A Fuzzy Linear Programming in Optimizing Meat Production

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Abstract— Linear programming is one of the possible methods in optimizing objective function bounded by constraints with coefficients are real numbers. However, fuzzy linear programming with single objective function and multi-objective function could be better served in vague and uncertainty phenomena where fuzzy numbers are its coefficients. Fuzzy linear programmings with single objective function and multi-objective function have been applied in various real life applications. This paper proposes optimal solutions and profits of red meat production problem using fuzzy linear programming with single objective function. Data from Veterinary Department of Malaysia were sought in identifying beef and mutton production and its production constraints. The original method of fuzzy numbers linear programming and multi-objective linear programming were modified specifically to solve a single objective function in maximizing red meat profit. The profits ranged from RM 38,806.25 million to RM 57,922.00 million were obtained. The profits obtained were depending on the flexibility of aspiration values. The trends of interval-valued of optimal solution were moved in tandem with aspiration values. The profit of red meat production was successfully obtained with the variability of fuzzy memberships in fuzzy linear programming.

Keyword- Optimization, Linear Programming, Fuzzy number, Meat Production

I. INTRODUCTION

Linear programming has been known for years as one of the tools in decision making. Linear programming is a powerful method for maximizing goal achievement or minimizing costs while satisfying all kinds of side conditions. For example, Sakawa et al., [1] investigate maximum profit problem in production and transportation planning using linear programming. They combined the two problems, which is the profit, cost problem and also the transportation problem. The basis of this real life problem was solved using linear programming. A general linear programming is typically defined as

$$\begin{aligned} \text{Max} \quad & Z = cx \\ \text{Subject to} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

where $c = (c_1,..,c_n)$, $b = (b_1,..,b_n)^T$ and $A = \lfloor a_{ij} \rfloor_{m\times n}$. In this maximizing linear programming problem, all of the coefficients are crisp.

However, in the real-world decision problems, a decision maker does not always know the exact values of the coefficients taking part in the problem, and that vagueness in the coefficients may not be of a probabilistic type. The decision maker can model the inexactness by means of fuzzy coefficients. The introduction of fuzzy coefficients in linear programming had been suggested since ninety seventies as the knowledge of fuzzy theory developed. Bellman and Zadeh [2] suggested that fuzzy mathematical programming applies fuzzy set theory and mathematical programming approaches in linear programming, nonlinear programming, dynamic linear programming, goal programming. The fuzzy knowledge is used to optimize the alternative of which constraints have fuzzy coefficients, fuzzy variables or fuzzy inequalities. The objective functions might also have fuzzy coefficients. Fuzzy linear programming has received significant contribution in solving optimization problem as the model has been formulated to allow flexibility of constraint and fuzziness in the objective function. In an attempt to substitute crisp coefficients with fuzzy numbers, many researchers have proposed the various approaches of fuzzy linear programming. Tanaka and Asai [3], Zimmermann [4] introduced fuzzy linear programming problem in fuzzy environment. Tong [5], Gasimov and Yenilmez [6] among others, considered single objective mathematical programming with all fuzzy parameters. The flexibility of fuzzy numbers have played pivotal role in fuzzy linear programming. Due to this flexibility, Su [7] introduced interval-valued fuzzy numbers to fuzzify the crisp linear programming to three cases. Rommelfanger [8] proposed a general model of linear programming problem with fuzzy coefficients in objective function and constraints. The general model of fuzzy linear programming problem is presented by the following system:

$$\tilde{C}_1 x_1 \oplus \tilde{C}_2 x_2 \oplus \ldots\ldots\ldots \oplus \tilde{C}_n x_n \rightarrow \max$$

Subject to $\quad \tilde{A}_{i1} x_1 \oplus \tilde{A}_{i2} x_2 \oplus \ldots\ldots\ldots \oplus A_{in} x_n \tilde{\leq} \tilde{B}_i \,,\ i = 1,\ldots,m$

$$x_1, x_2, \ldots\ldots, x_n \geq 0 .$$

$\tilde{A}_{ij}$, $\tilde{B}_i$, $\tilde{C}_j$, $i = 1,\ldots,m$; $j = 1,\ldots,n$, are fuzzy set in R. Symbol $\oplus$ represent the extended addition. The interpretation of the inequality relation $\tilde{\leq}$. Membership of fuzzy number $\tilde{A}$ is given by

$$\tilde{A} = \{(x, f_a(x)) \mid x \in R\} \text{ with } f_A(x) = \begin{cases} 1 & \text{if } x = a \\ 0 & \text{else,} \end{cases}$$

Since its inception, fuzzy linear programming has been applied in diverse application especially in decision making. Vasant et al., [9] courageously coined the idea of fuzzy linear programming as a modern tool in decision making. They used a special membership function to linear programming in solving in real life industrial problem of mix product selection. Wu [10] provided a framework of fuzzy linear programming model for function management division dealing with manpower allocation problem within matrix organization. Most of these researches used a single objective function with fuzzy numbers. Baba et al., [11] extended single objective function and employ fuzzy multi-objective linear programming to solve the multi-objective no-wait flow shop scheduling problem in a fuzzy environment. This research explains that the problem may have many objective functions and constraints. Liu [12] used conventional inventory models to determine the selling price and order quantity for a retailer's maximal profit with exactly known coefficients. The recent development in multi-objective functions does not connote any incapability in single objective function. The number of objective functions is totally depends on the objective of real life problems. Nasseri et al., [13] presented a new method for solving fuzzy linear programming problems using of linear ranking function which is look similar to simplex method. Although it looks straight forward with the embedded simplex method and single objective function, the method never been test in real life applications. Jana and Roy [14] and Li and Lai [15] presented a solution procedure of multi-objective fuzzy linear programming problem with mixed constraints and its application in solid transportation problem. Despite the successful applications of single and multiple objective functions of fuzzy linear programming in various real life problems, very few literatures focus its application in meat production. There was a research conducted by Kahraman et al., [16] aiming to provide an analytical tool to select the best catering firm and not directly to meat production. Fuzzy analytic hierarchy process was used to compare these catering firms and far little attention has been paid to the application of fuzzy linear programming to meat production.

Against all this background, this paper proposes a solution in maximizing profit of red meat production using single objective function of fuzzy linear programming. Specifically the objective of this paper is to solve fuzzy single objective linear programming of red meat production problem. With a single objective function of maximizing profit, this paper also examines the effect of alpha-cuts to the profit. The basic idea behind this fuzzy based method is the existence of intervals in meat production variables such as volume of consumption, price of meat and also livestock population. Inexactness in these variables paves the way to represent magnitude of meat production variables in fuzzy numbers. This is in line with definition of fuzzy number where it does not refer to one single value but rather to a connected set of possible values [17]. Unclear intervals of meat production variables in Malaysia become the basis of information in constructing fuzzy linear programming to obtain optimal values. This paper proceeds as follows. Section II discusses the general definition of fuzzy linear programming and its affiliates. Section III describes a case of using secondary data of meat production to the objective function of fuzzy linear programming. Section IV presents empirical analysis of fuzzy linear programming with meat production data. The paper ends with discussion and conclusion in Section V.

II. PRELIMINARIES

The following definitions are given to make this paper self-contained.

Definition1: Triangular Fuzzy Number [17].

Let $\tilde{m} = (l, m, r)$ be a triangular fuzzy number, where the memberships function $\mu_{\overline{m}}$ of $\tilde{m}$ is given by

$$\mu_{\overline{m}} = \begin{cases} \dfrac{x - l}{m - l} & (l \leq x \leq m) \\ \dfrac{r - x}{r - m} & (m \leq x \leq r) \end{cases} .$$

It is easy that a triangular fuzzy number $\tilde{m} = (l, m, r)$ is reduced to a real number m can be written as a triangular fuzzy number. $\tilde{m} = (m, m, m)$.

The fuzzy triangular numbers are used in defining fuzzy coefficients in linear programming.

Definition 2 : Alpha-Cuts [19].

The α-cut, $\alpha \in [0,1]$, of a fuzzy number A is crisp set defined as

$$A_\alpha = \{x \in R : \mu_A(x) \geq \alpha\}$$

Every α -cut of a fuzzy number A s a closed interval $A_\alpha = [A_L(\alpha), A_U(\alpha)]$, where

$$A_L(\alpha) = \inf\{x \in R : \mu_A(x) \geq \alpha\},$$
$$A_U(\alpha) = \sup\{x \in R : \mu_A(x) \geq \alpha\}.$$

In the fuzzy linear programming, alpha-cuts are termed as value of aspiration, λ. The value of aspirations are used to obtain the optimal solutions in the case study of meat production.

Definition 3: Fuzzy number linear programming [13]. A fuzzy number linear programming problem is defined as follows:

$$\text{Max } \tilde{z} = \tilde{c}x$$

$$\text{s.t.} \quad Ax=b$$
$$x \geq 0$$

where

$$b \square R^m, b \square R^n, A \square R^{m\times n}, \tilde{c}^T \square (F(R))^n$$

and $\Re$ is linear ranking function.

Since fuzzy number linear programming is also subfamily of fuzzy linear programming thus the latter is used throughout this paper.

Jana and Roy [14] proposed multi-objective linear programming problem to solve solid transportation problems with mixed constraints. With a simple modification to the multi-objective linear programming, the following definition is proposed to solve single objective linear programming problem.

*Definition 4***:** Single Objective Linear Programming Problem.

Minimize $$Z = \sum_{j=1}^{n} c_j x_j$$

Subject to $$\sum_{j=1}^{n} a_{ij} x_j \geq b_i, \qquad \text{for } i = 1,2,3,\ldots\ldots, m_1$$

$$\sum_{j=1}^{n} a_{ij} x_j \leq b_i \qquad \text{for } i = m_1+1, m_1+2, m_1+3, \ldots\ldots, m_2$$

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \qquad \text{for } i = m_2+1, m_2+2, m_2+3, \ldots\ldots, m_3$$

$$x_j \geq 0 \qquad j = 1,2,3,\ldots\ldots, n$$

This definition is extended to fuzzy linear programming problem where all coefficients are fuzzy numbers. These conceptual definitions are relevant in maximizing profit OF the following empirical study.

III.PRODUCTION DATA

Data of production of red meat are retrieved from Malaysian Department of Veterinary Service Annual Report [20]-[25]. Two categories of red meats viz. beef and mutton are considered in this empirical analysis. Table 1 shows the secondary data of consumptions, productions and availability of the red meat.

TABLE 1
Production, Consumption and Availability according to Meat Category

Category	Meat		
	Beef	Mutton	Availability
Value, RM million	464.59	33.51	
Recorded Slaughter (M.ton)	114280	19284	
Consumption (M.ton)	148546	15072	
Per Capita Consumption (M.ton)	5.81	0.59	
Live stock Population (M.ton)			1305376
Output of Livestock (M.ton)			27978

The information from Table 1 is translated into the fuzzy linear programming problem with the aim to obtain the maximum profit earned by the country. The next section describes the computation steps leading to optimal solution.

IV. OPTIMAL SOLUTION

Based on fuzzy simplex method and multi-objective function proposed by Nasseri, et al., [13] and Jana and Roy [14] the following steps are proposed aiming to obtain the maximum profit of red meat production in Malaysia.

Step 1: Write linear programming problem.

Let assign the variable to each x_1 and x_2; where $x_1 =$ beef and x_2 = mutton.

The objective function for the problem is created by adding profit of each types of meat for the studied year to the optimal profit.

The full linear programming form can be produced in following structure.

Maximize $Z = \tilde{c}_1 x_1 + \tilde{c}_2 x_2$

Subject to

$$\tilde{a}_{11} x_1 + \tilde{a}_{12} x_2 \leq \tilde{b}_1$$

$$\tilde{a}_{21} x_1 + \tilde{a}_{22} x_2 \geq \tilde{b}_2$$

$$\tilde{a}_{31} x_1 + \tilde{a}_{32} x_1 \leq \tilde{b}_3$$

$$x_1, x_2 \geq 0$$

The constraints can be defined as,

$\tilde{a}_{11}, \tilde{a}_{12}$ = recorded slaughter

$\tilde{a}_{21}, \tilde{a}_{22}$ = consumption

$\tilde{a}_{31}, \tilde{a}_{32}$ = consumption per capita

$\tilde{b}_1, \tilde{b}_2, \tilde{b}_3$ = availability.

Based on the empirical data, the problem can be written as follows,

Maximize

$Z = 464.59x_1 + 33.51x_2$

Subject to

$$114280x_1 + 19284x_2 \leq 1305376$$

$$148546x_1 + 15072x_2 \geq 27978$$

$$5.81x_1 + 0.59x_2 \leq 27978$$

$$x_1, x_2 \geq 0$$

Step 2: Obtain the optimal value of the problem in Step 1.

Using a linear programming software, the optimal values for the problem is obtained. Table 2 shows the optimal solution for linear programming of beef and mutton problem.

Product	**Beef , x_1**	**Mutton, x_2**			
Quantity to make	11.42261	0			
Objective function					
Profit (RM million)	464.59	33.51	5306.83		
Constraint					Available
Recorded Slaughter	114,280.00	19,284.00	1305376.00	<=	1305376.00
Consumption	148,546.00	15,072.00	1696783.19	>=	27978.00
Consumption per capita	5.81	0.59	66.37	<=	27978.00

TABLE 2
Optimal solutions for the linear programming

Step 3: Create Fuzzy numbers for Coefficients.

From the crisp amount in Table 1, the fuzzy numbers for each coefficient are calculated. The fuzzy numbers for each coefficient are presented in Table 3.

Coefficient	**Left side**	**Crisp number**	**Right side**
$\tilde{c}_1$	464.59	464.59	557.508
$\tilde{c}_2$	33.51	33.51	40.212
$\tilde{a}_{11}$	68568	114280	114280
$\tilde{a}_{12}$	11570.4	19284	19284
$\tilde{a}_{21}$	89127.6	148546	148546
$\tilde{a}_{22}$	9043.2	15072	15072
$\tilde{a}_{31}$	3.486	5.81	5.81
$\tilde{a}_{32}$	0.354	0.59	0.59
$\tilde{b}_1$	783225.6	1305376	1305376
$\tilde{b}_2$	16786.8	27978	27978
$\tilde{b}_3$	16786.8	27978	27978

TABLE 3
Fuzzy numbers of coefficients

Step 4: Form the fuzzy linear programming problem.

The fuzzy linear problem can be written as follows,

Maximize

$$Z = (464.59, 464.59, 557.508)x_1 + (33.51, 33.51, 40.212)x_2$$

Subject to

$$(68568, 114280, 114280)x_1 + (11570.4, 19284, 19284)x_2 \leq (783225.6, 1305376, 1305376)$$

$$(89127.6, 148546, 148546)x_1 + (9043.2, 15072, 15072)x_2 \geq (16786.8, 27978, 27978)$$

$$(3.486,5.81,5.81)x_1 + (0.354,0.59,0.59)x_2 \le (16786.8,27978,27978)$$

$$x_1, x_2 \ge 0$$

Step 5: Outline the sub-problem of objective linear programming problem.

The fuzzy linear programming problem is split into eight sub-problems of crisp linear programming. Table 4 lists all the sub-problems inherited from Step 4.

TABLE 4
Sub-problems for Crisp Linear Programming.

Maximize $Z^{11} = 464.59x_1 + 33.51x_2$ Subject to $114280x_1 + 19284x_2 \le 1305376$ $148546x_1 + 15072x_2 \ge 27978$ $5.81x_1 + 0.59x_2 \le 27978$ $x_1, x_2 \ge 0$	Maximize $Z^{12} = 557.508x_1 + 40.212x_2$ Subject to $114280x_1 + 19284x_2 \le 1305376$ $148546x_1 + 15072x_2 \ge 27978$ $5.81x_1 + 0.59x_2 \le 27978$ $x_1, x_2 \ge 0$
Maximize $Z^{13} = 464.59x_1 + 33.51x_2$ Subject to $114280x_1 + 19284x_2 \le 783225.6$ $148546x_1 + 15072x_2 \ge 16786.8$ $5.81x_1 + 0.59x_2 \le 16786.8$ $x_1, x_2 \ge 0$	Maximize $Z^{14} = 557.508x_1 + 40.212x_2$ Subject to $114280x_1 + 19284x_2 \le 783225.6$ $148546x_1 + 15072x_2 \ge 16786.8$ $5.81x_1 + 0.59x_2 \le 16786.8$ $x_1, x_2 \ge 0$
Maximize $Z^{15} = 464.59x_1 + 33.51x_2$ Subject to $68568x_1 + 11570.4x_2 \le 1305376$ $89127.6x_1 + 9043.2x_2 \ge 27978$ $3.486x_1 + 0.354x_2 \le 27978$ $x_1, x_2 \ge 0$	Maximize $Z^{16} = 557.508x_1 + 40.212x_2$ Subject to $68568x_1 + 11570.4x_2 \le 1305376$ $89127.6x_1 + 9043.2x_2 \ge 27978$ $3.486x_1 + 0.354x_2 \le 27978$ $x_1, x_2 \ge 0$
Maximize $Z^{17} = 464.59x_1 + 33.51x_2$ Subject to $68568x_1 + 11570.4x_2 \le 783225.6$ $89127.6x_1 + 9043.2x_2 \ge 16786.8$ $3.486x_1 + 0.354x_2 \le 16786.8$ $x_1, x_2 \ge 0$	Maximize $Z^{18} = 557.508x_1 + 40.212x_2$ Subject to $68568x_1 + 11570.4x_2 \le 783225.6$ $89127.6x_1 + 9043.2x_2 \ge 16786.8$ $3.486x_1 + 0.354x_2 \le 16786.8$ $x_1, x_2 \ge 0$

Step 6: Obtain the optimal solution for the sub-problems.

The optimal solutions of the sub-problems are obtained using linear programming solver.

$$x^{11*} = \left(x_1^{11*}, x_2^{11*}\right) = (11.42261, 0),\ Z^{11*}\left(x^{11*}\right) = 5306.83091$$

$$x^{12*} = \left(x_1^{12*}, x_2^{12*}\right) = (11.42261, 0),\ Z^{12*}\left(x^{12*}\right) = 6368.19709$$

$$x^{13*} = \left(x_1^{13*}, x_2^{13*}\right) = (6.85357, 0),\ Z^{13*}\left(x^{13*}\right) = 3184.09854$$

$$x^{14*} = \left(x_1^{14*}, x_2^{14*}\right) = (54.84773, 0),\ Z^{14*}\left(x^{14*}\right) = 30578.04887$$

$$x^{15*} = \left(x_1^{15*}, x_2^{15*}\right) = (19.03769, 0),\ Z^{15*}\left(x^{15*}\right) = 8844.71818$$

$$x^{16*} = \left(x_1^{16*}, x_2^{16*}\right) = (19.03769, 0),\ Z^{16*}\left(x^{16*}\right) = 10613.66181$$
$$x^{17*} = \left(x_1^{17*}, x_2^{17*}\right) = (11.42261, 0),\ Z^{17*}\left(x^{17*}\right) = 5306.83091$$
$$x^{18*} = \left(x_1^{18*}, x_2^{18*}\right) = (11.42261, 0),\ Z^{18*}\left(x^{18*}\right) = 6368.19709$$

Step 7: Find the upper and lower bound (U_k and L_k) from the objective value obtained in Step 6.

So, $1 \le r \le k \Rightarrow 1 \le r \le 1 \Rightarrow r = 1, 1 \le s \le 8$

Thus

$$L_1 = \min\left\{Z\left(x^{rs*}\right)\right\} = \min\left\{Z\left(x^{11*}\right), Z\left(x^{12*}\right), Z\left(x^{13*}\right), Z\left(x^{14*}\right), Z\left(x^{15*}\right), Z\left(x^{16*}\right), Z\left(x^{17*}\right), Z\left(x^{18*}\right)\right\}$$
$$= 3184.09854$$
$$U_1 = \max\left\{Z\left(x^{rs*}\right)\right\} = \max\left\{Z\left(x^{11*}\right), Z\left(x^{12*}\right), Z\left(x^{13*}\right), Z\left(x^{14*}\right), Z\left(x^{15*}\right), Z\left(x^{16*}\right), Z\left(x^{17*}\right), Z\left(x^{18*}\right)\right\}$$
$$= 30578.04887$$

Step 8: Create Initial Fuzzy Model with Value of Aspirations and Maximize It.

The crisp linear programming is formulated as follows;

Max λ,

Subject to $27393.95033\lambda - 464.59x_1 - 33.51x_2 + 30578.04887 \le 0$,

$$(114280 - 45712\lambda)x_1 + (19284 - 7713.6\lambda)x_2 - 522150.4\lambda - 1305376 \le 0$$
$$(148546 - 59418.4\lambda)x_1 + (15072 - 6028.8\lambda)x_2 - 11191.2\lambda - 27978 \le 0$$
$$(5.81 - 2.324\lambda)x_1 + (0.59 - 0.236\lambda)x_2 - 11191.2\lambda - 27978 \le 0$$
$$0 \le \lambda \le 1,\ x_1, x_2 \ge 0;$$

Step 9: Obtain Optimal Value.

By setting the value of aspiration, λ, the values of x_1, x_2 and Z can be obtained.

For example, when $\lambda = 1$, the optimal solutions are $x_1 = 116.95368$, $x_2 = 107.02741$ and $Z = 57{,}922.00$. The possible optimal solutions with their respected values of inspiration can be seen in Table 5.

TABLE 5

Optimal solution with different value of aspiration, λ.

Value of aspiration, λ	x_1	x_2	Z
0.3	38.48303	624.51281	38,806.25
0.5	75.38123	276.14620	42,881.25
0.7	94.42017	175.68286	49,753.80
0.9	109.8776	124.87532	55,232.60
1.0	116.95368	107.02741	57,922.00

Fuzzy linear programming yields a number of optimal solutions depending on the flexibility of aspiration values. The maximum profits of red meat are varies depending on the value of aspiration, and variables of red meat.

V. Discussion and Conclusion

It has been shown that fuzzy linear programming was successfully employed in obtaining the optimal solution of red meat production. Fuzzy linear programming problem used fuzzy numbers and memberships of aspiration values contributed to the optimal solution. In this empirical analysis, trends of the optimal profits against the value of aspiration are presented in Fig 1.

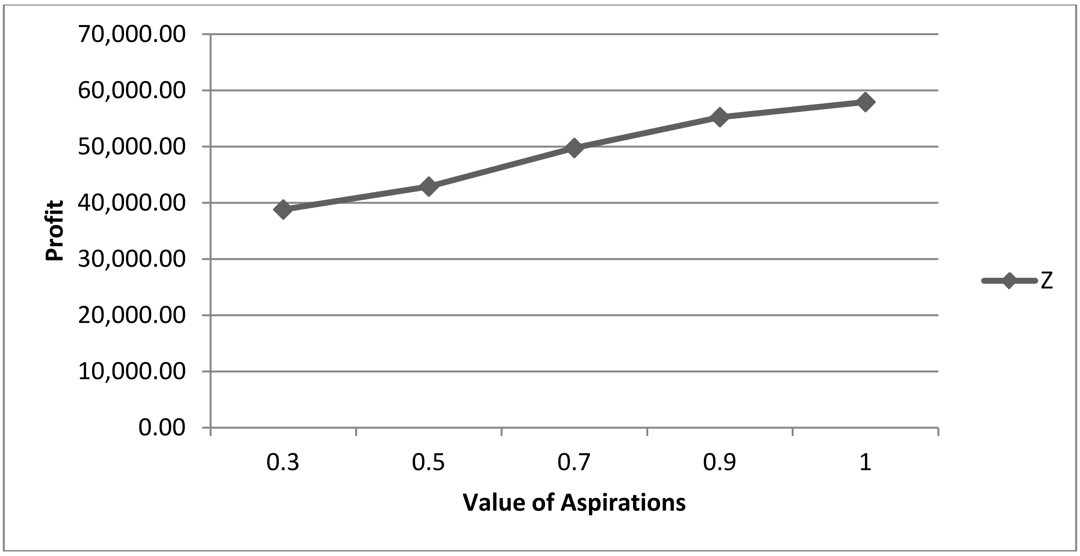

Fig 1 Trends of profit against the values of aspiration.

It can be seen that the profit of meat production was increased in tandem with the aspiration level.

It is also good to observe the trends of the beef and mutton productions against the value of aspirations. Fig 2 shows trends of the two variables which indicate that the gaps between two variables are getting smaller once the aspiration values approaching the highest value.

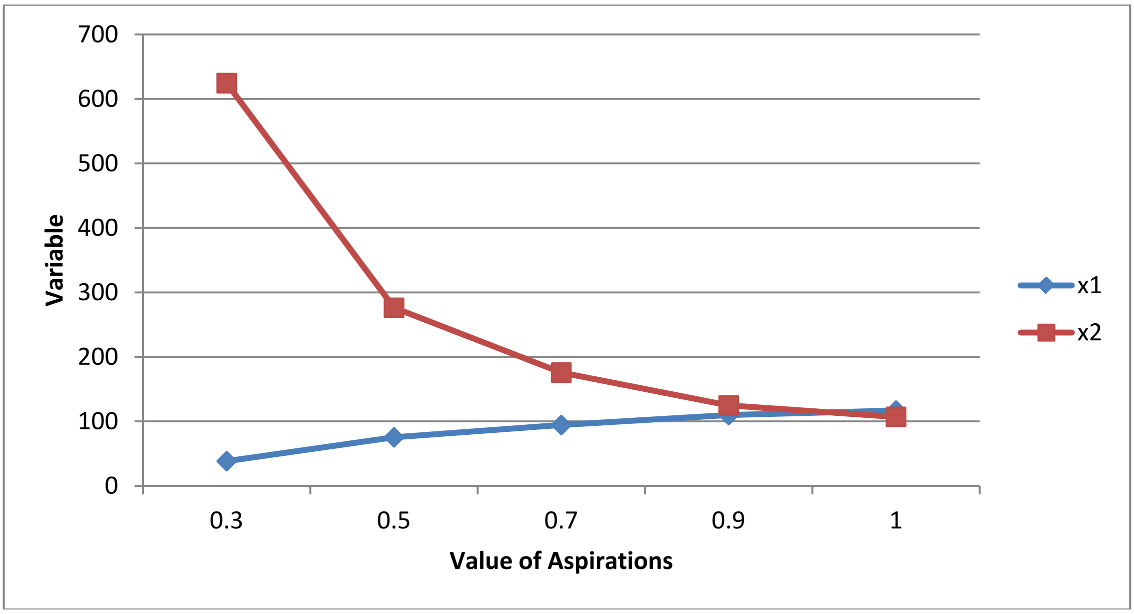

Fig 2 Trends of variables against value of aspiration level.

This paper has signified the application of fuzzy linear programming in maximizing profit earned by the country based on the red meat productions. The profit in closed interval of [RM 38.806.25 million, RM 57,922.00 million] was obtained once the beef and mutton were [3848303 tonne, 11695368 tonne] and [62451281 M.ton, 10702741M.ton] respectively. Furthermore, the optimal solution with different values of aspiration has also been investigated. Fuzzy linear programming produced the optimal solution when the value of aspiration in the interval [0.3, 1.0]. It is also good to note that the maximum profit of the same production problem using the crisp linear programming was RM 5306.83 million. This profit was felled in the closed interval of the fuzzy profit. The profits obtained using the fuzzy linear programming were given in an interval value. However, the advantage of the fuzzy results versus crisp results depends critically on the value of aspiration and the defined fuzzy numbers. Future research may be undertaken to include more variables and constraints to provide comprehensive results.

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