An Analysis of the Performance of Wireless Sensor Networks

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Abstract—Designing energy-efficient algorithms becomes an important factor for extending the lifetime of sensors while still meeting functional requirements in a Wireless sensor network. It is well acknowledged that clustering is an efficient way to save energy for sensor networks. In multi-hop networks clustering is very effective in reducing communications. Cluster head role rotation and duty-cycling of nodes are two effective ways to balance energy consumption. CH rotation, also called local delegation has minimum overhead thus minimizing delay in data transmission compared to the case when re-clustering is to be done globally during which period no data transmission can take place. We propose a theoretical investigation of the parameters for optimal deployment of WSN taking into account clustering and CH rotation. The optimality considered is in view of energy cost as well as network lifetime.

Keyword-Network lifetime, Poisson process, Cluster head delegation, Epoch, Energy Models

I. INTRODUCTION

Current research and development challenges in WSN implementation are to develop low-power communication with low-cost on-node processing and self organizing connectivity/protocols and the need for extended temporal operation of the sensing node despite a (typically) limited power supply. In WSNs energy is a scare resource. But WSNs have tighter requirements on network lifetime, and recharging or replacing WSN node batteries is not a viable option. Due to this, the impact of energy considerations on the entire system architecture is much deeper in WSNs than any other networks [1]. In view of the importance of the lifetime of the nodes in WSN, in this paper, we propose a theoretical study of the expected lifetime of a node in WSN under different assumptions. For this purpose we assume certain statistical behavior of the deployment of sensors in the field. Such studies will be useful to get an estimate of the effect of the different parameters on the average lifetime expected of a node in WSN and on the expected optimality of WSN deployment in terms of the parameters involved.

The main contribution of the paper is that we investigate the phenomenon of energy depletion in a wsn in an abstract frameworks using certain simplifying assumptions. Based on the investigation, we derived the expected normalized energy loss per epoch as a function of cluster configurations. The average (expected) lifetime of a node is also derived.

Organization of this paper is as follows: Related Work is discussed in section 2**.** Sensor Distribution as a Poisson process is discussed in section 3. Lifetime Energy Models is discussed in section 4. Simulation & Results are discussed in section 5. Section 6 gives the conclusion with a direction of the future work.

II. RELATED WORK

De Silva [2] presented an analytical framework to investigate the average energy expenditure of the global re-clustering method and a local delegation strategy in which the CH role is delegated to a member of the cluster without changing the initial cluster boundary. The author analyzed and found that the area of the partial local delegation strategy clearly shows that if the distance of the CH delegation radius is limited to a certain value prior to a global re-clustering proves to be a better strategy in CH rotation to achieve energy optimization within WSNs. In the mathematical modeling ofthe average energy expenditures for global re-clustering and local delegation,the average energy from transmission of all nodes in thecluster with radius R can be formulated by using Campbell's theorem.Kenji Leibnitz[3] presented an analytical model for performance valuation of a clustered sensor network.They examined the energy required for a single round of clustering for a biologically-inspired extension of the well-known LEACH clustering method. Methods from stochastic geometry were used to model the locations of the node clusters using a hard-core Matern cluster process. Based on the computation of the energy consumption of the whole clustering procedure the most energy efficient setting of the cluster radius is determined.

Liu, An-Feng [4] analyzed the node energy consumption in different regions through the differential analysis method. Thus, the optimal parameters which maximize the lifetime can be obtained and the detailed energy consumption in different regions at different time can be also obtained. Shastry [5] presented a theoretical framework to model routing behaviour and cooperative relay selection and used this information to predict the lifetime and energy consumption of the network. The analysis uses knowledge of the spatial distribution of nodes to determine the number of packets to be transmitted as a function of distance from a sink. This number is a sum of packets due to min-hop routing and due to cooperation. These numbers are then used in an energy analysis to determine the average energy used as a function of distance, thereby predicting network lifetime. An essential feature of the analysis here is that it does not assume a high node density. The theory presented quantifies the significant gains in network performance due to node cooperation. Haijun Wang [6] proposed an optimization model for the energy consumption of clustering protocol in WSN under the condition that a large number of sensors with random deployment obey Poisson distribution. Through calculation of the optimum number of CHs, it is shown that the node density under Poisson distribution affects the optimal cluster numbers, and results in the lowest energy consumption of clustering protocol.

III. SENSOR DISTRIBUTION AS A POISSON PROCESS

Distribution of sensors over an area can be characterized as a random phenomenon. Poisson model is the simplest and the most random way in which to describe such a phenomenon. The Poisson distribution can be seen as the limiting case of the Binomial distribution where the number of points is large and the probability of success is low. The Poisson distribution [7] is given as

$$\pi_{r(\mu)} = \mu^r \left\{ \frac{e^{-\mu}}{r!} \right\} \tag{1}$$

$r \geq 0$ and μ is the Poisson distribution parameter.

In the theory on Poisson processes, 'test sets' A_1, A_2 , .. are set down in the space of distributed points denoted by A The number of points falling in the set A , N(A) can be considered an integer–valued random variable (RV). Given that the sets A_1, A_2 , .. do not overlap, the random variables $N(A_j)$ can be considered to be statistically independent. The set $\{N(A_j)\}$ constitute a random process formally.

In the context of WSN, the 'test sets' A_j can be identified with the node clusters. The number of sensors in a cluster is identified with $N(A_j)$ and is a random variable. The expected value of $N(A_j)$ is $\mu(A_j)$.

More formally ,the probability space is defined as a triple (Ω, F, P) where Ω is the set of "elementary outcomes", F is a subset of Ω (called events) and P is the probability measure falling in the range [0,1] of the real line R for every event in F. A real valued RV, X mapping into R such that $\{\omega \, \varepsilon \, \Omega; X(\omega) \leq x\}$ belongs to F for every $x \in R$. A Poisson process with state space S defined on a probability space (Ω, F, P) is a function π from Ω into subsets of S. In the case of WSN, we can consider S to correspond to R^2 on which test sets (clusters) A_j are defined. If A is such a set , the number of points of π in A is $N(A) = \neq \pi(\omega \cap A)$, where the symbol # denotes the number of points in the set following it. N(A) is a RV. In the context of R^2 space, the parameter μ can be defined in terms of density $\lambda(x)$ where 'dx' stands for 'dx1 dx2 ' in R^2. In a homogeneous Poisson process λ can be assumed to be a constant so that $\mu_A = \lambda |A|$ where |A| stands for the magnitude of area A.

In the context of WSN, we may consider homogeneous Poisson process resulting in a constant density λ_j of sensors in cluster A_j, j=1,2,…k.

Any function $f : S \rightarrow R$ will induce a function of RV X as f(X) which itself is a RV.

According to Campbell's theorem [2]

$$E\left(f(X)\right) = \int_S f(x)\, \mu(dx) = \int_S f(x)\, \lambda(x)\, dx \tag{2}$$

When applied to a cluster of area A,

$$E\left(f(X)\right) = \int_A f(x)\lambda(x)dx \tag{3}$$

If λ is constant,

$$E\left(f(X)\right) = \lambda \int_A f(x)dx \tag{4}$$

IV. LIFETIME ENERGY MODELS

We need to find the expectation of Energy spent by each node per epoch. Epoch is defined as the period during which each cluster node get a turn to be cluster Head (CH)on a local delegation basis once exactly. For each CH period there can be 'x" data gathering rounds from each cluster node.

Assuming Poisson distribution let the cluster j have a density λ_j with overall density of λ. Let R_j be the radius of cluster j while R is the overall radius of the circular region where the WSN is deployed with the BS at the centre.

We have assumed a simple first order radio model for the analysis. Here the transmitter and the receiver dissipate energy to run the radio electronics but transmitter additionally expends some energy due to its signal amplifier. Moreover, the actual power dissipated depends on the distance between transmitter and receiver. Path loss can be modeled as proportional to inverse of square of the distance if the distance is small whereas it is taken inversely proportional to the fourth power of the distance if the distance is large. In this way, to transmit an L-bit message across a distance d, the energy expenditure $E_{tr}(L,d)$ can be modelled as [2]

$$E_{tr}(L,d) = E_{tr}elec(L) + E_{tr}amp(L,d) \qquad (5)$$

where $E_{tr}elec(L) = LE_{elec}$ and

$$E_{tr}amp(L,d) = L\varepsilon_{fs}d^2 \text{ ; for free space}$$

$$= L\varepsilon_{mp}d^4 \text{ for multipath} \qquad (6)$$

The values of E_{elec}, εfs, and ε_{mp} in the simulation are specified in Table 1.

Minimization of energy spent in WSN requires distribution of traffic load evenly among all nodes in a cluster and CH delegation reduces clustering overhead to nil while at the same time minimizing delay. This assumes a homogeneous network where all the CHs are uniformly loaded which may not be the case in practice. In such a case, local delegation may have to be abandoned to favor periodic re-clustering at the expense of delay. In any case CH local delegation case can be considered for getting benchmark values of expected energy spent and the expected lifetime of a node.

We consider energy spent by a node in an epoch where each cluster node takes turn to be CH including the node under consideration. The following cases are applicable.

(i) The chosen node sends data to CH which is rotated among all cluster nodes.

(ii) The chosen node acts as a CH receiving data from other cluster nodes and sending the consolidated data to BS.

(iii) Each CH gathers data for 'x' rounds

(iv) Since the CH selection is by rotation, for simplicity of analysis, the clustering overhead can be assumed to be zero.

With the above assumptions, we proceed to find the expected value of Energy spent by each node which may in turn lead to expected life time of a node.

Case (i)

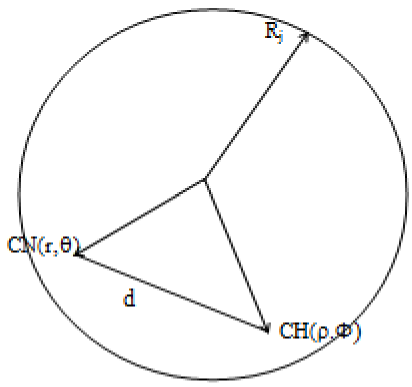

Fig.1. Cluster j with CH and CN Configuration

Consider cluster of radius R_j as shown in Fig.1. CN is assumed to be located at(r,θ). d is the distance between CH and CN. Let the CH be located at the polar location (ρ,Φ) with respect to the centre of the cluster[2].

CH (ρ,Φ) is assumed rotated through ρ: 0 to R and 0 to 2π. Cluster node (CN) at polar location (r, θ) sends data to CH at (ρ,Φ)in each case. The expected value of energy spent per node in CN sending data to CH is obtained by integrating energy cost involved over all possible locations of CN followed by all possible locations of CH, and dividing the result by the expected number of nodes in cluster j.

$$E_1 = \frac{1}{\pi R_j^2 \lambda_j} \int_0^{2\pi} \int_0^{R_j} \lambda_j \left\{ \int_0^{2\pi} \int_0^{R_j} \lambda_j \{(LE_{elec} + L \in d^n)\rho d\rho d\phi\} drd\theta \right\} \tag{7}$$

It can be shown that

$$E_1 = L\lambda_j \pi R_j^2 \left\{ E_{elec} + \varepsilon R_j^2 \right\}, \; n = 2 \tag{8}$$

$$E_1 = L\lambda_j \pi R_j^2 \left\{ E_{elec} + \frac{5}{3} \varepsilon R_j^4 \right\}, \; n = 4 \tag{9}$$

Taking energy lost per node in the cluster

$$E_1 / node = \begin{cases} \left\{ L\left(E_{elec} + \varepsilon R_j^4 \right), \; n = 2 \right. \\ \left\{ L\left(E_{elec} + \frac{5}{3} \varepsilon R_j^4 \right), \; n = 4 \right. \end{cases} \tag{10}$$

Case (ii)

As a CH, the chosen node collects data from all other nodes in the cluster and aggregates them into one packet to be sent to BS. These two components amount to the following.

$$E_2 = \lambda_j \int_0^{2\pi} \int_0^{R_j} \left\{ LE_{elec} + LE_p \right\} \lambda_j \, rdrd\theta = L\lambda_j (E_{lec} + E_p)(\pi R_j^2) \tag{11}$$

where R_j is the radius of the j^{th} cluster.

$$E_2 / node = \frac{E_2}{\pi R_j^2 \lambda_j} \; L(E_{elec} + E_p) \tag{12}$$

The third component is the transmission of data to BS which can be averaged over the overall domain of WSN of circular region with density λ and radius R since all the nodes tend to be CH in turn. That is,

$$E_3 / CH = \left(\frac{1}{\pi R_j^2 \lambda} \right) \int_0^{2\pi} \int_0^{R} \left\{ LE_{elec} + L\varepsilon r^n \right\} \lambda rdrd\theta \tag{13}$$

$$E_3 / CH = \begin{cases} LE_{elec} + L\varepsilon \frac{R^2}{2}, \; n = 2 \\ LE_{elec} + L\varepsilon \frac{R^2}{2}, \; n = 4 \end{cases} \tag{14}$$

where R is the radius of the network

Consider the total loss in the WSN per round of data transfer to CH, and assuming K clusters. Substituting for E_1, E_2 and E_3 , we have

$$E_{tot,K} = E_1 / node(N) + E_2 / node(N) + E_3 / CH(K)$$

Where N is the total number of nodes in the WSN and K is the number of cluster in the WSN.

$E_{tot,K}$ represents the total energy lost in the WSN per round of data gathering by each CH.

$$E_{tot,K} = \begin{cases} LN\left\{ E_{elec} + \varepsilon R_j^2 \right\}, \; n = 2 \\ LN\left\{ E_{elec} + \frac{5}{3} \varepsilon R_j^4 \right\}, \; n = 4 \end{cases} + LN(E_{elec} + E_p) \begin{cases} K\left(LE_{elec} + L\varepsilon \frac{R^2}{2} \right), \; n = 2 \\ K\left(LE_{elec} + L\varepsilon \frac{R^4}{2} \right), \; n = 4 \end{cases} \tag{15}$$

where K is the number of clusters in the network and $R_j \le R / K$

If each CH is allowed x rounds, then the total energy lost by the WSN will be ($x(E_{tot,K}$).

We call the period where every node get its turn to be CH and data is gathered from CN in the cluster and sent to BS as an Epoch.If we normalize the $E_{tot,K}$ as $E_{tot,K}$ $\hat{E}_{tot,K} = \frac{E_{tot,K}}{N}$ then $\hat{E}_{tot,K}$ represents the average energy lost per epoch per node with K clusters in WSN. If we invert $\hat{E}_{tot,K}$ we get average life time of a node Exp(T) as below [8]

$$\mathrm{Exp(T)} = \mathrm{Exp}\left(\frac{1}{\hat{\mathcal{E}}_{tot,K}}\right) = \frac{1}{\hat{\mathcal{E}}_{tot,K}} + \frac{1}{\hat{\mathcal{E}}_{tot,K}^{\;2}} + \frac{2}{\hat{\mathcal{E}}_{tot,K}^{\;3}} + \frac{6}{\hat{\mathcal{E}}_{tot,K}^{\;4}} + \frac{24}{\hat{\mathcal{E}}_{tot,K}^{\;5}} \qquad (16)$$

where Exp(T) is the expected value of energy lost per node per epoch for the case of K clusters

Using the above formula, the expected value of a node life time can be calculated in terms of epoch intervals using normalized value of Energy spent and the initial energy per node being assumed to be unity.

V. SIMULATION & RESULTS

The mathematical formulations discussed in Section IV have been tested for their practical effectiveness through MATLAB simulations. We analyzed a random network of 100 nodes spread over a area of radius of 80m and 50m in the simulation cases.Table 1 gives the specifications used in the simulation.

TABLE I
Specifications Adopted for the Simulated Network

Type	Simulation Parameter	Qty
Network	No. of Nodes	100
	Radius of the area	80
Radio model	E_{elec}	50nJ/bit
	ε_{fs}	10pJ/bit/m2
	ε_{mp}	0.0013pJ/bit/m4

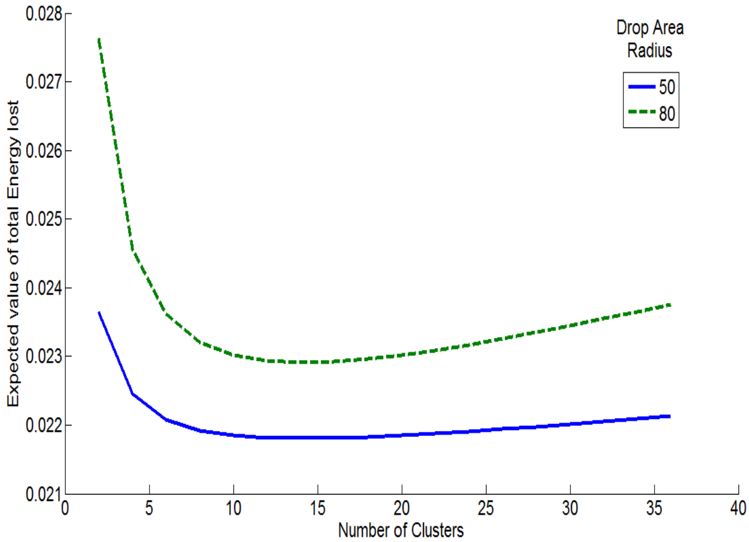

Fig .1 Average Energy Lost by a WSNin terms of Epoches

Fig.1 depicts the average energy lost by a WSN based on number of clusters. It is understood from the graph that formaltion of clusters helps in reducing the average energy loss by any individual node. This is because clustering aids better discovery and connectivity among neighborhood nodes. However an optimum cluster number is reached before the transmission cost of CH to BS increases with the number of clusters.

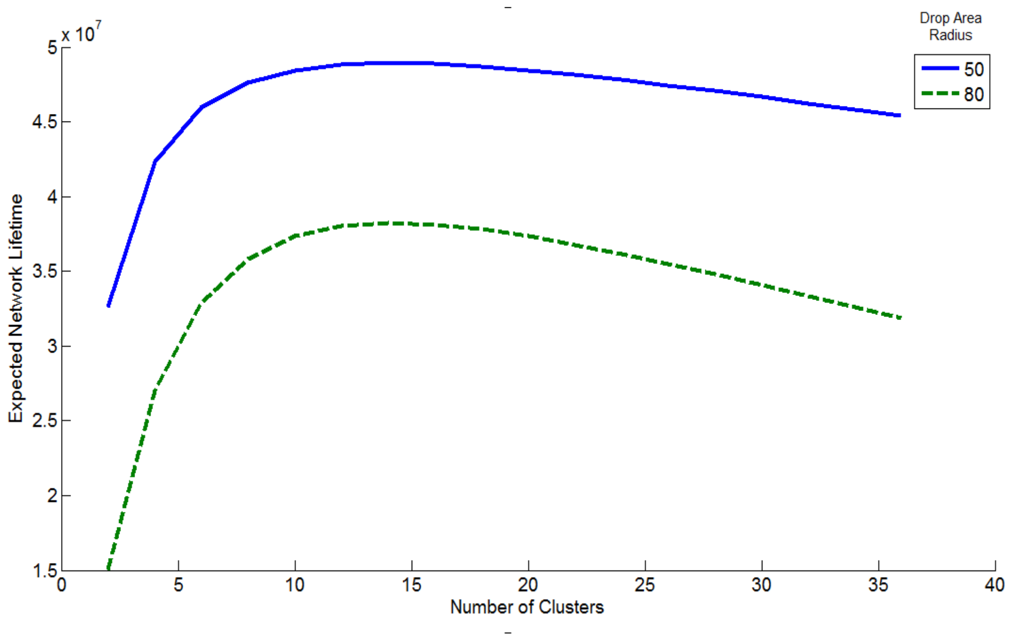

Fig.2 Expected Lifetime of a WSN interms of Epoches

Similarly, Fig.2 depicts the variation in expected lifetime of a WSN with respect to number of clusters. It is found from this graph that expected lifetime of any individual node is enhanced when there are more clusters but there again an optimum cluster number is expected for a given simulation parameters.

VI. Conclusions with Future Direction of Work

In this paper,we provided a theoretical investigation on the possible performanceof a WSN given a set of parameters involved and under certain assumptions.Theanalysis reveals that the expected energy lost and the average life time varywith number of clusters.Such analysis will be helpful in optimal deployment of WSN for simulation and implementation. In future, we would be extending this methodology of performance analysis on hierarchical multilevel wireless sensor networks.

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