# Voltage-Vector in Three-Dimensional Space Vector Modulation based on $\alpha \boldsymbol{\beta r}$ Coordinate for Four-Leg Active Filter 

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#### Abstract

This paper presents a concept of voltage-vector in three-dimensional space vector modulation for four-leg active filter which are developed based on Euler angle rotation method, and its produces a skewed rod shape consisting of twelve-pairs of asymmetrical tetrahedron nonzero switching vector in which voltage-vector reference is determined by four-pairs of its tetrahedron. Voltage-vector reference and calculation of duty cycle based forms of asymmetrical $\alpha \beta r$-coordinate are presented. Simulation results for converter topology provided a new understanding approach of harmonics reduced in 3D SVM which is different from so far.


Keywords-Three-dimensional space vector modulation, Euler angle rotation method, asymmetrical tetrahedron, voltage-vector reference, four-leg active filter

## I. INTRODUCTION

In power conditioning, an active filter with a controller is used to reduce harmonics by injecting an unwanted current into the system wherein the currents have same amplitude and opposite polarity. Control against currents compensation can be performed by the instantaneous active-reactive power theory in [1]. This theory becomes known as the pq-pqr theory using Clarke transformation for controlling of the currents compensation. For an active filter three-phase four-wire systems mapping matrices model is used related to currents flow in the neutral wire in [2],[3]. Power analysis for neutral current compensation uses p -axes, q -axes and r -axes (the pqrcoordinate). The existence of $r$-axes relates to events oscillations in the neutral wire [4], when a source connected with the nonlinear load on the neutral wire will be no currents. The three-phase four-leg converter as enhancements of three-phase four-wire systems is developed for harmonic compensation, reactive power compensation, load balances and neutral currents compensation [5]. Control strategy is done pulsewidth modulation (PWM) technique by current regulator output on the neutral wire three-phase four-leg systems. In a series of four-leg converter, three-phase currents which flow in the converter-R, S, T and neutral each must be controlled [6],[7]. A zero-sequence voltage vector causes neutral-zero sequence current flow in the neutral component. Three dimensional space vector modulations for three-phase four-leg active filter are performed by 16 switching voltage-vectors [8]. The active filter topology of upright rod-shaped with a neutral component at a symmetrical center consists of fourteen nonzero vector switching (NZSV) plus two zero switching vector (ZSV). Compensation current control is done by setting the sixteen voltage vectors.
In this paper, we discuss a concept of voltage-vector in three-dimensional space vector modulation by using Euler angle rotation method that was developed based on modified of Kim-Akagi model for control in threephase four-leg active filter. The projection pqr-coordinate result of rotation method in $\alpha \beta 0$-coordinate generates asymmetrical tetrahedron. This is in contrast to the upright rod-shaped models. R-axes as a harmonic indicator having a slope towards the vertical axes by a certain angle, and topology based on rotation methods result twelve-pairs of asymmetrical tetrahedron. The discussion conducts four-pairs of tetrahedron qualified as voltage vector reference.

## II. FOUR-LEG ACTIVE FILTER

Active filter experiences rapid development from year to year along with the widespread use of the IGBT (insulated gate bipolar transistor) as a converter replaces SCR and power MOSFET function. As its development of three-phase four-wire system [9], active filter three-phase four-leg system has many functions which can be used to current harmonic compensation, reactive power compensation, load balancing and neutral current compensation related to symmetrical component [5]. In three-phase four-wire system with DC-Link, where the neutral wire is connected to the center of split capacitor, is unstable control system. Converter-R, $\mathrm{S}, \mathrm{T}$ are not optimal in controlling a zero-sequence current flow in the neutral wire. Each current flow in converter$\mathrm{R}, \mathrm{S}, \mathrm{T}$ and neutral must be controlled. Therefore the fourth leg converter is developed to serve as a zerosequence current controller in active filter and, is used to eliminate it without using a DC-link voltage.


Figure-1.Active filter three-phase four-leg system with various nonlinier loads.
In figure-1 it appears that active filter consists of eight pieces IGBT connected to four pairs converter-R, $\mathrm{S}, \mathrm{T}$ and neutral, where each gate IGBT converter is controlled by the converter control modules. Gate control in IGBT converter is conducted by control modules with SVM rotation method. Converter-R, S, and T are connected to its source, while neutral converter connected to the neutral wire and is connected to a nonlinear load. Nonlinear load consists of six pairs of $20 \mathrm{~W} / 220 \mathrm{~V}$ LED lamps. If current on diode- 1 is greater than diode-2, then overcurrent flow in the neutral wire direction, and vice versa, if the current in diode- 1 is smaller than diode2, then the neutral wire will supply current to the branch point Kirchoff's with a certain magnitude so that results equal to zero. Active filters will inject current with magnitude of the inverse neutral current in its process. Akagi's theory consists of an IGBT converter series as an active filter three-phase three-wire system. Compensation current control on a combination of three pairs of IGBT converter and the split capacitor are connected to the neutral wire which produces six prisms comprising twelve of tetrahedron in three-dimensional space-vector.

## III. ROTATIONAL METHOD BASED-ON EULER ANGLES

Power transformation in a space-vector modulation (SVM) is a stationary reference frame transformation ( $\alpha \beta$ coordinate) move on a rotating reference frame (dq-coordinate). This transformation can be used to analysis the power theory and the control system on induction generator. Control in SVM by the instantaneous power theory has been developed in time domain. Development pqr-coordinate on the mapping matrices model determines the presence of r-axes component related to neutral in instantaneous power with the voltage or current source orthogonal vectors which is in tetrahedron space. If source is connected to non-linear load, then the neutral wire current is always there, if source is connected with linear load then the neutral wire current is zero. The mapping matrices model transformation is a rotation of twice and three-time of Euler angle [10].
A rotation can be done with continuous configuration between two Cartesian coordinate. Rotation, R ( $\theta_{1}, \theta_{2}$, $\theta_{3}$ ), is performed move on the rotation axis of the stationary reference frame to the rotating frame of reference. Three Euler angles $\left(\theta_{1}, \theta_{2}, \theta_{3}\right)$ is determined by the opposite rotation clockwise. The $\alpha \beta$-coordinate is a reference frame and a stationary pqr-coordinate as a reference rotating fame. Notation of Euler rotation angles can be determined as $\mathrm{R}\left(\theta_{1}, \theta_{2}, \theta_{3}\right)=\mathrm{R}_{\mathrm{z}}(\theta 3) \mathrm{R}_{\mathrm{N}}(\theta 2) \mathrm{R}_{\mathrm{z}}(\theta 1)$ where the boundary condition, $0<\theta_{1}, \theta_{2}<2 \pi$ and $0<\theta_{3}<\pi$. It can be seen that the rotation of the rotating reference frame is done continuously from stationary reference frame and rotating reference frame that can be done in a systematic incorporation axis.
Euler angle rotation matrix is formed from the multiplication of each rotation axis and can be described as follows
$R\left(\theta_{1}, \theta 2, \theta_{3}\right)=$

$$
\left[\begin{array}{ccc}
\cos \theta_{3} & \sin \theta_{3} & 0  \tag{1}\\
-\sin \theta_{3} & \cos \theta_{3} & 0 \\
0 & 0 & 1
\end{array}\right]\left[\begin{array}{ccc}
\cos \theta_{2} & 0 & \sin \theta_{2} \\
0 & 1 & 0 \\
-\sin \theta_{2} & 0 & \cos \theta_{2}
\end{array}\right] \times\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \theta_{1} & \sin \theta_{1} \\
0 & -\sin \theta_{1} & \cos \theta_{1}
\end{array}\right]
$$

Multiplication Euler angles for two variable angles $\theta 1, \theta 2$ are as follows

$$
\begin{align*}
& \mathrm{R}\left(\theta_{1}, \theta_{2}\right)=\mathrm{R}_{\mathrm{N}}\left(\theta_{2}\right) \mathrm{R}_{\mathrm{z}}\left(\theta_{1}\right) \\
& \quad=\left[\begin{array}{ccc}
\cos \theta_{2} \cos \theta_{1} & \cos \theta_{2} \sin \theta_{1} & \sin \theta_{2} \\
-\sin \theta_{1} & \cos \theta_{1} & 0 \\
-\sin \theta_{2} \cos \theta_{1} & -\sin \theta_{2} \sin \theta_{1} & \cos \theta_{2}
\end{array}\right] \tag{2}
\end{align*}
$$

in this case $R_{N}\left(\theta_{2}\right)=R y\left(\theta_{2}\right)$ is the rotation in the normal direction of rotation of the $y$-axis. This result is similar to mapping matrices model in [4]. Thus, the principle of rotation can be used to modify the mapping matrices
model in solving problems related to the pqr-coordinate. More on the coordinate pqr-transformation into the following equation:

$$
\begin{align*}
{\left[\begin{array}{l}
X_{p} \\
X_{q} \\
X_{r}
\end{array}\right] } & =\left[\begin{array}{ccc}
\cos \theta_{2} \cos \theta_{1} & \cos \theta_{2} \sin \theta_{1} & \sin \theta_{2} \\
-\sin \theta_{1} & \cos \theta_{1} & 0 \\
-\sin \theta_{2} \cos \theta_{1} & -\sin \theta_{2} \sin \theta_{1} & \cos \theta_{2}
\end{array}\right]\left[\begin{array}{l}
X_{\alpha} \\
X_{\beta} \\
X_{0}
\end{array}\right]  \tag{3}\\
& =\sqrt{\frac{2}{3}}\left[\begin{array}{ccc}
\cos \theta_{2} \cos \theta_{1} & \cos \theta_{2} \sin \theta_{1} & \sin \theta_{2} \\
-\sin \theta_{1} & \cos \theta_{1} & 0 \\
-\sin \theta_{2} \cos \theta_{1} & -\sin \theta_{2} \sin \theta_{1} & \cos \theta_{2}
\end{array}\right]\left[\begin{array}{ccc}
1 & -\frac{1}{2} & -\frac{1}{2} \\
0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{array}\right]\left[\begin{array}{l}
X_{a} \\
X_{b} \\
X_{c}
\end{array}\right] \tag{4}
\end{align*}
$$

in this case $X \equiv \mathrm{i}, \mathrm{V}$ (as current or voltage)

(a)

(c)

(b)

(d)

Figure-2. Coordinate Transformation: a,b).Kim-Akagi’s mapping matrices model, c,d).Euler angle rotational method [11]
A zero sequence voltage and a zero sequence current arise due to nonlinear loads in three-phase systems an analysis of sixteen voltage vector arises because of the fourth-wire so that the concept of the plane vector of two-dimension (coordinate- $\alpha \beta$, dq) turn into three-dimensional space vector (pqr-coordinate). Topologies of active filter so far have an upright shape stems for analysis pulsewidth modulation (PWM) four-leg converter system. Based on calculation of the Euler angles rotation method is given in the equation-3 above and then pqrcoordinate projected into coordinate- $\alpha \beta 0$ generates skewed shape such as following in the figure-2c and figure2d. Explicitly the angle of $\theta_{2}$ is specified but its existence can not be there. Therefore, for simplicity the angle of $\theta_{2}$ is tilted angle of vertical axes.


Figure-3. pqr-coordinates projection in (a).dq-coordinate, (b). $\alpha \beta$-coordinate (c). Skewed rod shape of regular hexagonal composing.

## A. Tetrahedron in $\alpha \beta$ r-coordinate

Skewed rod shape is composed of two pieces of regular hexagonal stacked with a shift on the center point, where the regular hexagonal top and bottom are not centralized. The top is the positive part that indicates switch "on" and the bottom is contrary which indicates switch 'off' from the IGBT converter. The central part of the regular hexagonal arrangement of two octagonal shape has two-pieces with a central point, namely ( $1,1,1,1$ ) and $(0,0,0,0)$ are not coincident with each other. Six sectors of regular hexagonal shape of tetrahedron are a glimpse of the couple together form upside down. Overall, the three-dimensional space vector of active filter three-phase four-leg system can be broken down into twelve-pair asymmetry tetrahedrons.
Sector-2 and sector-3 as voltage vector is regular hexagonal top, and a bottom of each is paired as well as sector-5 and sector-6 regular hexagonal top and bottom, where the couple has a two-fruit midpoint in figure-4. The combination of regular hexagonal top and bottom at the midpoint $(1,1,1,1)$ and $(0,0,0,0)$ produces four-pairs asymmetry tetrahedron which is used as the reference voltage-vector, while other sectors which do not have the qualifications to serve as a reference voltage-vector. Each tetrahedron is voltage-vector that qualifies as the reference voltage in three-dimensional space-vector modulation for four-leg active filter.
Asymmetrical shape between each pair tetrahedron voltage-vector indicates the occurrence of harmonic voltage. It is a harmonic indicator of $r$-axes in the $\alpha \beta r$-coordinate.

(a)


Figure-4. Pair of tetrahedron voltage-vector, a). Sector-2 up and sector-3 down, b). Sector-3 up and sector-2 down, c). Sector-6 up and sector-5 down, d). Sector-5 up and sector-6 down.

## B. Voltage vector reference

Voltage-vector reference is determined by the boundary conditions between the tangents to a circle with a point outside. Reference voltage-vector for the three-dimensional space modulation resembles a tetrahedron [12]. Tetrahedron shape can be obtained by slices of the cube and prism. Sliced prism can produce two pieces of symmetrical shape that have four planes for cover tetrahedron and five planes to cover pyramid as described in figure-5, parameter of prisms for length unit of the specified line length and the angle formed between two lines based on Pythagoras law.

ABCD point is an ideal tetrahedron form which is used to formalize voltage-vector reference, consists of fourpieces that each point can be determined by plane equations. The description for the pyramid-ACDEF is done in the same way as the tetrahedron. Outside the diagonal length and length of diagonal in the plane is determined by reference to the long side of which is equal to units. In detail, in figure-5, calculation results are given below each of parameter unit tetrahedron.


Figure-5: A sliced prism to form tetrahedron and pyramid
Plane equations of the four pairs of tetrahedron in figure-4 above can be derived by using a mathematical description that refers to figure- 5 sliced prisms and the following result as:

$$
\begin{align*}
& 2 \mathrm{~V}_{\beta}+\sqrt{3} \mathrm{~V}_{\gamma}-6 \sqrt{3}=0  \tag{5a}\\
& \sqrt{3} \mathrm{~V}_{\alpha}+\mathrm{V}_{\beta}=0  \tag{5b}\\
& \sqrt{3} \mathrm{~V}_{\alpha}-2 \mathrm{~V}_{\beta}+3 \sqrt{3}=0  \tag{5c}\\
& \mathrm{~V}_{\gamma}-2=0  \tag{5d}\\
& \sqrt{3} \mathrm{~V}_{\alpha}-\mathrm{V}_{\beta}+2 \sqrt{3}=0  \tag{5e}\\
& \sqrt{3} \mathrm{~V}_{\alpha}-\mathrm{V}_{\beta}=0  \tag{5f}\\
& 2 \sqrt{3} \mathrm{~V}_{\beta}+3 \mathrm{~V}_{\gamma}-6=0  \tag{5g}\\
& 3 \mathrm{~V}_{\alpha}-\sqrt{3} \mathrm{~V}_{\beta}+3 \mathrm{~V}_{\gamma}-6=0 \tag{5h}
\end{align*}
$$

## C. Calculation of duty cycle.

A zero sequence voltage and/or a zero sequence current are decomposed into vector position both at the midpoint of $\alpha \beta 0$-coordinate and $\alpha \beta r$-coordinate in [13]. $\mathrm{V}_{0}$ and $\mathrm{V}_{11}$ at the midpoint of the 0 -axis and r-axes in the opposite direction in which the two can form a voltage-vector reference as following equation;

$$
\begin{equation*}
\mathrm{V}_{\text {ref }}=\mathrm{d}_{\alpha} \cdot \mathrm{V}_{\alpha}+\mathrm{d}_{\beta} \cdot \mathrm{V}_{\beta}+\mathrm{d}_{\gamma} \cdot \mathrm{V}_{\gamma} \tag{6}
\end{equation*}
$$

Four-pair tetrahedron which are defined at the position are selected in turn determines the duty cycles with normalization vector in the voltage vector reference to determine the position of $V_{0}$ or $V_{11}$ last is to determine the time for each pulse pattern.

## IV. RESULTS AND DISCUSSION

An active filter three-phase four-leg system as shown in figure-1 above can be described in a Simulink/Matlab diagram as shown in figure-6 below like as in [14]. This approach is used to give result according to the model created by a pair of IGBT converter with various nonlinear loads such as LED lamps depicted in the circuit.


Figure-6.Simulink/Matlab diagram for four-leg active filter
For a measurement quality standard either excess or lack for four-leg active filter model's which is developed, we can measurement the performance of parameters such as total harmonics distortion (THD) and distortion factor (DF) comparison with the existing model [15].
Where THD and DF value is given as the following equation

$$
\begin{align*}
& T H D=\frac{1}{V_{o 1}}\left(\sum_{h=2,3, . .}^{n} V_{0 h}^{2}\right)^{1 / 2}  \tag{7}\\
& D F=\frac{1}{V_{o 1}}\left(\sum_{h=2,3, . .}^{n}\left(\frac{V_{o h}}{h^{2}}\right)^{2}\right)^{1 / 2} \tag{8}
\end{align*}
$$

Based on this developed model was obtained as follows THD $=18.23 \%$ and $\mathrm{DF}=2.37 \%$. Measurement and simulation result for four-leg active filter in 3D SVM rotation method provides a new understanding approach of harmonic reduced.

## V. CONCLUSION

A concept of voltage-vector in 3D SVM has been discussed whereby the Euler angle rotation methods has been used, and generates skewed rod shape that can be broken down into twelve pairs of asymmetrical tetrahedron. Here is discussed four pairs of the asymmetrical tetrahedron shape are related to voltage vector reference. The plane equations of four pairs tetrahedron that limits that one is described and can be used to determine the duty cycle of the voltage vector references. Modeling on the active filter three-phase four-leg system by using a Simulink/Matlab produces value of total harmonics distortion (THD) and distortion factor (DF) are interesting to be explored in the further, and which is different from so far.

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