Improved Debye Model for Experimental Approximation of Human Breast Tissue Properties at 6 GHz Ultra-Wideband Centre Frequency

Ikram-e-Khuda[#1], Sabira Khatun*[#2], Khondker Jahid Reza[#3], Md. Mijanur Rahman[#4], Md. Moslemuddin Fakir [#5]

[#] School of Computer and Communication Engineering, Universiti Malaysia Perlis
Pauh Putra Main Campus, 02600 Arau, Perlis, Malaysia.

[1] ikramekhuda@gmail.com

[2] sabira@unimap.edu.my

[3]jahid_rifat@yahoo.com

[4]mijanur@unimap.edu.my

[5]shahrulnizam@unimap.edu.my

[##]Institute of Engineering Mathematics, Universiti Malaysia Perlis (UniMAP),

Pauh Putra Main Campus, 02600 Arau, Perlis, Malaysia.

[6]moslem@unimap.edu.my

Abstract—This paper investigates complexity of breast tissue structure and mathematical equations to describe their dielectric properties. Experimental breast tissue dielectric properties are compared to theoretical results from mathematical postulates to find their gaps. Polynomial fitting is then used to propose an analytical model (viz. improved first order relaxation Debye model) to bring theoretical results as close as possible to experimental results. The proposed model is able to bridge the gap efficiently by reducing 5.9 % average error. This will help researchers to consider actual tissue properties to develop real-like breast phantom for early breast cancer detection based researches. This would aid in faster experimental / clinical research output and practical-implementation which would save precious human life.

Keyword-Dielectric properties, breast tissue, phantom, breast model, polynomial fitting, linear regression analysis.

I. Introduction

Biological tissues are multifarious blend of various materials and quantities such as water, ions, membranes and macromolecules with a broad variety of profiles. Human breast is a complex biological tissue. Fig. 1 shows an image of breast with associated tissues, such as glandular, adipose and fibrous tissues etc [1].

Adipose tissues are the fatty tissues of the breast. The glandular tissue is usually dense and lumpy because of the presence of mammary glands, lobules and ducts. Fibrous tissues connect the adipose and glandular tissues and also balance the density of the breast. Breast cancer is a prominent disease among women worldwide. It usually crops up in the mammary ducts or in the tiny lobules. Initially, the presence of cancer is called tumor and it adds up another layer of tissue within the breast.

Each tissue type has their respective dielectric properties which are used by the biomedical engineers to make the artificial models or phantoms. Usually the tissue dielectric properties are: conductivity (S/m), permittivity (F/m) and permeability (H/m). Since tissues are non-magnetic, therefore, their permeability is assumed to be the same as that in free space [2], which is negligible. So, researchers use permittivity and conductivity to make breast phantoms, which mimic the behavior of breast. Phantoms are used to investigate electric field response through it by means of transmitted and received signals. So, the behavior of actual tissues in static and time dependent electric fields needs to be known clearly. Hence, phantoms must be developed with correct dielectric properties.

Debye [3] and Cole-Cole models [4-5] are considered to be the basis for modeling dielectric properties of human breast for making breast phantoms [6-15]. Debye model is used when single frequency is considered for modeling of dielectric values. But Cole–Cole model extends over a wide range of frequency scales than Debye model. Whether Cole-Cole or Debye theoretical modes are in use, gap exists between research to research followed by analytical and practical values.

Based on the dielectric properties in Debye and Cole-Cole models [3-5], many breast phantoms have been developed numerically [6-12] and practically [13-15]. One of the numerical phantoms using first order or single relaxation Debye model [3] presented in [6] considered permittivity values of breast skin, adipose, glandular and tumor tissue layers at UWB center frequency of 6 GHz. This numerical model is very close to the experimental phantom model presented in [13] for the whole UWB range including the same 6 GHz center frequency. Here for each individual homogeneous breast tissue and their corresponding dielectric values were measured in the ultra-wide band (UWB) frequency using HP 85070B dielectric probe attached to a vector network analyzer. Then breast phantoms were developed for each individual homogeneous breast tissues using their corresponding dielectric values. Though the considered dielectric properties in [6] and [13] are similar, still there exists a gap which needs to be mended for a realistic phantom model development, and this is the aim of this paper.

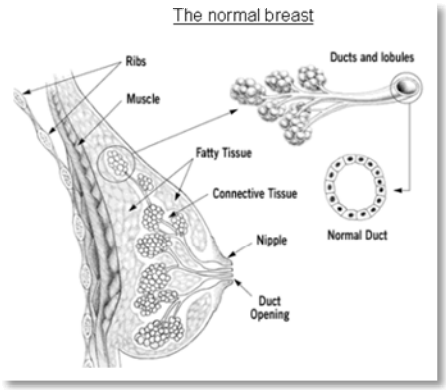

Fig. 1. Different types of breast tissues [1]

In this paper, a polynomial fitting approach is used to propose an analytical model to reduce the gap between the theoretical [6] and experimental [13] breast model results in terms of absolute permittivity.

II. First Order or Single Relaxation Debye Model Details

Dielectric relaxation is the response of a dielectric medium under the influence of an external electric field in microwave frequencies. It is an exponential decay of the polarization in a dielectric medium after the removal of the applied electric field. This phenomenon has an important effect in determining the permittivity of the medium, which characterize the dispersive and lossy nature of breast tissues. The analytical permittivity model is shown in Equation (1) and the related conductivity model is presented in Equations (2-3). The polarization of the medium is relaxed towards the steady state as a first order process characterized by a single time constant τ.

$$\varepsilon^* = \varepsilon' - j\varepsilon'' \qquad (1)$$

Here ε' is the real part and ε'' is the imaginary part that represents the dielectric loss factor. The complex conductivity is,

$$\sigma = \sigma' + j\omega\varepsilon_0\varepsilon' \qquad (2)$$

Where, ω is the angular frequency and the real part of conductivity σ' is related to the loss factor ε'' as:

$$\varepsilon'' = \frac{\sigma'}{\omega\varepsilon_0} \qquad (3)$$

The Equation (4) showing the relationship between single relaxation time τ and permittivity is called first order Debye model.

$$\varepsilon^* = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)}{1 + j\omega\tau} \qquad (4)$$

Where, ε_s and ε_∞ are the low and high frequency constraints of the dielectric constant respectively. A simple and comprehensive derivation of Equation (4) is provided in Appendix A. Process described by Equation (4) can be separated into real and imaginary parts as follows,

$$\varepsilon' = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)}{1 + (\omega\tau)^2} \qquad (5)$$

$$\varepsilon'' = \frac{(\varepsilon_s - \varepsilon_\infty)\omega\tau}{1 + (\omega\tau)^2} \qquad (6)$$

The centre relaxation frequency, f_c is related to relaxation time τ as follows

$$f_c = 1/(2\pi\tau) \qquad (7)$$

Because of the current flow, as group of ions move under the influence of a regular field, the effect of static conductivity, σ_s, is included in the Debye model, Equation (4) as,

$$\varepsilon^* = \varepsilon_\infty + \frac{(\varepsilon_S - \varepsilon_\infty)}{1 + j\omega\tau} - j\frac{\sigma_S}{\omega\varepsilon_0} \quad (8)$$

Rearrangement gives,

$$\varepsilon^* = \varepsilon_\infty + \frac{(\varepsilon_S - \varepsilon_\infty)}{1 + (\omega\tau)^2} - j\left[\frac{(\varepsilon_S - \varepsilon_\infty)\omega\tau}{1 + (\omega\tau)^2} + \frac{\sigma_S}{\omega\varepsilon_0}\right] \quad (9)$$

Equations (1 and 9) are similar and comparable.

For a special case: $f = f_c$ and $\omega\tau = 1$, Equation (9) becomes,

$$\varepsilon^* = \frac{(\varepsilon_S + \varepsilon_\infty)}{2} - j\left[\frac{(\varepsilon_S - \varepsilon_\infty)}{2} + \frac{\sigma_S}{\omega\varepsilon_0}\right] \quad (10)$$

Here, we observed that the real part is the mean value for low (or static) and high frequency permittivity. The imaginary part is the combination of static, infinite frequency permittivity and the static conductivity. In the absence of static conductivity we have,

$$\varepsilon' = \frac{\varepsilon_S + \varepsilon_\infty}{2} \quad (11)$$

And

$$\varepsilon'' = \frac{\varepsilon_S - \varepsilon_\infty}{2} \quad (12)$$

For a material that possesses non-linear relaxation process with a range of spectral shapes, Cole-Cole mathematical formulation is then used [4, 5]. The mathematical formulations for the Cole-Cole model are shown in Equations (13-15),

$$\varepsilon(\omega) = \varepsilon_\infty + \frac{(\varepsilon_S - \varepsilon_\infty)}{1 + (j\omega\tau)^{1-\alpha}} \quad (13)$$

$$\varepsilon'(\omega) = \varepsilon_\infty + (\varepsilon_S - \varepsilon_\infty)\frac{1 + (\omega\tau)^{1-\alpha}\sin\left(\frac{1}{2}\alpha\pi\right)}{1 + 2(\omega\tau)^{1-\alpha}\sin\left(\frac{1}{2}\alpha\pi\right) + (\omega\tau)^{2(1-\alpha)}} \quad (14)$$

$$\varepsilon''(\omega) = (\varepsilon_S - \varepsilon_\infty)\frac{(\omega\tau)^{1-\alpha}\cos\left(\frac{1}{2}\alpha\pi\right)}{1 + 2(\omega\tau)^{1-\alpha}\sin\left(\frac{1}{2}\alpha\pi\right) + (\omega\tau)^{2(1-\alpha)}} \quad (15)$$

Exponent parameter α, in the above equations, takes a value between 0 and 1, which allows describing different spectral shapes. For biological materials, α ranges from 0.3 to 0.5. When $\alpha = 0$, the Cole-Cole model becomes the first order Debye relaxation model, which is considered in this paper and its application to derive breast tissue dielectric values towards theoretical breast phantom [6] development is presented in the following section.

III. NUMERICAL BREAST PHANTOM USING THEORETICAL DIELECTRIC VALUES

Converse et al. [6] developed an anatomically realistic two-dimensional (2-D) finite-difference time-domain (FDTD) breast phantom model to simulate the absorbed Electro-Magnetic (EM) power density distributions. They used Debye parameter set of the human breast tissues from Equation (9) for skin, breast fat, glandular and tumor at 6 GHz UWB center frequency to calculate their respective absolute permittivity values. These values are shown in Table I with associated parameters accordingly.

TABLE I
Debye Parameters at 6 GHz UWB Centre Frequency [6]

Medium	**Debye Parameter Values**				**Absolute Permittivity**
	ε_∞	ε_s	σ_s	τ *(ps)*	ε
Skin	4.00	37.00	1.10	7.37	36.62
Tumor	3.99	54.00	0.70	7.23	52.75
Normal Breast Tissue (More Fat)	6.57	16.29	0.23	7.0	15.96
Normal Breast (More Gland)	5.28	35.14	0.46	7.0	34.42

To observe the characteristics of Debye equation (Equation (16)) or to validate whether the considered center frequency 6 GHz is appropriate or not, we have used the following parameters from Table I for fatty tissues to calculate the real part of complex permittivity vs. frequency using Matlab and shown in Fig. 2.

$\varepsilon_s = 16.29,\ \varepsilon_\infty = 6.57,\ f_c = 6\ GHz$ and $\sigma_s = 0.23$

It shows that the real part of complex permittivity has amplitude of 0.5 (the solid line) when operating with relaxation or center frequency, f is 6 GHz (Fig. 2) and either less or greater than 0.5 for other frequencies (2, 4, and 8 GHz). This result satisfies Equation (16), which states that relative permittivity value at relaxation frequency (center frequency) is average of low and high frequency permittivity values. It is also clear from this figure that the model in [6] is valid for 6 GHz only. If we consider relaxation for other frequencies then the slope of the curve will be changed and the results will not be accurate as well.

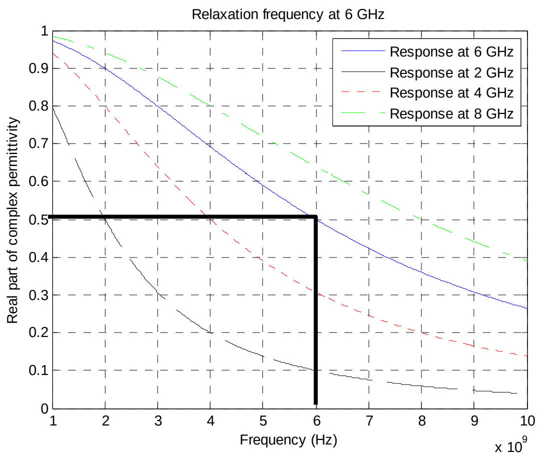

Fig. 2. Behaviour of real part of complex permittivity.

IV. EXPERIMENTAL BREAST PHANTOM

Porter, et al. [13] developed practical heterogeneous phantoms by considering individual homogeneous phantoms for fat, skin, gland and tumor initially. Relative permittivity and conductivity for each phantom were measured at UWB microwave frequencies. They also described a proper methodology and procedure to merge four phantoms into a single hemispherical breast phantom to make it heterogeneous. The final heterogeneous breast phantom comprised of 2mm layer of skin enveloping a mixture of fat, gland and one or more tumors. It has a radius of 0.65 mm with 0.015 mm tumor inside the gland. For comparison with numerical model in [6], we have considered the permittivity values in this work [13] at 6 GHz UWB center frequency only. *Table 2* shows the permittivity values of breast tissues considered by Porter (the detail measurement procedure and values can be found in [13]).

It is apparent from Table I and II that the absolute permittivity values of Debye and Porter models are different showing some gap between them. The formulation to mend this gap is presented in the next section.

TABLE II
Porter Parameters at 6 GHz UWB Centre Frequency [13]

Medium	**Absolute Permittivity**
	ε
Skin	33.53
Tumor	50.30
Fatty Breast Tissue	13.48
Fibro Glandular Tissue	33.86

V. POLYNOMIAL FITTING OF DEBYE MODEL

In this section we have compared the theoretical absolute permittivity values [6] to the corresponding experimental values [13] to obtain an analytical model (polynomial fit) to mend their gap.

For this purpose, we need to determine the statistical correlation between the permittivity results shown in Tables I and II to substantiate modelling the data. So, we have considered absolute permittivity values in Table I and Table II as *x* and *y* respectively. Then, correlation coefficients have been found by Pearson correlation equation [16] using Matlab. Pearson correlation coefficient, Γ_{xy} between two sets of data *x* and *y* is given as,

$$\Gamma_{xy} = \frac{Cov(x,y)}{\sigma_x \sigma_y} \tag{16}$$

Where, $\mathrm{Cov}(x, y)$ is the covariance of *x* and *y*, σ_x is the standard deviation in *x*, σ_y is the standard deviation in *y*. Equation (16) can be equally used to calculate correlation coefficient between similar sets of data, i.e. between *x* and *x* , Γ_{xx} or *y* and *y*, Γ_{yy}.

The correlation test result in terms of matrix is shown in Equation (17). The detail analytical derivation to obtain Equation (17) is shown in Appendix B.

$$\Gamma = \begin{bmatrix} \Gamma_{xx}\Gamma_{xy} \\ \Gamma_{yx}\Gamma_{yy} \end{bmatrix}$$

Which is

$$\Gamma = \begin{bmatrix} 1 & 0.9973 \\ 0.9973 & 1 \end{bmatrix} \tag{17}$$

The diagonal matrix elements correspond to the perfect correlation of each variable with itself which is equal to 1. The off-diagonal elements are approximately equal to 1, signifying that there is a strong statistical and linear correlation between the two sets of data in Table I and Table II. We fit this data sets and evaluate the polynomial equaton of the regression curve using polyfit command in Matlab shown in Equation (18).

$$p(x) = 0.9978x - 2.0687 \tag{18}$$

Where *P(x)* is the proposed permittivity value (the new realistic one), *x* is the Debye model value in Table I. The regression curve of Equation (18) is shown in Fig. 3, which fits very closely with the desired permittivity values in Table II; thus indicating the usefulness of our polynomial fitting model.

The slope (0.9978) and intecept constant (-2.0687) in Equation(18) are obtained using Least Square (LS) fitting criteria (*polyfit*, command in Matlab) as follows,

$$[p, s] = polyfit(x, y, n)$$

Where, *x* is the Debye model value in Table I, *y* are the corresponding experiental values in Table II and *n* is the order of the polynomial. This command determines the coefficients for a polynomial *P(x)*of degree *n* that fits the data *x* with *y*, using least squares logic.The outcome of *polyfit* is a row vector of length *n+1* containing the polynomial coefficients in descending powers. The second output 's' is a structure which is used to find error estimates in conjuction with the *polyval* command in Matlab. We tested with *n=1*, because of good correlation results of correlation matrix in Equation (17).

In [17] simple formulations have been derived to transform least square logics into covariances for finding slope and intercepts. Appendix C is provided to illustrate the evaluation of slope and intercept constant values in Equation (18).

To obtain new permittivity values, we used Matlab, command *polyval* as:

$$[y, delta] = polyval(p, x, s)$$

Here, *delta* is estimate of the standard deviation of the error in predicting a future observation at *x* by *P(x)*. With *polyval* we have obtained the new (targeted) values of polynomial *P(x)* (i.e., the desired permittivity values) for diffferent values of *x* and presented in Table III.

It can be seen from Table III that the desired permittivity values obtained from proposed regression analytical model are very accurate compared to that of original 1st order Debye equation. The last entry in Table III however shows an error value greater than its counterpart old Debye value. The reason for this is that, in [6] different breast models have been used for different proportion of adipose and glandular content. Changing to some other breast model for glandular, could improve the error without effecting the result of others.

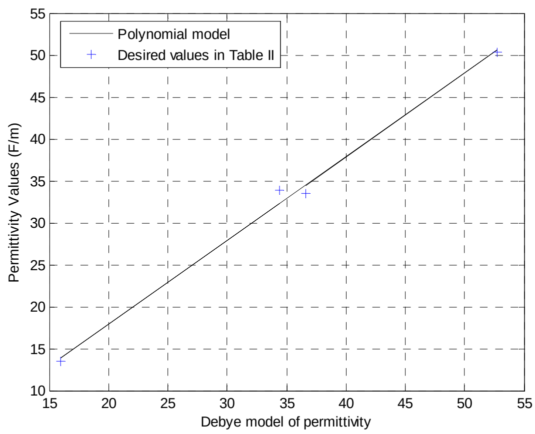

Fig.3. Polynomial fit to provide approximation of the Debye data

TABLE III
Comparison of New and Old Debye Model Values

Medium	Experimental Permittivity Values [13] (a)	First Order Debye Model Permittivity Values [6] (b)	% Error $\frac{\|a-b\|}{a} \times 100$	Proposed Permittivity Values Using Equation (18) (c)	% Error $\frac{\|a-c\|}{a} \times 100$
Skin	33.53	36.62	9.21	34.47	2.80
Tumor	50.30	52.75	4.87	50.56	0.51
Fatty Breast Tissue	13.48	15.96	18.39	13.85	2.74
Fibro Glandular Tissue	33.86	34.42	1.65	32.27	4.68

Table III shows that percentage errors of skin, tumor, fatty and fibro glandular breast tissues permittivity values for Debye [6], experimental [13] and proposed analytical models. Average percentage error considering all the four breast tissues permittivity values are 8.53% and 2.63% for [6] and our proposed model compared to the experimental one. Thus our proposed model exhibits less average error with better accuracy and usefulness.

VI. CONCLUSION

This paper proposes an analytical polynomial fitting model(viz. improved first order relaxation Debye model) to bridge the gap between experimental and analytical 1st order Debye permittivity values for breast tissues at 6 GHz UWB centre frequency. This new polynomial fitting model efficiently reduces the gap by reducing the relative error approximately 5.9%. Hence the proposed model could be very useful to determine real-like breast tissue permittivity values directly without wasting further investigation time duration. Presently, we are working to extend the fitting analysis for the whole UWB frequency range.

ACKNOWLEDGMENT

This work is supported by the Long-term Research Grant Scheme, LRGS (Code no.: LRGS/TD/2011/UKM/ICT/02/05/, Grant no.: 9012-00005) of Ministry of Higher Education, Malaysia and partially supported by Universiti Malaysia Perlis (UniMAP) short grant no.: 900100430.

APPENDIX A

The transient response of single time constant can be depicted as in Fig. A.1

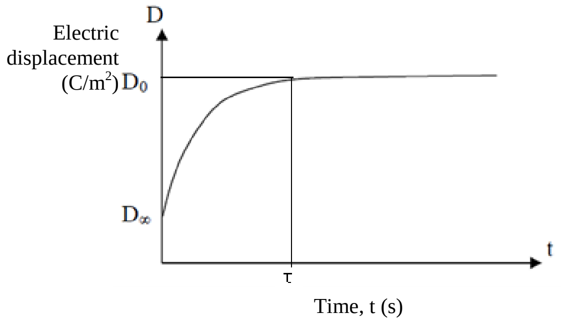

Fig. A.1. Transient response of a dielectric medium

In Fig. A.1, D is the electric displacement. This can be written as,

$$\mathrm{D} = \mathrm{D}_\infty + (\mathrm{D}_0 - \mathrm{D}_\infty)\left(1 - e^{-\frac{t}{\tau}}\right) \qquad A.1$$

Here

τ is the relaxation time constant

t is the time axis

D_0 is the final value of D

D_∞ is the initial value of D

According to the definition,

$$\mathrm{D} = \varepsilon^* \varepsilon_0 \mathrm{E} \qquad A.2$$

$$\mathrm{D}_\infty = \varepsilon_\infty \varepsilon_0 \mathrm{E} \qquad A.3$$

$$\mathrm{D}_0 = \varepsilon_s \varepsilon_0 \mathrm{E} \qquad A.4$$

Where

ε^* is the complex relative permittivity

ε_s is the steady state (static) relative permittivity

ε_∞ is the relative permittivity at infinite frequency

Using $A.2$ to $A.3$, we can re-write $A.1$ as,

$$\varepsilon^* \varepsilon_0 E = \varepsilon_\infty \varepsilon_0 E + (\varepsilon_s \varepsilon_0 E - \varepsilon_\infty \varepsilon_0 E)\left(1 - e^{-\frac{t}{\tau}}\right) \qquad A.5$$

On simplification, we get,

$$\varepsilon^* = \varepsilon_\infty + (\varepsilon_s - \varepsilon_\infty) - (\varepsilon_s - \varepsilon_\infty) e^{-\frac{t}{\tau}} \qquad A.6$$

Converting $A.6$ into Laplace transform(s) for frequency domain, we get (for t > 0),

$$\frac{\varepsilon^*}{s} = \frac{\varepsilon_\infty}{s} + \frac{(\varepsilon_s - \varepsilon_\infty)}{s} - \frac{(\varepsilon_s - \varepsilon_\infty)}{s + {}^1/_\tau} \qquad A.7$$

Multiplying $A.7$ by s and after re-arrangement, we get

$$\varepsilon^* = \varepsilon_\infty + \frac{(\varepsilon_s - \varepsilon_\infty)}{1 + j\omega\tau} \qquad A.8$$

Equation $A.8$ is the first order or single relaxation Debye model equation.

APPENDIX B

The data set available from Table I and Table II is,

x = [36.62 52.75 15.96 34.42]

y = [33.53 50.30 13.48 33.86]

The length of dataset, n = 4 here.

Using the definitions in [16], correlation coefficients are given by,

$$\Gamma_{xy} = \frac{Cov(x,y)}{\sigma_x\sigma_y} \qquad B.1$$

$$\Gamma_{xx} = \frac{Cov(x,x)}{\sigma_x\sigma_x} \qquad B.2$$

$$\Gamma_{yy} = \frac{Cov(y,y)}{\sigma_y\sigma_y} \qquad B.3$$

Where

$$Cov(x,y) = \sum_{i=1}^{n} x_i y_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i \qquad B.4$$

Standard deviation of x,

$$\sigma_x = \sqrt{\sum_{i=1}^{n} {x_i}^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2} \qquad B.5$$

Standard deviation of y,

$$\sigma_y = \sqrt{\sum_{i=1}^{n} {y_i}^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2} \qquad B.6$$

$$Cov(x,x) = \sum_{i=1}^{n} x_i x_i - \frac{1}{n}\sum_{i=1}^{n} x_i \sum_{i=1}^{n} x_i \qquad B.7$$

$$Cov(y,y) = \sum_{i=1}^{n} y_i y_i - \frac{1}{n}\sum_{i=1}^{n} y_i \sum_{i=1}^{n} y_i \qquad B.8$$

Substituting the values of x and y into Equations (B. 4) to (B. 8), we obtain the following results,

$$Cov(x,y) = 679.0437 \qquad B.9$$

$$\sigma_x = 26.0870 \qquad B.10$$

$$\sigma_y = 26.0992 \qquad B.11$$

$$Cov(x,x) = 680.5293 \qquad B.12$$

$$Cov(y,y) = 681.1687 \qquad B.13$$

Using the results from Equations (*B.9*) to (*B.13*) in Equations (*B.1*) to (*B.3*) we obtain the required values of correlation coefficients, as,

$\Gamma_{xx} = 1, \Gamma_{yy} = 1$

$\Gamma_{xy} = 0.9973, \Gamma_{yx} = 0.9973$

Hence correlation coefficient matrix becomes

$$\Gamma = \begin{bmatrix} \Gamma_{xx}\Gamma_{xy} \\ \Gamma_{yx}\Gamma_{yy} \end{bmatrix}$$

Which is

$$\Gamma = \begin{bmatrix} 1 & 0.9973 \\ 0.9973 & 1 \end{bmatrix} \qquad B.14$$

APPENDIX C

The data set available from Table I and Table II is,

x = [36.62 52.75 15.96 34.42]

y = [33.53 50.30 13.48 33.86]

According to [17] slope and intercept of a polynomial equation for linear regression is given as follows,

slope, *m*→

$$m = \frac{Cov(x,y)}{{\sigma_x}^2} \qquad C.1$$

Using results from Appendix B, slope is found to be equal to,

$$m = 0.9978 \qquad C.2$$

Similarly according to [17], the intercept term, b, is given as,

$$b = \hat{y} - m\hat{x} \qquad C.3$$

where

$$\hat{y} = mean\ value\ of(y) = 32.7925 \qquad C.4$$

and

$$\hat{x} = mean\ value\ of\ (x) = 34.9375 \qquad C.5$$

Substituting Equations (C. 2), (C. 4) and (C. 5) in Equation (C. 3), we get the value of intercept, *b*, as

$$b = -2.0687 \qquad C.6$$

Hence, the model equation is

$$P(x) = mx + b \qquad C.7$$

$$P(x) = 0.9978x - 2.0687 \qquad C.8$$

REFERENCES

[1] *The Cancer Council Victoria*, Available at http://www.cancervic.org.au.

[2] M.A.F. Feitosa, E. Fontana, "Prospects for the development of a magnetic stimulation device for human tissue," *IEEE MTT-S International Conference on Microwave and Optoelectronics, SBMO* , vol., no., pp.521-524, 25-25 July 2005

[3] P. Debye, Polar Molecules, The Chemical Catalog Company, Inc., New York, 1929. Section 18, pp. 89-95

[4] K. Cole and R. Cole, "Dispersion and absorption in dielectrics I. Alternating current characteristics," J. Chem. Phys., vol. 9, pp. 341–52, 1941.

[5] D. Davidson and R. Cole, "Dielectric relaxation in glycerol, propylene glycol, and n-propanol," J. Chem. Phys., vol. 19, pp. 1484–1490, 1951.

[6] M. Converse, J.B. Essex, S.C. Hagness, B.D. Van Veen, "Ultrawide-band microwave space-time beamforming for hyperthermia treatment of breast cancer: a computational feasibility study," *IEEE Transactions on Microwave Theory and Techniques*,, vol.52, no.8, pp.1876-1889, Aug. 2004

[7] E. Zastrow, S.K. Davis,M. Lazebnik, F. Kelcz,B.D.Van Veen, S.C.Hagness, "Development of Anatomically Realistic Numerical Breast Phantoms With Accurate Dielectric Properties for Modeling Microwave Interactions With the Human Breast," *IEEE Transactions on Biomedical Engineering*,, vol.55, no.12, pp. 2792-2800, Dec. 2008

[8] M.O'Halloran, R. Conceicao, D. Byrne, M. Glavin, E. Jones, "FDTD modeling of the breast: A review", *Progress in Electromagnetics Research B*, vol. 18, pp.1-24, 2009

[9] J.D.Shea, S.C.Hagness, B.D. Van Veen,"Scattering matrix decomposition in microwave imaging of realistic numerical breast phantoms," *IEEE International Symposium on Antennas and Propagation (APSURSI)*, vol., no., pp.718-721, 3-8 July 2011.

[10] Arnold Reynaldi, Samuel Lukas, Helena Margaretha, "Backpropagation and Levenberg-Marquardt Algorithm for Training Finite Element Neural Network," *6th European Modelling Symposium UKSim-AMSS*, 14-16 Nov 2012, pp. 89 - 94, 2012.

[11] Tabatabaee Y.H., Mehrnejad. M, Kazem Shekofteh. S, "Cancer detection based on experimental sampling by genetic-fuzzy classification system," *International Review on Computers and Software, vol. 7*, issue 3, 2012, pp. 1062-1069.

[12] A. Yilmaz, K. Ayan, "Early diagnosis in cancer disease by using fuzzy logic and the effects of stress on cancer," *International Review on Computers and Software*, *vol. 6*, issue 3, 2011, Pages 347-354.

[13] E. Porter, J. Fakhoury, R. Oprisor, M. Coates, M. Popović, "Improved tissue phantoms for experimental validation of microwave breast cancer detection," *Proceedings of the Fourth European Conference on Antennas and Propagation (EuCAP)*, pp.1-5, 12-16 April 2010

[14] J.C.Y Lai, C.B. Soh, E. Gunawan, K.S. Low,"Homogeneous and heterogeneous breast phantoms for ultra-wideband microwave imaging applications," *Progress In Electromagnetics Research, PIER*, vol.100,pp. 397-415, 2010

[15] S. Alshehri, S. Khatun, Z. Awang,**"**Homogeneous and heterogeneous breast phantoms for UWB imaging,"ISABEL'11, Proceedings of the 4th International Symposium on Applied Sciences in Biomedical and Communication Technlogies, Barcelona, Spain, ACM, 2011.

[16] J. Benesty, C. Jingdong, Y.A. Huang, "On the Importance of the Pearson Correlation Coefficient in Noise Reduction," *IEEE Transactions on Audio, Speech, and Language Processing*,, vol.16, no.4, pp.757-765, May 2008

[17] Weisstein, W. Eric "Least Squares Fitting." From MathWorld-AWolfram Web. Resource: http://mathworld.wolfram.com/LeastSquaresFItting.html