Design of Decentralised PI Controller using Model Reference Adaptive Control for Quadruple Tank Process

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***Abstract*— Multivariable systems exhibit complex dynamics because of the interactions between input variables and output variables. This paper presents an approach to design auto tuned decentralized PI controller using ideal decoupler and adaptive techniques for controlling a class of multivariable process with a transmission zero. Using the decoupler, the MIMO system is transformed into two SISO systems. The controller parameters are adjusted using the Model Reference Adaptive reference Control (MRAC). A benchmark quadruple tank system (QTS) is considered to illustrate the benefits of the design paradigm. The performance of this set up was studied for reference tracking and disturbance rejection cases. Simulation results confirm the effectiveness of the proposed system.**

Keyword- Multivariable control, Decoupler, PI controller, Quadruple Tank process, Relative gain array, Model reference adaptive control.

I. Introduction

A typical process control plant may contain more number of input and output signals. There exist complicated couplings between the input and output signals. Many control methods have been applied to control multivariable processes with interactions. Centralized controller design for MIMO systems suffer from potential problems associated with complex computations, maintenance due to the size and a high risk of failure even though it provides better performance. In turn decentralized strategies, based on or mathematical analysis provide flexible and scalable solutions with simple single-input single-output (SISO) controllers [1], [2].

This paper presents a design methodology for auto tuned decentralized PI controller using decoupling and MRAC techniques to solve the problem of interactions in quadruple tank process, which is a bench mark multivariable process used in control literature. To control a quadruple tank system, one essential problem is how to handle the interactions among two loops. An effective approach is to apply the so called decentralized control strategy: each loop is controlled by one controller independently based on local information and local actions.

The control system design procedure for a multivariable plant involves several steps [3]. Before designing the controller design, the control configuration is selected using Relative Gain Array (RGA) method. The first part of this step is to find a set of variables to manipulate input signals and a set of variables to control and measure output signals i.e. the problem of input/output selection. Next, the control configuration selection has to be performed, where the connections between the inputs and the outputs are decided. Finally the PI controller parameters are automatically adjusted by designing a Model Reference Adaptive Control and the results are compared.

The concepts behind this study are organized as follows: Section 2 gives the description of quadruple tank process and the linearised mathematical modelling of the system. The design of decoupler is explained in Section 3. The Section 4 briefs the Relative Gain Array concepts for selecting input/output pairing. The next one, Section 5 explains the design of MRAC technique followed by Results and Discussions in Section 6. The conclusion is given in Section 7.

II. QUADRUPLE TANK SYSTEM

A quadruple tank apparatus which was proposed in [4] has been used in chemical engineering laboratories to illustrate the performance limitations for multivariable systems posed by ill-conditioning, right half plane transmission zeros and model uncertainties.

The quadruple tank system consists of four interconnected tanks and two pumps. The schematic of the quadruple tank equipment is presented in Fig.1.

The process inputs are u_1 and u_2 (input voltages to the pumps), and the outputs are y_1 and y_2 (voltages from level measurement devices). The target is to control the level of the lower two tanks with inlet flow rates. The output of each pump is split into two by using a three- way valve. Thus each pump output goes to two tanks, one lower and another upper, diagonally opposite and the ratio of the split up is controlled by the position of the valve. With the change in position of the two valves, the system can be appropriately placed either in the minimum phase or in the non-minimum phase.

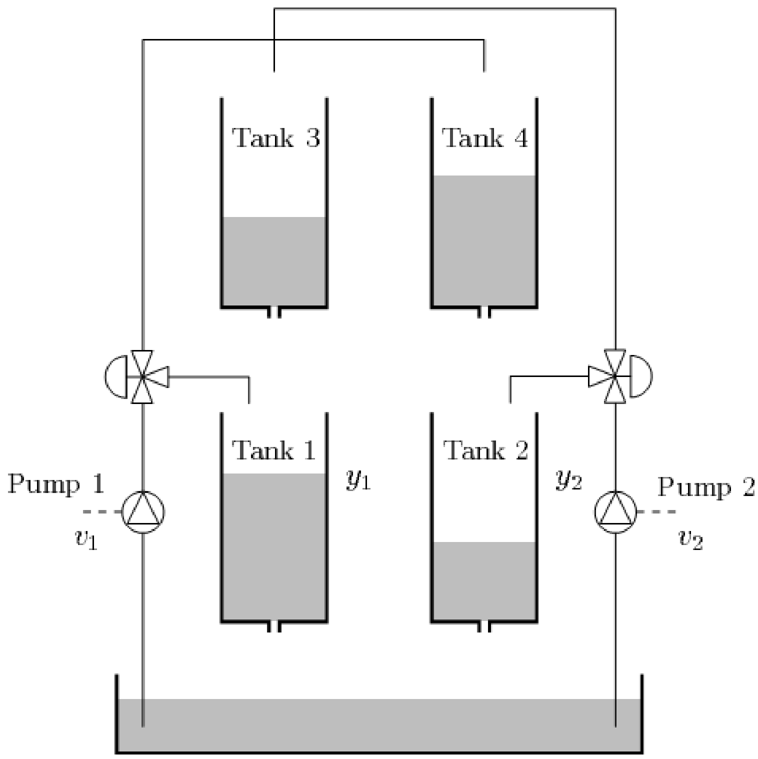

Fig.1. Schematic of Quadruple Tank System

Let the parameter $\boldsymbol{\gamma}$ be determined by how the valves are set. If $\boldsymbol{\gamma}_1$ is the ratio of flow to the first tank, then (1 - $\boldsymbol{\gamma}_1$) will be the flow to the fourth tank. Similarly if $\boldsymbol{\gamma}_2$ is the ratio of flow to the second tank, then (1 - $\boldsymbol{\gamma}_2$) will be the flow to the third tank. The voltage applied to Pump 'i' is V_i and the corresponding flow rate is K_iV_i. The parameters $\boldsymbol{\gamma}_1$ $\boldsymbol{\gamma}_2$ [0, 1] are determined from how the valves are set prior to an experiment. The flow to tank 1 is $\boldsymbol{\gamma}_1K_1V_1$ and the flow to tank 4 is (1- $\boldsymbol{\gamma}_1$) K_1V_1 and similarly for Tank 2 and Tank 3. The acceleration of gravity is denoted as 'g'. The measured level signals are $y_1 = k_ch_1$ and $y_2= k_ch_2$ [4].

The state equations of the four tank system are given in equation (1)

$$\frac{dh_1}{dt} = -\frac{a_1}{A_1}\sqrt{2gh_1} + \frac{a_3}{A_1}\sqrt{2gh_3} + \frac{\gamma_1 k_1}{A_1}u_1$$
$$\frac{dh_2}{dt} = -\frac{a_2}{A_2}\sqrt{2gh_2} + \frac{a_4}{A_2}\sqrt{2gh_4} + \frac{\gamma_2 k_2}{A_2}u_2$$
$$\frac{dh_3}{dt} = -\frac{a_3}{A_3}\sqrt{2gh_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2$$
$$\frac{dh_4}{dt} = -\frac{a_4}{A_4}\sqrt{2gh_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1 \quad (1)$$

where A_i is cross sectional area of Tank ' i '

a_i is cross section of outlet hole of Tank ' i '

h_i is water level in Tank ' i '

The linearised state space model is given by equation (2).

$$\frac{dX}{dt} = \begin{bmatrix} -\frac{1}{T_1} & 0 & \frac{A_3}{A_1T_3} & 0 \\ 0 & -\frac{1}{T_2} & 0 & \frac{A_4}{A_2T_4} \\ 0 & 0 & -\frac{1}{T_3} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_4} \end{bmatrix} X + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0 \\ 0 & \frac{\gamma_2 k_2}{A_2} \\ 0 & \frac{(1-\gamma_2)k_2}{A_3} \\ \frac{(1-\gamma_1)k_1}{A_4} & 0 \end{bmatrix} U \tag{2a}$$

$$Y = \begin{bmatrix} k_c & 0 & 0 & 0 \\ 0 & k_c & 0 & 0 \end{bmatrix} X \tag{2b}$$

The time constants are calculated using equation (3)

$$T_i = \frac{A_i}{a_i}\sqrt{\frac{2h_i^0}{g}}, \qquad \mathrm{i} = 1,2,3,4. \tag{3}$$

The parameter values and steady state operating points of the process are assumed as per the system given in literature [5]. The transfer function matrix is given in equations (4) and (5) for minimum phase and non-minimum phase operating points.

$$G_{-}(s) = \begin{bmatrix} \frac{2.6}{1+62s} & \frac{1.5}{(1+23s)(1+62s)} \\ \frac{1.4}{(1+30s)(1+90s)} & \frac{2.5}{1+90s} \end{bmatrix} \tag{4}$$

$$G_{+}(s) = \begin{bmatrix} \frac{1.5}{1+63s} & \frac{2.5}{(1+39s)(1+63s)} \\ \frac{2.5}{(1+56s)(1+91s)} & \frac{1.6}{1+91s} \end{bmatrix} \tag{5}$$

The transfer matrix G has two zeros one of them is always in the left half of s-plane, but the other can be located either in left half or right half of s-plane based on the position of three way valves. So, the system is in minimum phase, if the values of γ_1 *and* γ_2 satisfy the condition $0 < \gamma_1 + \gamma_2 < 1$ and in non-minimum phase, if the values of γ_1 *and* γ_2 satisfy the condition $1 < \gamma_1 + \gamma_2 < 2$.

III. DECOUPLER DESIGN

A popular approach to deal with control loop interactions is to design non-interacting or decoupling control schemes. The objective of this control is to eliminate completely the effects of loop interactions. This is achieved through the specification of compensation networks known as "Decoupler". The role of decoupler is to decompose a multivariable process into a series of independent single-loop sub-systems. If such a controller is designed, complete or ideal decoupling occurs and the multivariable process can be controlled using independent loop controllers [5]. The following Fig.2 shows the general decoupling control structure.

Ideal decoupler is selected because the decoupling elements are independent of the forward path controllers and therefore on line tuning of the controllers does not require redesign of the decoupler elements. Moreover this technique can address both servo and regulator problem because decoupling occurs between the forward path control signals and the process levels and not between the set points and the outputs.

The ideal decoupler is designed by the method of Zalkind and Luyben [6], [7].

According to ideal decoupling procedure, the diagonal elements of the decoupler is taken unity thus $D_{11} = D_{22} = 1$. Hence the decoupler matrix D will become

$$D = \begin{bmatrix} 1 & D_{12} \\ D_{21} & 1 \end{bmatrix}$$

For an ideal decoupling, the off diagonal elements of G*D matrix are equal to zero so by taking the product of the matrices D * G and equating the diagonal elements of the product, the equations for D_{12}, D_{21}.

The Decoupler design equations are

$$D_{11} = D_{22} = 1$$

$$D_{12} = \frac{-G_{12}D_{22}}{G_{11}}$$

$$\text{and } D_{21} = \frac{-G_{21}D_{11}}{G_{22}}$$

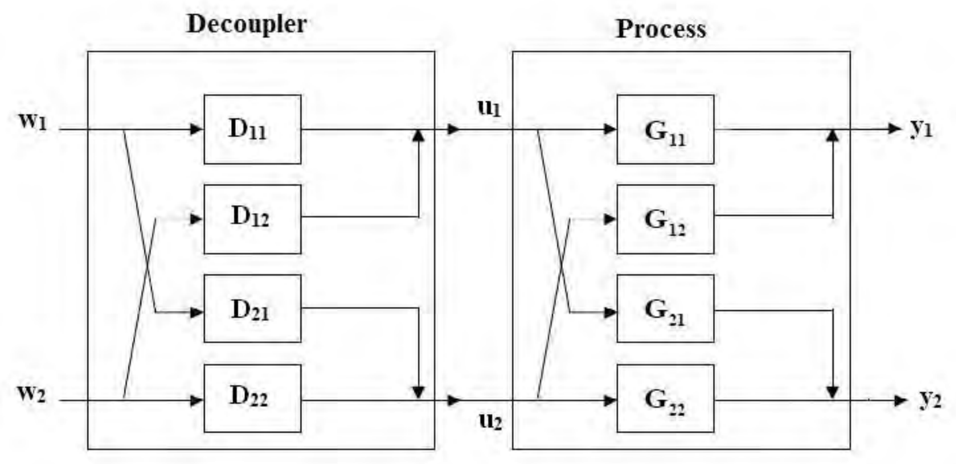

Fig.2. Architecture of System with decoupler

The decoupler matrices designed for minimum phase and non minimum phase systems are given in equation (6) and (7) respectively.

$$D_{-}(s) = \begin{bmatrix} 1 & \frac{-0.577}{(1+23s)} \\ \frac{-0.5}{(1+30s)} & 1 \end{bmatrix} \tag{6}$$

$$D_{+}(s) = \begin{bmatrix} 1 & \frac{-1.667}{(1+39s)} \\ \frac{-1.5625}{(1+56s)} & 1 \end{bmatrix} \tag{7}$$

IV. RELATIVE GAIN ARRAY

For designing control system for MIMO systems with interactions, it is required to select proper input-output pairing. To determine proper pairs it is essential to evaluate the degree of interaction between variables. The Relative Gain Array (RGA) concept is employed to determine the input-output pairing for both minimum phase and non minimum phase conditions [8].

According to the proposal given by Bristol [9], RGA is given by equation (8)

$$\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} \tag{8}$$

For minimum phase system λ_{11} is obtained as 0.63 which falls in the range $0.5<\lambda_{11}<1$, so the pairing is determined as $y_1 - u_1$ and $y_2 - u_2$. But for the non minimum phase system λ_{11} is obtained as 0.375 which falls in the range $0<\lambda_{11}<0.5$, so the suitable pairing is found as $y_1 - u_2$ and $y_2 - u_1$.

V. MODEL REFERENCE ADAPTIVE CONTROL

MRAC is an adaptive control technique where the performance specifications are given in terms of a reference model. The model is selected in such a way that it gives the ideal response of the process which is desired. The controller parameters are automatically adjusted by the adaptation mechanism in such a way that the performance of the process output matches with that of the model [10]. The block diagram of the proposed control structure using adaptive control technique is given in Fig. 3.

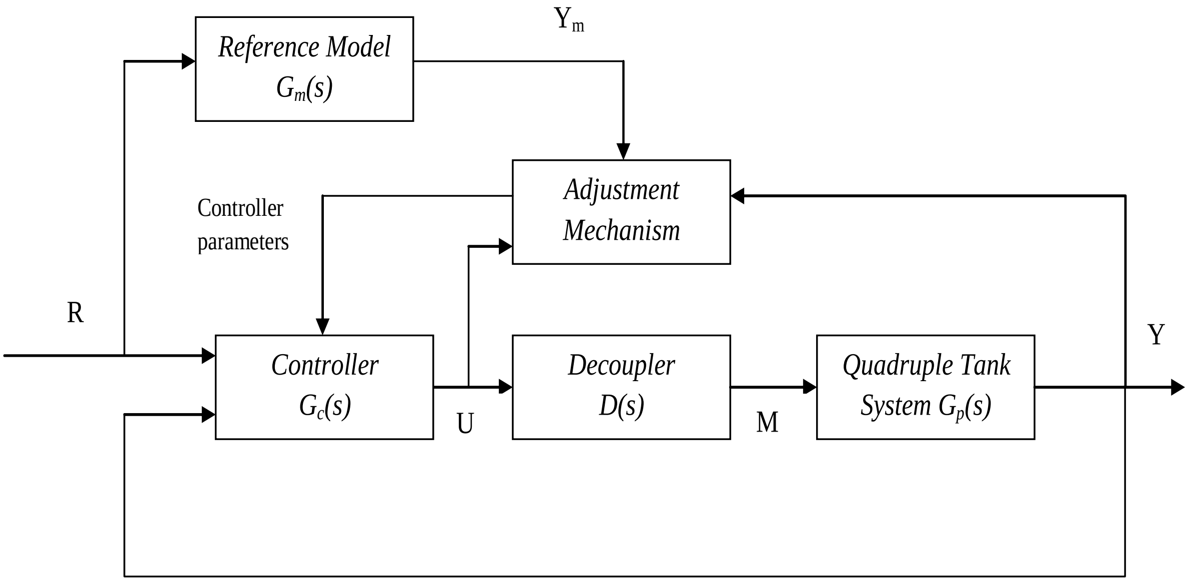

Fig. 3. Structure of MRAC based control system for QTS

The architecture of adaptive controller has two loops:

1. The inner loop is the simple feedback loop consisting of the process, decoupler and the controller.

2. The outer loop is employed to adjust the controller parameters in such a way that the deviation between actual process output and that of the reference model is small [11].

For the first order process the adaptation laws are framed based on the MIT rule as follows:

$$\textbf{For process} \quad \frac{dy}{dt} = -ay + bu$$

$$\textbf{For the model} \quad \frac{dy_m}{dt} = -a_m y_m + b_m u$$

$$\textbf{For controller} \quad u = \theta_1 u_{c-} \theta_2 y$$

The cost function for adaptation is given by (9)

$$J(\theta) = \frac{1}{2} e^2(\theta) \tag{9}$$

where $e = y - y_m$

MIT rule with negative gradient approach is given by the following expression

$$\frac{d\theta}{dt} = -\gamma \frac{\delta J}{\delta\theta} = -\gamma e \frac{\delta e}{\delta\theta} \tag{10}$$

where γ is adaptation gain

From MIT rule and equation (10) the adaptation laws can be found in terms of the original controller parameters as follows:

$$\frac{d\theta_1}{dt} = -\gamma e u_c$$

$$\frac{d\theta_2}{dt} = -\gamma e y \tag{11}$$

where θ_1 and θ_2 are K_p and K_i in this case because the PI algorithm is used as the controller here.

The following expressions (12) and (13) are obtained as decoupled transfer function matrix for minimum phase and non minimum phase operating points respectively.

$$GD_{-}(s) = \begin{bmatrix} \frac{2}{1+62s} & 0 \\ 0 & \frac{1.85}{1+42s} \end{bmatrix} \tag{12}$$

$$GD_{+}(s) = \begin{bmatrix} \frac{5.4}{1+146s} & 0 \\ 0 & \frac{2.4}{1+64s} \end{bmatrix} \tag{13}$$

The block diagram of adaptation control mechanism using MRAC is shown in Fig. 4.

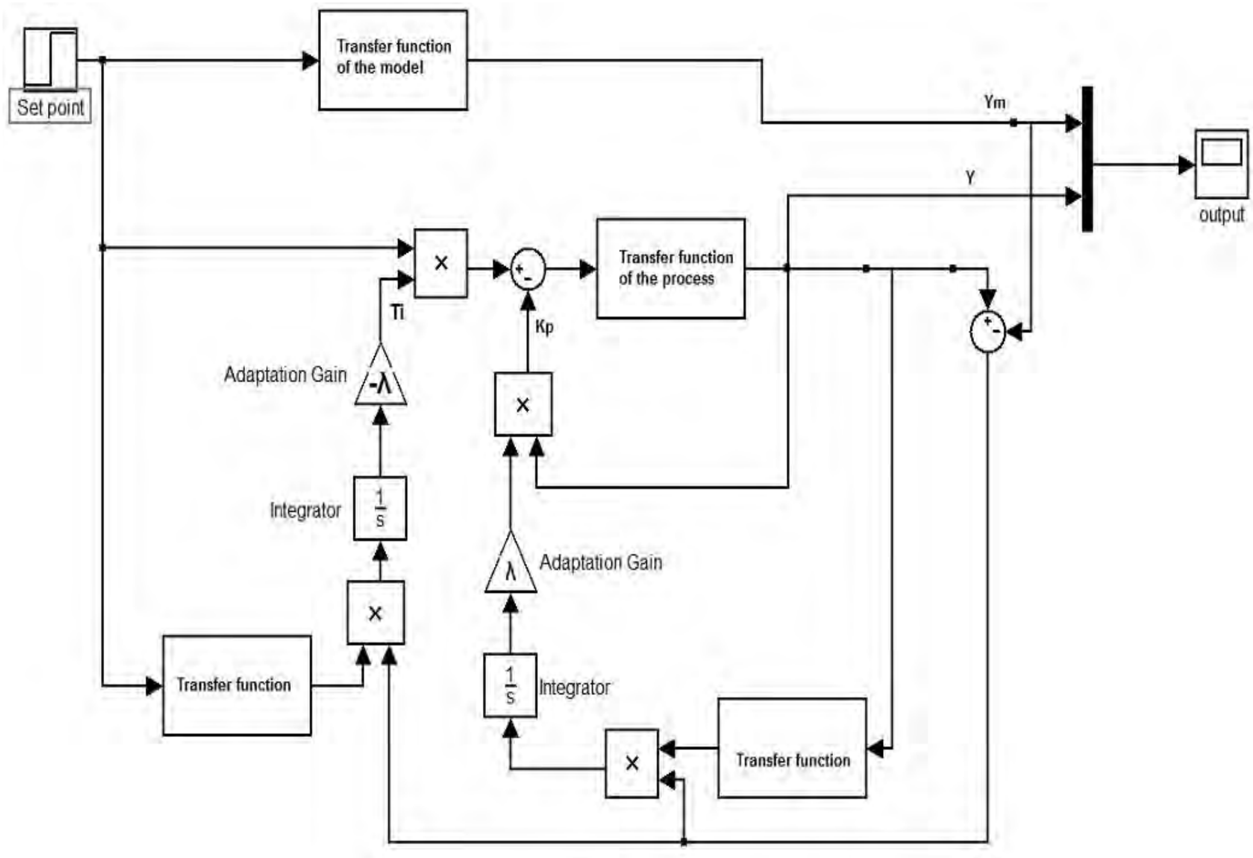

Fig. 4. Adaptation mechanism for a single control loop using MRAC

It has been noted that the two input two output quadruple tank system is now considered as two SISO systems acting without interactions. To control these processes two PI controllers are employed. The controller parameters K_p and K_i are automatically adjusted to force the process outputs (levels) as specified by the reference model.

The reference model is selected in such a way to get the closed loop response with overshoot 10% and settling time 10 sec. The adaptation gains (λ) are selected as given in Table I.

TABLE I
Adaptation Gain values for both the Operating Points

	γ Values	
	For adapting K_p	**For adapting T_i**
PI Controller for Level 1	5000	1000
PI Controller Level 2	3000	1000

VI. RESULTS AND DISCUSSIONS

The responses of the quadruple tank process for both minimum phase and non minimum phase operating points are obtained using PI controller, decoupled PI controller and adaptive PI controller. The minimum phase responses are shown in Fig. 5, 6 and 7 while the non minimum phase responses are shown in Fig. 8, 9 and 10.

It is observed that the process exhibits inverse response when it is operated in non minimum phase condition. It is not possible to provide better control to the process with simple PI controller in this operating condition [12].

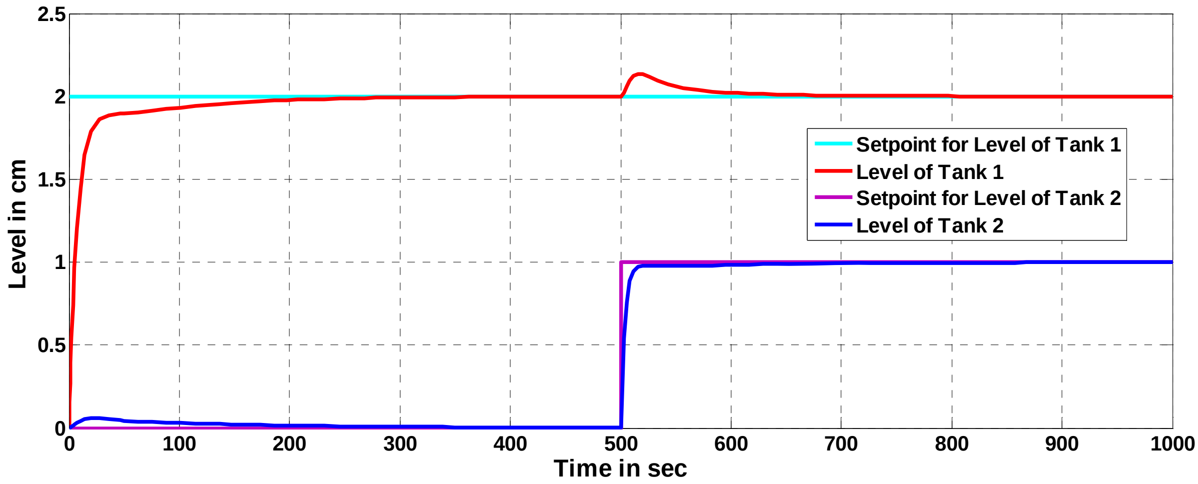

Fig.5. Response of PI controller in Minimum Phase

From the graphs it has been observed that there exist strong interactions with PI controller in both the operating conditions. But the interactions are minimized in non minimum phase and nullified in minimum phase with the help of decouplers. Adaptive controller strongly suppresses interactions in both the operating conditions and guarantees robust performance, but the overshoots are there in the response. Proper selection of the adaptation gain will lead to better response with tolerable overshoot.

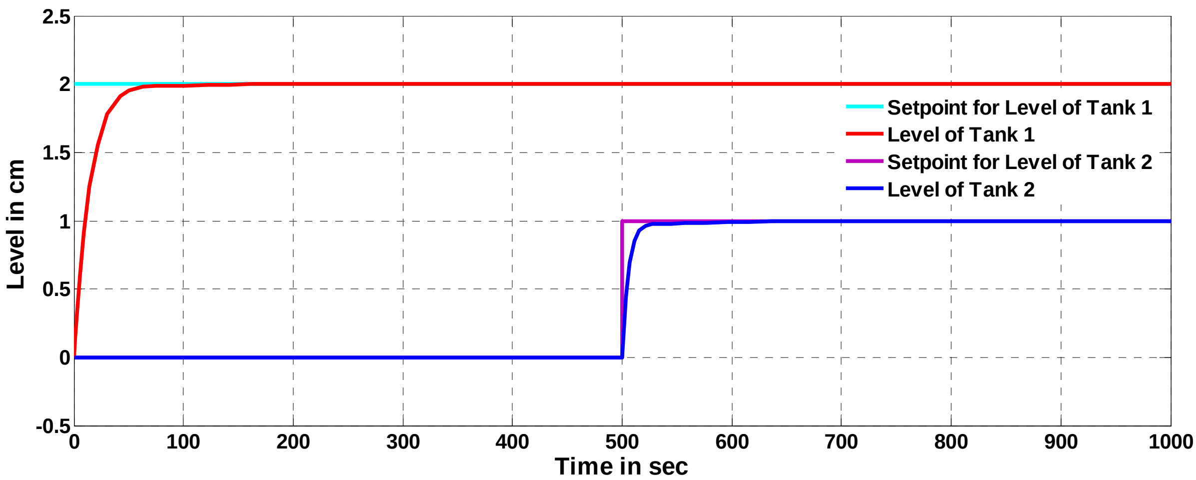

Fig.6. Response of PI controller with decoupler in Minimum Phase

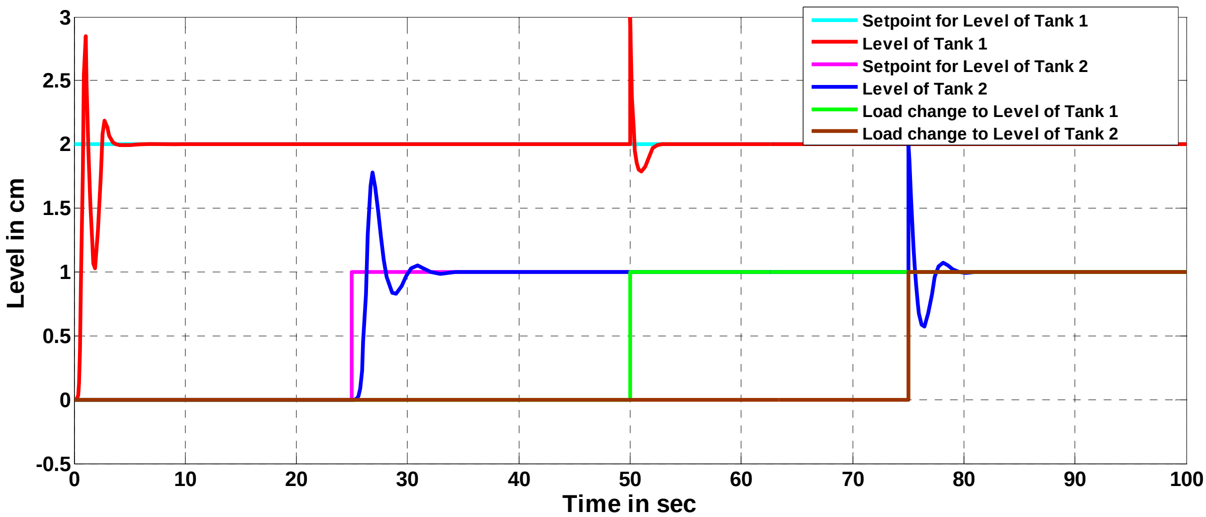

Fig.7. Response of the PI controller with MRAC in Minimum Phase

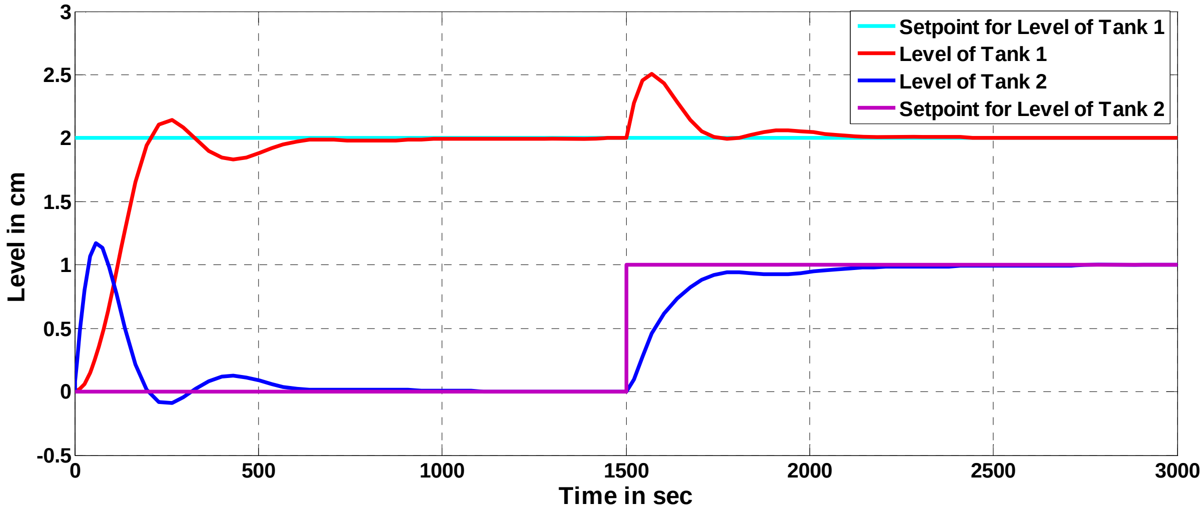

Fig. 8. Response of PI controller in Non-Minimum Phase

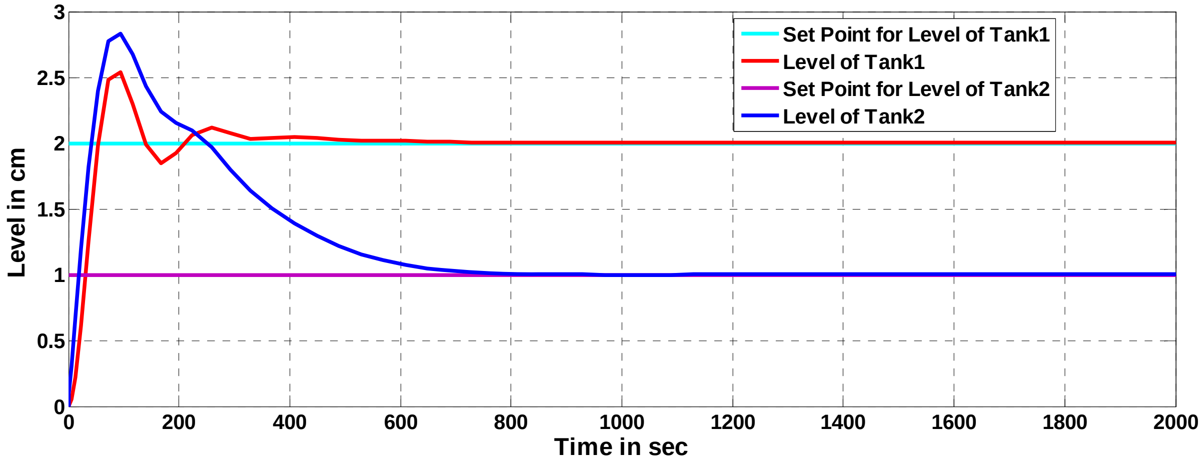

Fig. 9. Response of PI controller with decoupler in Non-Minimum Phase

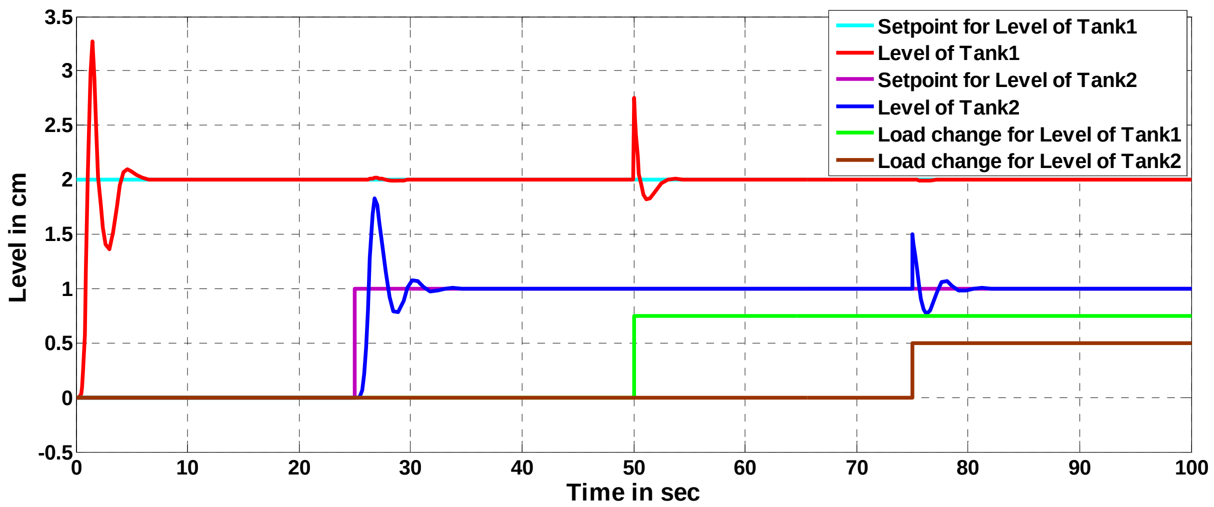

Fig. 10. Response of the PI controller with MRAC in Non Minimum Phase

It is also observed that level of tank 1 is not affected if the set point or load for level of tank 2 is changed. Similarly the change in set point or load for level of tank 1 is not affecting the level of tank 2. Thus the interactions are eliminated. More over the response time is faster and settling time is shorter.

Table II presents the quantitative comparison of the performances of PI controller, decoupled PI controller and adaptive decoupled PI controller.

TABLE II
Quantitative Comparison of Controller Performance

Controller	**Parameters**	**Minimum Phase**		**Non-Minimum Phase**	
		Level 1	***Level 2***	***Level 1***	***Level 2***
PI Controller	Settling Time	250 sec	150 sec	1380sec	1380sec
	Peak Overshoot	1%	2%	10%	25%
	Rise Time	15 sec	10 sec	240 sec	210 sec
Decoupled PI Controller	Settling Time	90 sec	50 sec	600	800
	Peak Overshoot	1%	2%	30%	20%
	Rise Time	12 sec	10 sec	80 sec	65 sec
Auto tuned PI Controller using MRAC	Settling Time	6 sec	8 sec	7 sec	10 sec
	Peak Overshoot	30%	70%	30%	40%
	Rise Time	2 sec	1 sec	3 sec	2 sec

VII. CONCLUSION

The linearized model of quadruple tank system has a multivariable transmission zero and it is much more difficult to control the system in non minimum phase condition than minimum phase condition. In this study, a design of auto adjustable decentralized PI controller using MRAC techniques for quadruple tank process has been described. The proposed controller can adjust the controller parameters in response to changes in plant uncertainties and disturbances based on the specified reference model and prevent the system from interaction between process variables. It is shown by the simulation results that MRAC technique solves the dynamic problem of the quadruple tank process and it is convenient for controller design under the requirement of the system. In future optimization techniques may be used for selecting the adaptation gains so that better performance can be ensured.

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