Blind Identification of Transmission Channel with the Method of Higher-Order Cummulants

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Abstract-The modern telecommunication systems require very high transmission rates, in this context, the problem of channels identification is a challenge major. The use of blind techniques is a great interest to have the best compromise between a suitable bit rate and quality of the information retrieved.

In this paper we are interested to learn the algorithms for blind channel identification.

Keywords— *channel, telecommunication systems, blind identification and equalization, higher order cumulants, moments, rls.*

1. Introduction

The actual progress in resolving systems became more important in the telecommunication systems, especially the blind identification channel, including the requirement of modern telecommunications systems that seek to use very high transmission rates. In this context, the application of the method of higher-order cumulants is a technique now commonly addressed by digital telecommunication systems. The use of blind identification has a great interest to have a good estimate of the channel, and therefore a good quality of information retrieved.

In this work we develop algorithms for blind identification based on four order cumulants [1]. The objective is to make a comparative study of these algorithms, in order to have a better channel estimation by noisy white Gaussian noise.

2. Problem statement

The channel is modeled by a FIR filter whose impulse responses are H (t) according to the diagram of Figure 1.

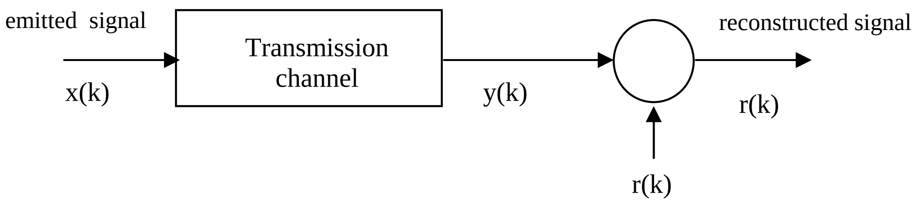

Figure 1. Channel model

x(t) : represents the sample of the input signal to the channel.

y(t) : the response of the channel.

The problem is to determine h(t) from a statistical analysis of y(t) (the channel response) received no information about the input signal x(t).

Higher order cumulants or equal to 3 for a Gaussian signal is zero, which justifies the use of statistical analysis using higher order cumulants.

Often we take samples of finished size to reduce the execution time; however, the distribution is far from Gaussian, where the higher-order cumulants are different from zero. Hybridization between algorithms reduces this error to the size of the sample (see Figure 2.)

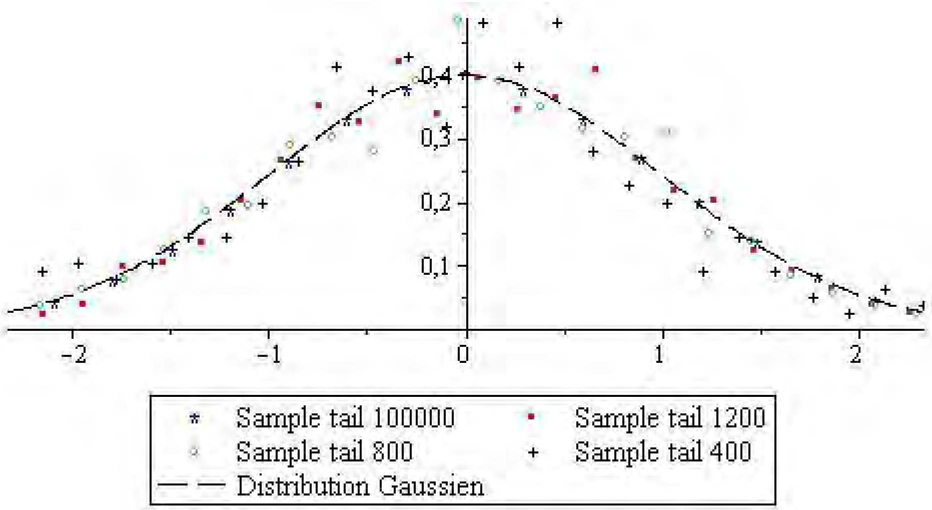

Figure 2. Different samples of different sizes around the Gaussian distribution

According to the figure, note that the sample of the large size (100 000, blue) coincides perfectly with the Gaussian distribution. In contrast, other distributions are far away.

3. Algorithm Based Cumulants

We would like to mention in passing that there are several models that are classified into several categories. In this work, structure of the model is generally used single variable, discrete time, invariant in time.

3.1.Moment and Cumulant

In this section we present some definitions of higher order statistics, moments and cumulants.

Let {x(k), avec k∈ ℤ}, is a real process discrete stationary, so its moment of order m is given by [4] [5] [6] [7] [2]:

$$M_{m,x}(t_1, t_2, \ldots, t_{m-1}) = E\{x(k)x(k + t_1)x(k + t_2) \ldots x(k + t_{m-1})\} \quad (1)$$

With E{.} represents the mathematical expectation.

The cumulant of order n of a non-Gaussian stationary process is given by:

$$C_{m,x}(t_1, t_2, \ldots, t_{m-1}) = M_{m,x}(t_1, t_2, \ldots, t_{m-1}) - M_{m,G}(t_1, t_2, \ldots, t_{m-1}) \quad (2)$$

With $M_{m,x}(t_1, t_2, \ldots, t_{m-1})$ represents the moment of order m and $M_{m,G}(t_1, t_2, \ldots, t_{m-1})$ is the time of a signal equivalent Gaussian which has the same function as the autocorrelation signal $\{x(k)\}$

3.2.Estimating moment and cumulants

3.2.1.Estimating moment

Let $X = \{x_i\}_{i=1,\ldots,k}$ a random variable representing scalar centered N samples of a stationary signal.

The simplest estimator of order k appointed conventional estimator is given by:

$$m_{k,x}(t_1, t_2, \ldots, t_{k-1}) = \frac{1}{N}\sum_{i=1}^{N} x(i)x(i + t_1)x(i + t_2) \ldots x(i + t_{k-1}) \quad (3)$$

3.2.2.Estimating cumulants

A detailed presentation of the theory of cumulants estimation can be found in [9], [4]. As cumulants are expressed in terms of moments, the estimates of cumulants are obtained as follows:

$$\hat{C}_{2,x}(t_1) = \hat{C}_2(t_1) = m_2(t_1)$$

$$\hat{C}_{3,x}(t_1, t_2) = m_3(t_1, t_2)$$

$$\hat{C}_4(t_1, t_2, t_4) = m_4(t_1, t_2, t_4) - m_2(t_1)m_2(t_2 - t_3) - m_2(t_2)m_2(t_1 - t_3) - m_2(t_3)m_2(t_1 - t_2) \quad (4)$$

Algorithm based on four order cumulant

Generally, most of the methods use different order cumulants, in this work we develop algorithms based on a single order cumulants.

4.1 Algorithm Based on 3th Order Cumulant: Alg. 1

The m order cumulant of the system output may be expressed as a function of the coefficients of the impulse response {h (i)} [4] by:

$$C_{my}(t_1, \dots, t_{m-1}) = \gamma_{mx} \sum_{i=-\infty}^{+\infty} h(i)h(i+t_1) \dots h(i+t_{m-1}) \quad (5)$$

With γ_{mx} is the order cumulant m behind the input sequence.

And q is the number of channels

If m= 4, equation (5) becomes:

$$C_{4y}(t_1, t_2, t_3) = \gamma_{4x} \sum_{i=0}^{q} h(i)h(i+t_1)h(i+t_3) \quad (6)$$

Similarly, if m = 2, the equation (5) becomes:

$$C_{2y}(t_1) = \gamma_{2x} \sum_{i=0}^{q} h(i)h(i+t_1) \quad (7)$$

Applying the Fourier transform of (6) and (7) we obtain:

$$S_{2y}(\omega) = \sigma^2 H(\omega)H(-\omega) \quad (8)$$

$$S_{4y}(\omega_1, \omega_2, \omega_3) = \gamma_{4x} H(\omega_1)H(\omega_2)H(\omega_3)H(-\omega_1 - \omega_2 - \omega_3) \quad (9)$$

By applying the inverse Fourier transform [1] we have:

$$\sum_{i=0}^{q} h(i)C_{4y}(t_1 - j, t_2 - j, t_3 - j) = \epsilon \sum_{i=0}^{q} h(i)h(i + t_2 - t_1)\, h(t_3 - t_1)C_{2y}(t_1 - i) \quad (10)$$

With $\epsilon = \frac{\gamma_{4e}}{\gamma_{2e}}$

For $t_3 = t_2$ and $t_1 = 2q$(with h (0) =1)

The autocorrelation function of a system to FIR vanishes for all values as: |t|>q

As the system is supposed causal (h (i) =0 for i<0 and i>q).

L'équation (10) devient :

$$\sum_{i=0}^{q} h(i)C_{4y}(2q - j, t_2 - j, 2q - j) = \epsilon h(q)h(t_2 - q)C_{2y}(q) \quad (11)$$

The choice of t_2 requires that ($t_2 > 2q$) then $q \le t_2 \le 2q$, and for $t_1 = t_2 = -q$ in equation (10).

And if we use the cumulants proprieties [4] [11]. We have:

$$\epsilon = \frac{C_{4y}(q,0,0)}{C_{2y}(q)} \quad (12)$$

So we can present the system in matrix form:

$$\begin{pmatrix} C_{4y}(2q-1, 2q-1, q-1) & \cdots & C_{4y}(q,q,0) \\ C_{4y}(2q-1, 2q-1, q) - \epsilon' & \cdots & C_{4y}(q,q,1) \\ \vdots & \ddots & \vdots \\ 0 & \cdots & C_{4y}(q,q,0) - \epsilon' \end{pmatrix} \begin{pmatrix} h(1) \\ \vdots \\ h(q) \end{pmatrix} = \begin{pmatrix} \epsilon' - C_{4y}(2q,2q,q) \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad (13)$$

With $\epsilon' = \frac{C_{4y}(q,q,q)C_{4y}(q,0,0)}{C_{4y}(q,q,0)}$

We can also write the system in the following form:

$$Mh_q = d \quad (14)$$

The resolution of the system in the sense of least squares is given by:

$$h_q = (M^T M)^{-1} M^T d \quad (15)$$

4.2 Algorithm Based on 4th Order Cumulants: Alg. 2

From the Eq. (3), the m[th] and n[th] cumulants of the output signal, {y(n)}, and the coefficients {h(i)}, where n > m, are linked by the following relationship:

$$\sum_{j=0}^{p} h(j)C_{ny}(j+t_1,\dots,j+t_{m-1},t_m,\dots.t_{n-1} = \frac{\gamma ne}{\gamma me}\sum_{i=0}^{p} h(i)\left[\prod_{k=m}^{n-1} h(i+t_k)C_{my}(i+t_1,\dots,i+t_{m-1}\right] \quad (16)$$

If we take n = 4 and m = 3 into Eq. (16), we find the basic relationship developed in [12], [13]. If we take n = 3 and m = 2 into Eq. (16), we find the basic relationship of the algorithms developed in [14] as follows:

$$\sum_{j=0}^{q} h(j)[\prod_{k=1}^{r} h(j+t_k)]C_{ny}(\beta_1,\dots,\beta_r,j+\alpha_1,\dots,\alpha_{n-r-1} = \sum_{i=0}^{q} h(i)[\prod_{k=1}^{r} h(i+\beta_k]\, C_{ny}(t_1,\dots,t_r,i+\alpha_1,\dots,\alpha_{n-r-1} \quad (17)$$

Where $1 \leq r \leq n-2$.

From the equation (17), and for n=4 we obtain the relationship :

$$\sum_{i=0}^{q} h(i)h(i+t_1)\, h(i+t_2)C_{4y}(\beta_1,\beta_2,i+\alpha_1) = \sum_{j=0}^{q} h(j)h(j+\beta_1)h(j+\beta_2C_{4y}(t_1,t_2+\alpha_1) \quad (18)$$

If $t_1 = t_2 = q$ and $\beta_1 = \beta_2 = 0$ the relationship (18) becomes:

$$h(0)h^2(q)C_{4y}(0,0,i+\alpha_1) = \sum_{j=0}^{q} h^3(j)\, C_{4y}(q,q,j+\alpha_1) \quad (19)$$

As the system, RIF is supposed causal with order q, so the $j+\alpha_1$ will necessarily be in the interval [0, q]. So, to determine the range of variation of the parameter α_1 *is* proceeded as follows:

We have $0 \leq j+\alpha_1 \leq q \;\; ==> -i \leq \alpha_1 \leq q-j$.

In other hand we have: $0 \leq j \leq q$

From the tow last inequalities we obtain:

$$-q \leq \alpha_1 \leq q$$

If we suppose h(0)=1 h(p) 6= 0 and the cumulant $C_{my}(t_1,\dots,t_{m-1}) = 0$, if one of the variables $t_k > p$, where k = 1, . . . ,m−1; the system of Eq. (19) will be written as follows:

$$\begin{pmatrix} 0 & \cdots & 0 & C_{4y}(q,q,0) \\ \vdots & & & \vdots \\ 0 & \ddots & & C_{4y}(q,q,q) \\ C_{4y}(q,q,0) & \cdots & & 0 \\ \vdots & \ddots & & \vdots \\ C_{4y}(q,q,q) & \cdots & & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{h^2(q)} \\ \vdots \\ \frac{h^3(i)}{h^2(q)} \\ \vdots \\ \frac{h^3(q)}{h^2(q)} \end{pmatrix} = \begin{pmatrix} C_{4y}(0,0,-q) \\ \vdots \\ C_{4y}(0,0,0) \\ \vdots \\ C_{4y}(0,0,q) \end{pmatrix} \quad (20)$$

We can also write the system in the following form:

$$Mh_q = d \quad (21)$$

The resolution of the system in the sense of least squares is given by:

$$h_q = (M^T M)^{-1} M^T d \quad (22)$$

This solution give us an estimation of the quotient of the parameters $h_3(i)$ and $h_3(q)$, i.e., $b_{p_2}(i) = \left[\widehat{\frac{h_3(i)}{h_3(q)}}\right]$, i=1…q.

So, in order to obtain an estimation of the parameters $\hat{h}(i)$, i = 1… p, we can use the following equation:

$$\hat{h}(i)=\text{sign}\left[\hat{b}_{p_2}(i) \quad \hat{b}_{p_2}(i)^2\right]\left\{abs(\left[\hat{b}_{p_2}(i) \quad \hat{b}_{p_2}(i)^2\right])\right\}^{1/3} \quad (23)$$

Where :$sign(x) = \begin{Bmatrix} 1 & if & x > 0 \\ 0 & if & x = 0 \\ -1 & if & x < 0 \end{Bmatrix}$

And $abs(x) = |x|$ means the absolute value of x.

The $\hat{h}(p)$ parameters is estimated as follows :

$$\hat{h}(p) = \frac{1}{2} sign[\hat{b}_{p_2}(p)] \left\{ abs\,(\hat{b}_{p_2}(p)) \quad \left(\frac{1}{\hat{b}_{p_2}(1)}\right)^{1/2} \right\} \tag{24}$$

4.3 RLS algorithm

The objective of this algorithm is the estimation of h parameters of the adaptive filter by using the criterion least squares:

$$\varepsilon(n) = \sum_{i=0}^{n} \lambda^{n-i} e(i) = \sum_{i=0}^{n} \lambda^{n-i} \big(d(i) - h^T(n).x(i)\big) \tag{25}$$

$$\sum_{i=0}^{n} \lambda^{n-i} \big(d(i) - h^T(n).x(i)\big) = 0 \tag{26}$$

λ is a weighting factor that always takes a positive value $0 < \lambda < 1$.

This factor is also called forgetting factor because it forget the data corresponding to a remote past. The special case λ = 1 corresponds to an infinite memory. Minimizing the cost function ε (n) allows us to determine the coefficients h (n), this amounts to calculate the partial derivative relative to h (n). Finally the following expression for w (n) is obtained (Bellanger 1989):

$$h(n) = h(n-1) + \frac{\lambda^{-1} R^{-1}(n-1) x(n)}{1 + \lambda^{-1} x^T(n) R^{-1}(n-1) x(n)} e(n) \tag{27}$$

with

$$e(n) = d(n) - x^T(n) h(n-1) \tag{28}$$

4.4. Zhang algorithm

Using equation (3), Zhang and Al. [10] developed an equation based on the cumulants of order m given by:

$$\sum_{i=0}^{q} h(i) C_{ny}^{n-1}(i-t, q, \dots, 0) = C_{ny}(t, 0, \dots, 0) C_{ny}^{n-3}(q, 0, \dots, 0) C_{ny}(q, q, \dots, 0) \tag{29}$$

For n=4 we obtain from equation (16) the following equation:

$$\sum_{i=0}^{q} h(i) C_{4y}^{3}(i-t, q, 0) = C_{4y}(t, 0,0) C_{4y}(q, 0,0) C_{4y}(q, q, 0) \tag{30}$$

For t=-q, -q+1, …, q

The system can be represented in the following matrix form:

$$\begin{pmatrix} C_{4y}^3(i+q, q, 0) & \cdots & C_{4y}^3(2q, q, 0) \\ C_{4y}^3(i+q-1, q, 0) & & C_{4y}^3(2q-1, q, 0) \\ \vdots & \ddots & \vdots \\ C_{4y}^3(i-q, q, 0) & \cdots & C_{4y}^3(0, q, 0) - \epsilon' \end{pmatrix} \begin{pmatrix} h(0) \\ \vdots \\ h(q) \end{pmatrix} = \epsilon \begin{pmatrix} C_{4y}(-q, 0,0) \\ C_{4y}(-q+1,0,0) \\ \vdots \\ C_{4y}(q, 0,0) \end{pmatrix} \tag{31}$$

With $\epsilon = C_{4y}(q, 0,0) C_{4y}(q, q, 0)$.

The system can be represented in a simplified form as follows:

$$Mzh_q = d \tag{32}$$

To estimate the parameters $h(i)_{i=1,\dots,q}$ we can use the method of least squares:

$$h_q = (Mz^T Mz)^{-1} Mz^T d \tag{33}$$

5. Simulation

In this section we will make a comparative study between five algorithms(Alg1, Alg2, Zhang, Zhang_Alg1, RLS), the Zhang_Alg1 is an hybridation between tow algorithms previously cited (ALG1, Zhang) [1], we showed that the ALG2 algorithm is better than other algorithms in terms of wandering (MSE), which implies, of course, a good estimate of the channel.

5.1. Order channel 2

To validate this algorithm, we applied it to different channels of a different order; remember that this algorithm is based on cumulants of order 4 that means that the Gaussian noise vanishes at cost on.

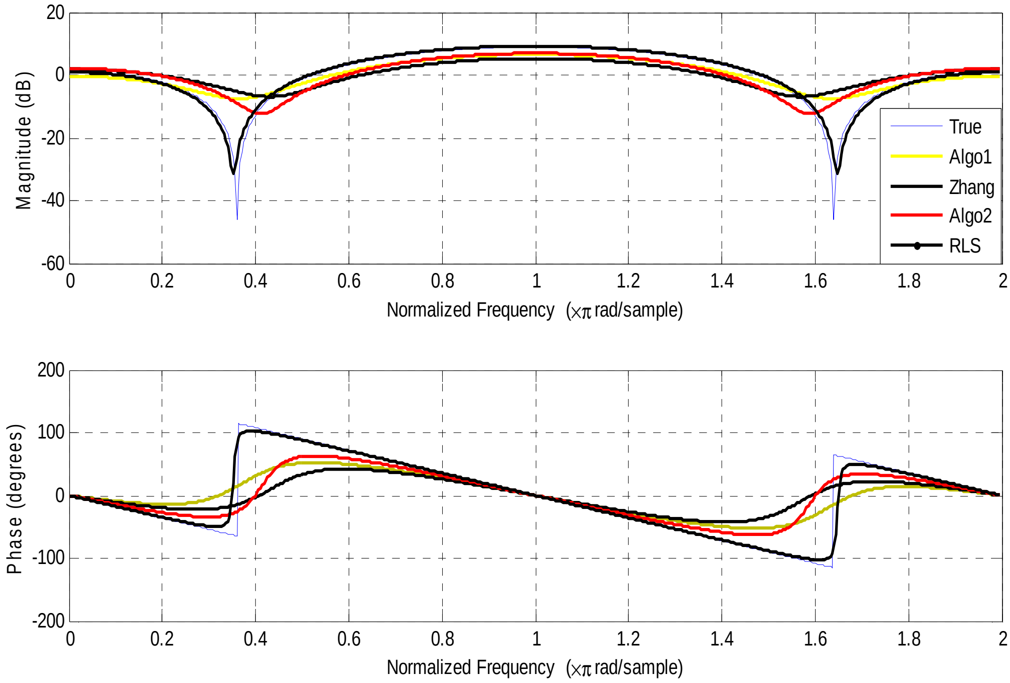

Figure 3. Estimation of the impulse responses in amplitude and phase algorithms for N=100 samples.

According to Figure 3, it's obvious that the shape of the channel estimated using RLS algorithm is nearer to the look of the desired channel, because it's a supervised algorithm, nevertheless, the algorithm Alg2 is better than other algorithms.

Table 1. Channel estimation by the different algorithms

N	Algorithme	EQM	Canal estimé		
	Canal désiré	---------	1.0000	-0.8500	1.0000
100	Alg1	0.1970	1.0000	-0.6724	0.6027
	Alg2	0.1592	1.0000	-0.4525	0.7793
	Zhang	0.2064	1.0000	-0.4229	0.3920
	RLS	0.0136	1.0426	-0.8268	0.8449
400	Alg1	0.0494	1.0000	-0.7542	0.8129
	Alg2	0.0151	1.0000	-0.4898	0.6791
	Zhang	0.1138	1.0000	-0.4516	0.6982
	RLS	0.0038	0.9964	-0.8397	0.9948
800	Alg1	0.0102	1.0000	-0.8080	0.8830
	Alg2	0.0046	1.0000	-0.6171	0.8231
	Zhang	0.0422	1.0000	-0.8279	0.9807
	RLS	1.3372e-04	1.0019	-0.8466	1.0003
1200	Alg1	0.0077	1.0000	-0.7894	0.8903
	Alg2	0.0024	1.0000	-0.7610	0.9401
	Zhang	0.0272	1.0000	-0.6896	0.9819
	RLS	8.2613e-04	0.9985	-0.8486	1.0007

Table 1 groups the estimated values of the channel of the five algorithms, for different sample sizes, i.e. N = 100, N = 400, N = 800 and N = 1200.

According to the comparative table, we note that the algorithm ALG2 gives a good estimation of the parameters of the studied channel, other algorithms give good results, but with a larger error compared to algorithm ALG2, this last gives better performance for different sample sizes.

We note that the mean squared error decreases when the sample size increases, then we get an error of order 10-4 with the proposed algorithm.

6. Conclusion

In this paper, we have cited the characteristics of higher order cumulants with their properties that has a great interest in the field of signal processing with and without noise. The use of higher-order cumulants gives more detailed analysis of signals; the success of these techniques is that they can solve the problem of blind identification without any information about the input RIF systems.

In this paper we made a comparative study between four algorithms, we have shown the algorithm ALG2 is better than other algorithms in terms of the mean square error, as it gives a good estimation of system parameters RIF for channel order two and for different sample sizes.

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