$ ; two being in parallel, but  $R_Y$  and  $R_Z$  appear in the IZ. This means two new bridging elements are added. This will make the method more complex



except when new bridging elements appear across some of the existing bridging elements. Hence star delta transformation is not suitable for reduction purpose. However, when the junctions cannot be arranged as mentioned earlier, some junctions may appear isolated. These isolated junctions may be removed by star delta transformation first and then proceed.

TABLE IX  
UTM after bridged-T is converted into equivalent delta

Junction	2	...	$p$	...	$i$	$i+2$	...		$N$
1									
⋮									
$p$					$R_Z$	$R_Y$			
$p+1$									
⋮									
$i$						$R_X$			
$i+2$						$R_{i,i+2}$			
⋮									
$N-1$									

Consider the network shown in Fig. 7(a) where the resistance is to be determined between the junctions 1 and 8. The junctions are numbered as shown and the bridged-ladder form of the network is drawn in Fig. 7(b). Note that junction 6 is not directly connected to the next junction 7 in serial order. Converting the star at junction 7 into equivalent delta, we get the new bridged-ladder network as shown in Fig. 7(c). Though it has added one additional bridging element between the junctions 3 and 5, all the junctions are now connected.

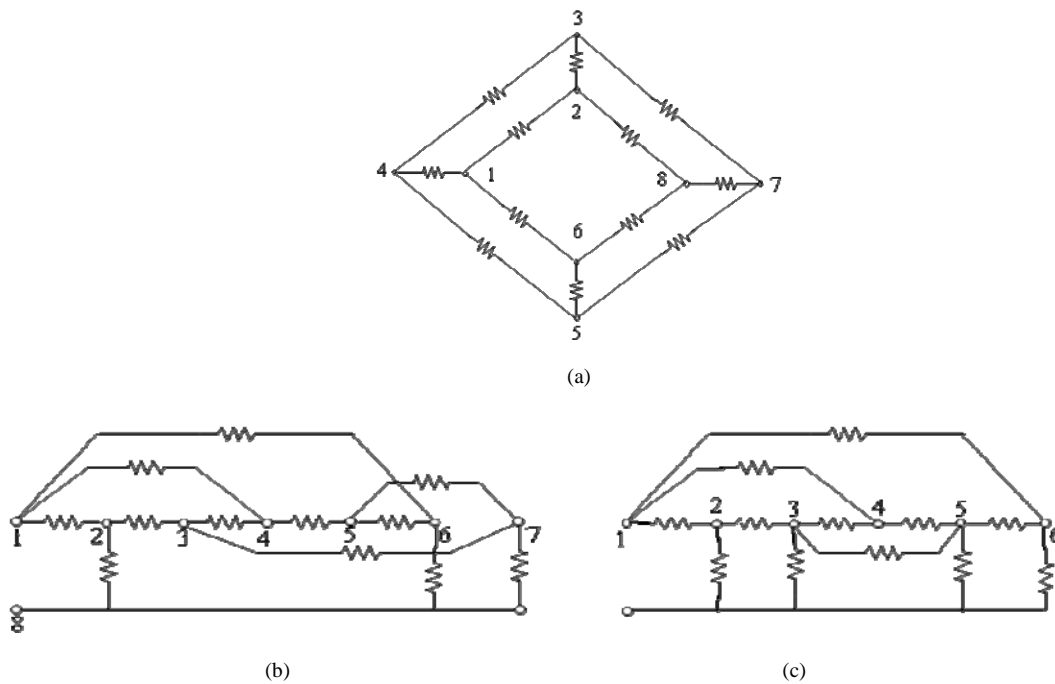


Fig. 7: Network for Example 6, (b) bridged-ladder equivalent of (a), (c) New bridged-ladder

**Procedure for evaluating the DPR**

The following procedure is formulated for evaluating DPR of any 2-terminal network by the proposed matrix reduction method.

1. Number the junctions.
2. Write the complete UTM.
3. Remove all the isolated junctions using star delta transformation.
4. Apply series and parallel reduction rules to reduce UTM.
5. Apply bridged-T to T reduction rule.
6. Apply parallel, series, and/or series parallel reduction rule.
7. Repeat steps 5 and 6 until UTM reduces to [UTM]1.

The procedure will now be illustrated for finding the DPR with two typical examples.

**Example 6:** Consider the circuit shown in Fig. 5(b). Assume all resistance values as 60. Its UTM is

$$[UTM]_6 = \begin{array}{c|cccccc} \text{Junction} & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 60 & \infty & 60 & 60 & \infty & \infty \\ 2 & & 60 & 60 & \infty & \infty & \infty \\ 3 & & & 60 & \infty & \infty & 60 \\ 4 & & & & 60 & 60 & 60 \\ 5 & & & & & 60 & \infty \\ 6 & & & & & & 60 \end{array}$$

There are no parallel and series matrices present in  $[UTM]_6$ , however, there are two BT-T matrices indicated by red color. Applying the BT-T to T reduction rule, we get the UTM as

$$[UTM]_6 = \begin{array}{c|cccccc} \text{Junction} & 2 & 3 & 4 & 5 & 6 & 7 \\ \hline 1 & 60 & \infty & 60 & 80 & \infty & \infty \\ 2 & & 20 & \infty & \infty & \infty & \infty \\ 3 & & & 20 & \infty & \infty & 80 \\ 4 & & & & 20 & \infty & 60 \\ 5 & & & & & 20 & \infty \\ 6 & & & & & & 60 \end{array}$$

There are two series connections (violet). Applying series reduction rule, we get

$$[UTM]_4 = \begin{array}{c|cccc} \text{Junction} & 3 & 4 & 5 & 7 \\ \hline 1 & 80 & 60 & 80 & \infty \\ 3 & & 20 & \infty & 80 \\ 4 & & & 20 & 60 \\ 5 & & & & 80 \end{array}$$

By BT-T (red) BL-R (green) to T reduction rule

$$[UTM]_4 = \begin{array}{c|cccc} \text{Junction} & 3 & 4 & 5 & 7 \\ \hline 1 & 30 & \infty & 90 & \infty \\ 3 & & 7.5 & \infty & 90 \\ 4 & & & 7.5 & \infty \\ 5 & & & & 30 \end{array}$$

By series reduction rule, we get

$$[UTM]_3 = \begin{array}{c|ccc} \text{Junction} & 3 & 5 & 7 \\ \hline 1 & 30 & 90 & \infty \\ 3 & & 15 & 90 \\ 5 & & & 30 \end{array}$$

Applying BT-T to star reduction rule,

$$[UTM]_3 = \begin{array}{c|ccc} \text{Junction} & 3 & 5 & 7 \\ \hline 1 & 20 & \infty & \infty \\ 3 & & 10 & 280/3 \\ 5 & & & 30 \end{array}$$

By series parallel reduction rule,

$$[UTM]_1 = \begin{array}{c|c} \text{Junction} & 7 \\ \hline 1 & 48 \end{array}$$

Thus the DPR of the given network is 48. This is the same obtained by manual calculations.

**Example 7:** Consider the network shown in Fig. 7(a) with all resistance values equal to 60. Its UTM is shown below. Star resistances appear in the row and column corresponding to the isolated junction 7 and shown in black color.

$$[UTM]_7 = \begin{array}{c|ccccccc} \text{Junction} & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ \hline 1 & 60 & \infty & 60 & \infty & 60 & \infty & \infty \\ 2 & & 60 & \infty & \infty & \infty & \infty & 60 \\ 3 & & & 60 & \infty & \infty & 60 & \infty \\ 4 & & & & 60 & \infty & \infty & \infty \\ 5 & & & & & 60 & 60 & \infty \\ 6 & & & & & & \infty & 60 \\ 7 & & & & & & & 60 \end{array}$$

After converting the star around junction 7 into equivalent delta, resulting UTM is

$$[UTM]_6 = \begin{array}{c|cccccc} \text{Junction} & 2 & 3 & 4 & 5 & 6 & 8 \\ \hline 1 & 60 & \infty & 60 & \infty & 60 & \infty \\ 2 & 60 & 60 & \infty & \infty & \infty & 60 \\ 3 & & 60 & 180 & \infty & \infty & 180 \\ 4 & & & 60 & \infty & \infty & \infty \\ 5 & & & & 60 & \infty & 180 \\ 6 & & & & & 60 & 60 \end{array}$$

BT-T and BT-L to T reductions:

$$[UTM]_6 = \begin{array}{c|cccccc} \text{Junction} & 2 & 3 & 4 & 5 & 6 & 8 \\ \hline 1 & 72 & \infty & 72 & \infty & 72 & \infty \\ 2 & & 36 & \infty & \infty & \infty & 36 \\ 3 & & & 36 & \infty & \infty & \infty \\ 4 & & & & 36 & \infty & \infty \\ 5 & & & & & 36 & \infty \\ 6 & & & & & & 36 \end{array}$$

Series reduction:

$$[UTM]_4 = \begin{array}{c|cccc} \text{Junction} & 2 & 4 & 6 & 8 \\ \hline 1 & 72 & 72 & 72 & \infty \\ 2 & & 72 & \infty & 36 \\ 4 & & & 72 & \infty \\ 6 & & & & 36 \end{array}$$

BT-T to T reduction:

$$[UTM]_4 = \begin{array}{c|cccc} \text{Junction} & 2 & 4 & 6 & 8 \\ \hline 1 & 24 & \infty & 72 & \infty \\ 2 & & 24 & \infty & 60 \\ 4 & & & 72 & \infty \\ 6 & & & & 36 \end{array}$$

Series reduction:

$$[UTM]_3 = \begin{array}{c|ccc} \text{Junction} & 2 & 6 & 8 \\ \hline 1 & 24 & 72 & \infty \\ 2 & & 96 & 60 \\ 6 & & & 36 \end{array}$$

BT-T to T reduction:

$$[UTM]_3 = \begin{array}{c|ccc} \text{Junction} & 2 & 6 & 8 \\ \hline 1 & 9 & \infty & \infty \\ 2 & & 36 & 72 \\ 6 & & & 36 \end{array}$$

Series parallel reduction:

$$[UTM]_1 = \begin{array}{c|c} \text{Junction} & 8 \\ \hline 1 & 45 \end{array}$$

Thus  $r_{1,8} = 45$ .

The method is successfully tested with several examples both manually as well as by running a computer program [8].

### III. CONCLUSION

A computerized matrix method has been proposed. It can easily be used for manual calculations. It proceeds step by step in a logical manner to reduce the given network into a single resistance. The method has been successfully tested with several examples both manually as well as by running a computer program.

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Dr T S Rathore was born in Jhabua (M P, India) on Oct. 29, 1943. He received the B Sc (Electrical Engineering), M E (Applied Electronics & Servo-mechanisms), and Ph D (by research on Passive and Active Circuits) degrees in Electrical Engineering from Indore University, Indore, India in 1965, 1970 and 1975, respectively.

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