Solving Maximal Covering Location with Particle Swarm Optimization

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***Abstract*— The use of ambulance location model is significant in determining the best ambulance locations to ensure efficient emergency medical services (EMS) delivery. Maximal Covering Location Problem (MCLP) is one of the most common location models. It is an NP-hard problem and the objective is to maximize the coverage by a fixed number of ambulances. In this study, the demand zones are distributed in a grid based hypothetical region and each zone can host at most one ambulance only. The effectiveness of using Particle Swarm Optimization (PSO) algorithm in finding the best solution for MCLP problem is investigated. The result is compared with the random search technique. It was found that the proposed method manages to identify global optimal solution at a reasonable search time.**

Keyword- Maximal Covering Location Problem, Particle Swarm Optimization

I. INTRODUCTION

Strategic placement of ambulance using a location model is critical to ensure good ambulance response time (ART). Ambulance models can be categorized into deterministic, probabilistic and dynamic [1]. Deterministic model does not consider stochastic nature of EMS.

Strategic placement of ambulance location is very important as it determines how fast an ambulance can arrive at accident scene. Although extra ambulances can be placed to reduce ART, it is not a resource optimized solution. It leads to higher idle time of ambulances and thus underutilizes the resources. MCLP, which is a type of deterministic model, is one of the earliest location models introduced by Church and Revelle [2]. Given a fixed number of facilities, MCLP model is used to save resources by seeking the maximization of total coverage. A demand point is considered covered if it is within the defined time distance to any of the available facilities, while demand points that are farther than the time distance is considered as not covered. This model is useful for the decision manager with limited resources to allocate facilities efficiently.

MCLP has been successful implemented in reducing ART and facility cost. For instance, in 1984 in Austin, Texas, the reorganization of EMS using MCLP has resulted in a saving of $3.4 million of construction cost and $1.2 million of operating cost annually [3].

Since its introduction, various extensions and modifications to the MCLP has been made by several researchers. Davari *et al.* [4] consider the travel times between any nodes as fuzzy variable and use simulated annealing (SA) to obtain the optimal solution. Tandem equipment location model (TEAM) and facility-location, equipment-emplacement technique (FLEET) are both derived from MCLP [1]. MCLP has also been used for solving real life problems in hierarchical health system design [5, 6], congested service systems [7] and bus stops allocation [8].

MCLP is a NP-hard problem. Various approaches, such as exact method, heuristic and meta-heuristic methods can be used to solve a MCLP problem. Exact method can guarantee on getting the most optimal solution but may take longer time to solve, especially for a large problem. On the other hand, meta-heuristic method does not guarantee on getting the most optimal solution, though a near optimal solution for large problem can be achieved with a reasonable time. Some literature has reported solving the MCLP or its extension using linear programming and heuristic methods such as Greedy Adding (GAd) algorithm [2], tabu search [9, 10], Lagrangean and Surrogate Relaxations [11], Myopic or Greedy heuristic [12], Heuristic Concentration [13] and Genetic Algorithm (GA) [14]. In this paper, we present the work of solving the MCLP problem using a conventional PSO.

In the next section, some basic on PSO algorithm is briefly introduced. In Section III, the formulation of fitness function based on MCLP problem is explained. Section IV explains the PSO implementation. The result and discussion are given in Section V. Lastly, Section VI concludes the findings.

II. Literature Review

PSO is a metaheuristic population based search technique proposed by Eberhart and Kennedy in 1995 from the observation on swarm of bird flocking and fish schooling [15]. The swarm consists of particles that fly in a search space to find potential solutions. Each particle in PSO changes its position based on its own and other particles experience. It stores its best position in memory, which is known as *pbest*. The best among *pbest* of particle is stored into a global best memory, which is called *gbest*. Both *pbest* and *gbest* are updated through fitness function that is defined according to the objective of the problem to be solved. Conventional PSO for global best model can be mathematically expressed as:

$$s_i^{k+1} = s_i^k + v_i^{k+1} \tag{1}$$

$$v_i^{k+1} = wv_i^k + c_1 r_1 \left(pbest_i - s_i^k\right) + c_2 r_2 \left(gbest - s_i^k\right) \tag{2}$$

where (1) is the equation for calculating the particle position and (2) is the equation for particle velocity. v and s denote the particle's velocity and position respectively. i is the particle number and k is the number of current iteration. $pbest_i$ is the best solution for the *i-th* particle while *gbest* is the best solution among all *pbest*. w, c_1 and c_2 are the weight factors. r_1 and r_2 are random number [0, 1].

The inertia weight, w is initially set to a high value to facilitate global exploration. Through iterations, w is gradually decreased to encourage local exploration, where the range used is commonly within [0.9, 0.4]. This technique is first proposed by Eberhart [16]. The values of w for each iteration can be calculated by using the following equation:

$$w_k = w_{max} - \left(\frac{w_{max} - w_{min}}{k_{max}} \times k\right) \tag{3}$$

where w_{max} and w_{min} are the limiting values determined by the user.

The velocity of each particle is updated according to (2) in each iteration. On the right-hand side of (2), there are three terms. The first term indicates the way inertia weight affects the velocity of each particle; the second term on how respective personal best affects the particle's velocity; and the last term describes the effect of global best. The updated velocity from (2) is then used to update the particle's position in (1). As the number of iteration increases, the particles move within the search space from an initially randomized position toward new *pbest* and *gbest*. The iteration is repeated until the particles converge toward *gbest*. The pseudo-code of PSO is as following:

```
Initialize all particles
Evaluate fitness for all particles
while stopping condition == false
    for each particle
        if fitness better than pbest
            pbest = fitness
    end for
    gbest = the best pbest
    for each particle
        update velocity using (2)
        update position using (1)
    end for
end while
```

PSO has been successfully used to solve various NP-complete problems [17]. There are many advantages of using PSO over other optimization algorithms such as GA and SA. These include having fewer numbers of parameters and higher convergence rate compared to GA [18]. Besides, PSO also uses less computational power compared to GA and SA in solving parameter estimation problems [19]. Nevertheless, one disadvantage of PSO is that it may easily get trapped at local optima [18].

Some facility location problems have been solved by using PSO. Ghaderi *et al.* [18] use Hybrid PSO to solve Continuous Uncapacitated Facility Location Problem. Yapicioglu *et al.* [21] solved semi-desirable facility location problem using bi-objective PSO. Sevkli and Guner [22] use PSO with local search to solve

Uncapacitated Facility Location Problem. In this paper, the effectiveness of PSO algorithm in solving MCLP problem is investigated.

III. PROBLEM FORMULATION

In this paper, we consider only one fitness function that aims to maximize the total demand coverage by ambulances. MCLP model is chosen to maximize the total demand coverage. It can be expressed as:

$$Maximize \sum_{i \in V} d_i y_i \tag{4}$$

$$Subject\ to \sum_{j \in W_i} x_j \geq y_i \quad i \in V \tag{5}$$

$$\sum_{i \in W_i} x_j = p \tag{6}$$

$$x_j \epsilon \{0,1\} \quad j \epsilon W \tag{7}$$

$$\mathrm{y_j} \in \{0,1\} \quad \mathrm{i} \in \mathrm{V} \tag{8}$$

where V and W denote the demand nodes and potential ambulance location sites respectively. d_i represents the demand at point i. y_i is a binary variable that will be 1 if and only if point i is covered by any ambulance within a distance D. x_j is a binary variable that will be 1 if there is an ambulance allocated at site j. p is the number of ambulances to be located.

IV. PSO MODELLING

A hypothetical region is created for the simulation. The demand zones are divided by using grids and the demand points are randomly generated using random number generator. In the problem, each demand point can host at most one ambulance only. Each solution of MCLP is represented by a particle in PSO. In each particle there are n vectors, where each vector represents an ambulance location in the demand space. For example, if a 10 dimensional particle is used for 115 potential ambulance locations, there would be ${}^{115}C_{10} = 7.4 \times 10^{13}$ possible combinations to place the ambulances. Therefore, it is nearly impossible to solve the problem with exact method.

Each ambulance location is given an integer number. A potential solution is represented by a combination of different ambulance locations. This can be modeled in particle's position as shown in (9).

$$s = [1^{st} \text{ ambulance location}, 2^{nd} \text{ ambulance location}, \ldots, n^{th} \text{ ambulance location}] \tag{9}$$

where n is the number of ambulance locations. In the following example, each number denotes an ambulance location in the demand space.

s = [4, 21, 81, 25, 82, 101, 16, 38, 77, 93]

Ten ambulances are used in the simulation to find the best locations. Distance used for an ambulance to cover a demand node is set to 2.5 point. The inertia weight, w used is a decreasing weight from 0.9 to 0.4. We limit to 3000 iterations in the simulation, as our initial finding shows that using higher number of iterations does not provide any significant improvement in the obtained result.

Some constraints are applied to the original PSO formulation. As we are using 115 possible ambulance locations, the velocity of each vector in a particle is capped at +/- 115. Also, the range of 1 to 115 is set for each vector of particle's position.

V. RESULTS AND DISCUSSION

An Intel core-i5 iMac running on Mac OSX Lion is used to host the simulation program written in objective-c. Each set of different PSO parameters as listed in Table 1 are simulated for 10 times to get an averaged result. Fig. 1 shows the best solution obtained using PSO algorithm. Note that PSO has the disadvantage of easily get trapped at local optima [18]. If that happens, the solution cannot get any improvement even with further iterations. Two such examples are shown in Figs. 2 and 3. We found that the best PSO result can be obtained by using c_1 and c_2 equal to 1.9 as compared to other values. Meanwhile, there is no noticeable difference in the obtained results by increasing the number of particles to 100.

TABLE III
Results of PSO and Random Search

Algorithm	Parameters					Result				Gap with global optima (%)
	iteration	particles	w	C_1	C_2	Best (%)	Mean (%)	Worst (%)	Time (s)	
PSO	3000	50	0.9-0.4	1.9	1.9	92.52	89.93	87.56	18	0
	3000	50	0.9-0.4	1.6	1.6	91.69	88.74	86.83	18	0.83
	3000	50	0.9-0.4	1.4	1.4	89.88	87.29	84.37	18	2.64
	3000	100	0.9-0.4	1.9	1.9	92.52	90.35	87.08	41	0
Random Search	300000	-	-	-	-	75.44	71.44	67.99	0.161	17.08

For comparison purpose, we tested the performance using the random search algorithm with 300000 iterations. The results are listed in Table 1. As can be seen, the best result from random search is only 75.44%, which is much lower than the best PSO result of 92.52%. However, the random search method requires the least processing time.

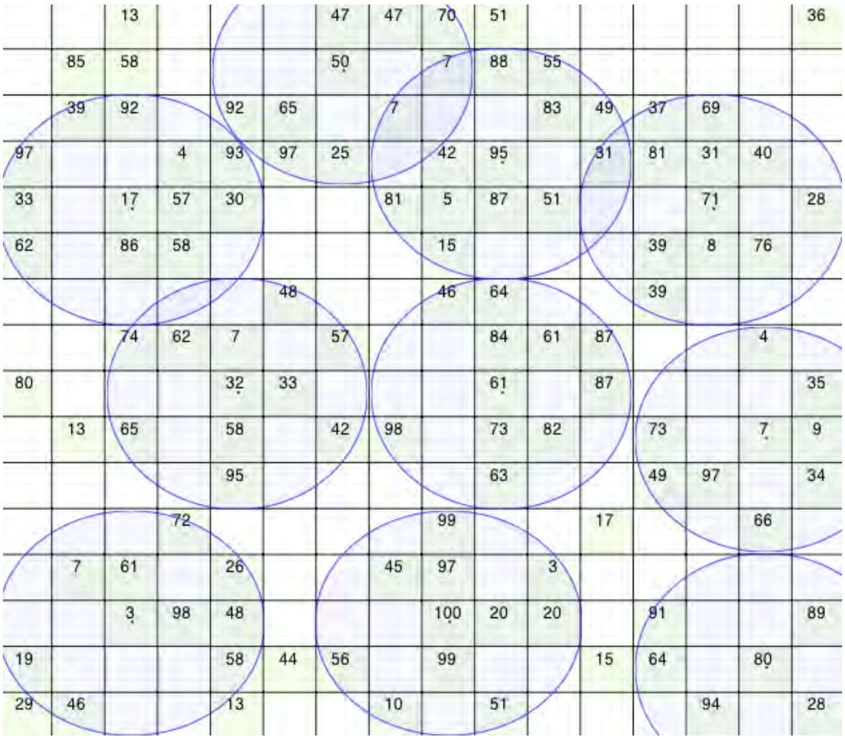

Fig. 1. Local optima, area covered = 92.52%.

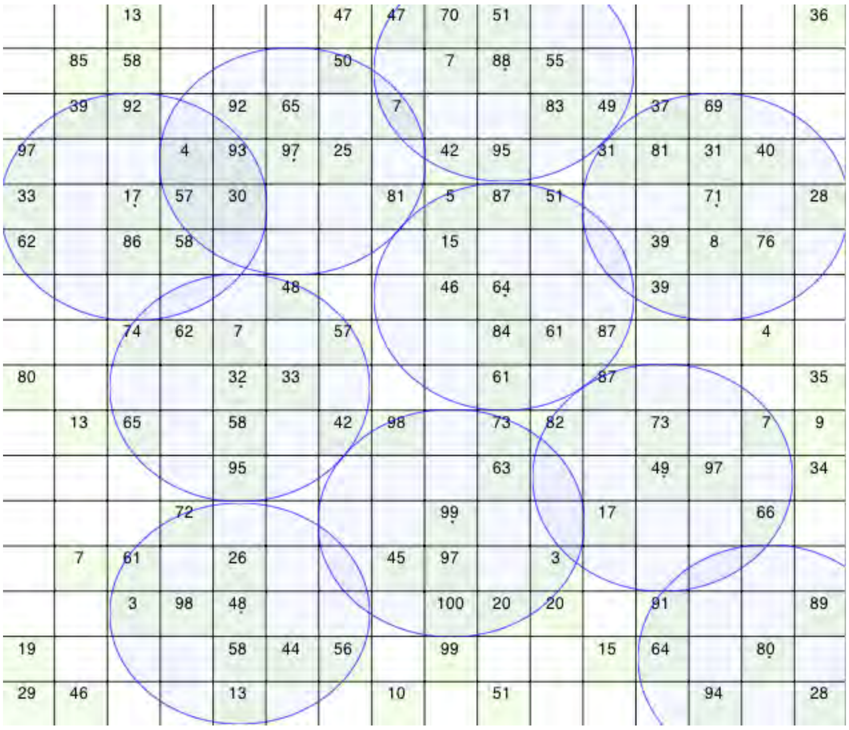

Fig. 2. Local optima, area covered = 88.30%.

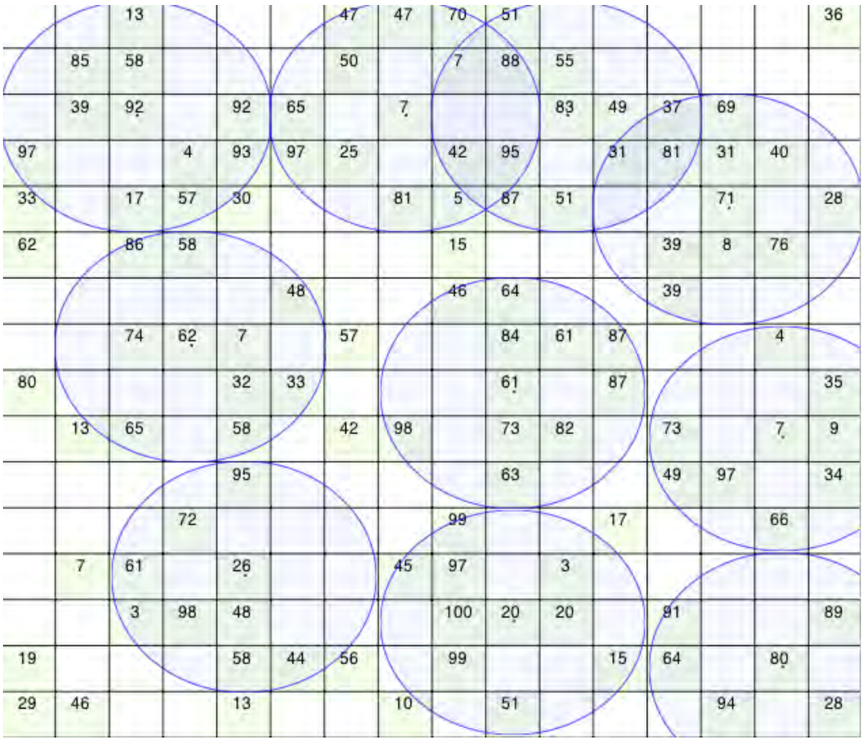

Fig. 3. Local optima, area covered = 90.62%.

VI. Conclusion

In this paper, PSO algorithm has been successfully applied to solve MCLP. It can produce a good result at a reasonable search time. Further work can be carried out to implement a modified or hybrid PSO that can avoid trapping in a local optima.

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