I-PUMA: Fast Phase Unwrapping Via IBFS Graph Cuts

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Abstract— Phase unwrapping (PU) is the process of recovering the absolute phase ϕ from the wrapped phase ψ and it is key problem in interferometry SAR processing (INSAR) and Sonar (In SAS), MRI (medical imaging) & signal processing applications, etc. José's phase unwrapping via graph cut (PUMA) is one of the efficient algorithms which can solve different kinds of phase unwrapping problems successfully. However, computation speed and memory consumption often times limit the effective use of PUMA especially for large images / interferogram's. To overcome this drawback and to reduce the complexity of the PUMA Algorithm, a new dynamic phase unwrapping algorithm (IPUMA) is proposed. In the proposed algorithm, instead of Boykov-Kolmogorov (BK) max flow solver, we use the incremental breadth-first search (IBFS) max-flow algorithm as an optimization step. By doing so, our algorithm I-PUMA outperforms PUMA with an speed increase of about 20-40% and the error norm is same. This proposed algorithm is described in detail and tested on simulated and real image (interferometry SAR data). Results show that the proposed algorithm works faster than the José's proposed phase unwrapping via graph cut (PUMA).

Keyword- Phase Unwrapping, PUMA, Interferometry SAR, Magnetic Resonance Imaging, Graph cuts

I. INTRODUCTION

Estimation of an absolute (true, ϕ) phase from the measured phase (wrapped, principle, ψ) is a key problem for many imaging techniques. Phase unwrapping (PU) [1] is one of the well known and widely adopted techniques for phase estimation. For instance, In remote sensing applications [2] like Synthetic aperture radar (SAR) or Sonar (SAS), phase difference between the terrain and the radar is captured by two or more antennas .The measured phase by SAR or SAS is in the interval of $[-\pi, \pi]$.

With this measured phase we cannot construct a topographical DEM (Digital Elevation Map). So in order to reconstruct DEM, absolute phase (ϕ) has to be recovered first. By using the Phase unwrapping process, we can obtain the absolute phases from the measured one (ψ). Similarly for MRI (Magnetic Resonance imaging) PU technique is used to determine magnetic field deviation maps, chemical shift based thermometry, and to implement BOLD contrast based venography. PU also acts as a necessary tool for the three-point Dixon water and fat separation. In optical interferometry, phase measurements are used to detect objects shape, deformation, and vibration.

Phase unwrapping (PU) [1] is however, an ill-posed problem, if no further information is added. There are different methods to unwrap the phase and can be broadly classified as Path following, Minimum L^p norm and Bayesian/regularization methods.

Bayesian methods [3][4][5] are also known as statistical methods. Bayesian methods depend on a data observation mechanism model and utilize prior knowledge of the phase. Bayesian approaches can be optimal from the information-theoretic point of view but they are unable to restore uniqueness of the solution.

Minimum norm [6][7] is a global minimization approach where all the observed phases are utilized to compute the solution. It find's the phase solution ϕ for which the L^P norm of the difference between absolute phase differences and wrapped phase differences is minimized. If p=2, then we have Least square (L^2) method. The disadvantage of least square method is they tend to smooth even the dis-continuities. L^1 deals well with dis-continuities when compared to L^2 solution .The main advantage of minimum norm method when compared with the Bayesian method is, it does not require any prior knowledge of the phase.

Path following algorithms apply line integration schemes over the wrapped phase image, and basically rely on the assumption that Itoh condition holds along the integration path. Wherever this condition fails, different integration paths may lead to different unwrapped phase values. This approach is not a global approach as it does not make use of all observed phase to determine the phase. These methods are less robust to noise as it does not follow the global approach.

During the last three decades, redundant of algorithms were proposed for Phase unwrapping. Among them, PUMA belongs to Minimum norm method is one of the best algorithm for unwrapping. Most of the Minimum norm algorithms faces difficulty while unwrapping phase especially at dis-continuities, but PUMA even though belongs to Minimum Norm method, it unwraps the phase properly even at discontinuities. PUMA [8] is the first technique which uses graph cut as an optimization technique for phase unwrapping.

Later, several algorithms belong to different approaches of phase unwrapping use graph cut as an optimization step. For instance, algorithms like [3],[4],[5] use graph cuts as a optimization step .But these algorithm requires prior step and consume more memory, time than PUMA Method . For instance, [5] faces issue while unwrapping especially at discontinuities. Among all available algorithms, PUMA is the optimum solution in the both the aspects of discontinuity preserving and utilizing system resources. However, computation speed and memory consumption often times limit the effective use of PUMA especially for large images .To overcome this and to reduce the complexity of the PUMA Algorithm, a new dynamic phase unwrapping algorithm (I-PUMA) is proposed.

In this letter, we analyze the complexity of PUMA and then propose the Phase Unwrapping via IBFS Graph cuts (I-PUMA) method. In our algorithm, we use the same energy minimization framework as in PUMA to unwrap the phase. We change the optimization step of the PUMA. The significant advantage of the proposed method with minimal changes of PUMA is maximum reduction of algorithm’s complexity and effective utilization of system resources is attained. So that the proposed method is more intelligent and works faster than PUMA for both convex and non-convex potentials. From the experiments in Section V, our proposed method achieves faster running times than PUMA unwrapping method.

The remaining of the letter is organized as follows. Section II, we present the PUMA phase unwrapping algorithm .In Section III ,we present the smarter IBFS graph cut method; Section IV, we briefly presents our new algorithm I-PUMA .In Section V, A set of experiments and results to compare I-PUMA with PUMA in all aspects and we conclude this letter in Section VI.

II. PUMA : PHASE UNWRAPPING VIA MAX FLOW

Phase unwrapping (PU) is the process of recovering the absolute phase from the wrapped phase, formally as

$$\phi = 2\pi k + \psi \tag{1}$$

Phase unwrapping via graph cuts [8] is a novel technique which uses graph cut as an optimization step. PUMA mainly consists of two algorithms and it is classified according to its clique potential, as an energy minimization framework for PU. The clique is a set of sites that are mutually neighbors. If the clique potential is greater than one, then such cliques are named as Convex potential ($p \geq 1$) and PUMA has an exact energy minimization algorithm. For non-convex clique potentials ($p \leq 1$), PUMA has an approximation solution owing to its discontinuity preserving ability. Both algorithms solve optimization problems by computing sequence of binary optimization, each one solved by graph cut techniques.

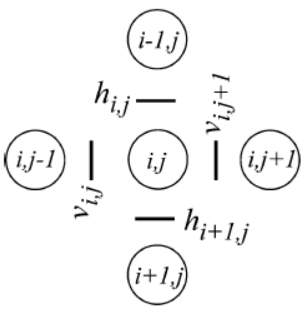


Fig.1 Representation of the site (i,j) and its first-order neighbors along with the variables h and v signalling horizontal and vertical discontinuities, respectively.

Let us define the energy for a site of (i,j) as shown in Fig.1 as (2)

$$E\left(\frac{k}{\boldsymbol{\psi}}\right) \equiv \sum_{\mathrm{ij\epsilon G0}} V\left(\Delta\Phi\,^{\mathrm{h}}_{ij}\right)\upsilon\mathrm{ij} + V(\Delta\Phi\,^{\upsilon}_{ij})hij \qquad \ldots(2)$$

where k is an image of integers, denoting 2π multiples, the so-called wrap-count image ψ , V(.) is the clique potential, a real-valued function,1 and (.)h and (.)υ denote pixel horizontal and vertical differences given by (3) to (6) . Our purpose is to find the integer image k that minimizes energy (2), k being such that $\phi = 2\pi k + \psi$

$$\Delta\Phi_{ij}^{h} = \left[2\pi\left(k_{ij} - k_{ij-1}\right) - \Delta\,\psi_{ij}^{h}\right],\ k \in \mathbb{Z} \quad \ldots(3)$$

$$\Delta\Phi_{ij}^{v} = \left[2\pi\left(k_{ij} - k_{i-1\,j}\right) - \underline{\Delta}\psi_{ij}^{v}\right], k \in \mathbb{Z} \quad \ldots(4)$$

$$\Delta\psi_{ij}^{h} = \psi_{ij-1} - \psi_{ij} \quad \ldots(5)$$

$$\Delta\psi_{ij}^{v} = \psi_{i-1j} - \psi_{ij} \quad \ldots(6)$$

A. ENERGY MINIMIZATION BY A SEQUENCE OF BINARY OPTIMIZATIONS: CONVEX POTENTIALS

By using the proof of Equivalence between Local and Global Minimization, Convergence Analysis and Mapping Binary Optimizations onto Graph Max-Flows, the author's [8] rewrites the energy equation (2) as (7)

$$E(k^t + \delta|\psi) = \sum_{ij\epsilon G0} V\ \left[2\pi\left(\delta_{ij} - \delta_{ij-1}\right) + a^h\right]v_{ij} + V\left[2\pi\left(\delta_{ij} - \delta_{i-1j}\right) + a^v\right]h_{ij} \quad \ldots(7)$$

Authors of PUMA, for the sake of simplicity, rename the equation (7) to (8)

$$E(k^t + \delta|\psi) = \sum_{ij\epsilon G0} E^{ij}(\delta_i, \delta_j) \quad \ldots(8)$$

The minimization of the equation (7) w.r.t δ is now mapped onto a max-flow algorithm. For graph construction , Authors[8] exploits a one to one map existing between the energy function (7) and the cuts on a directed graph $G(V, \mathcal{E})$ with non-negative weight, the graph has two specials vertices, namely the source 's' and the sink 't'. The number of vertices V is 2 + NxM (two terminals, the source and the sink, plus the number of pixels or nodes).An s-t cut or min cut is a partition of vertices V into two disjoint sets S and T, such that s ∈ S and t ∈ T with min cost .Cost of the cut is the sum of costs of all edges between S and T.

As per [9], a function of F^2 class of functions is graph representable, i.e., there exists a one-to-one relation between configurations δ ∈ {0, 1} MN and s-t cuts on that $E(k^t + \delta|\psi)$ graph, if and only if it satisfy the regularity condition (9).

$$E_{ij}(0{,}0) + E_{ij}\,(1{,}1) \leq E_{ij}\,(0{,}1) + E_{ij}(1{,}0) \quad \ldots(9)$$

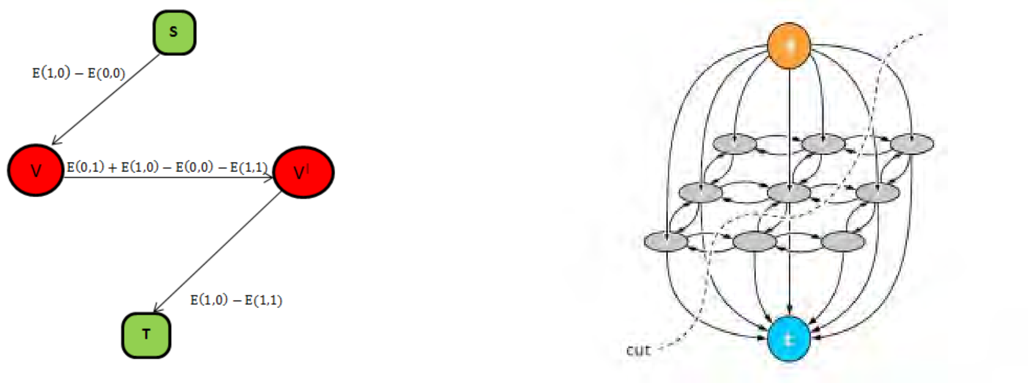
$$E^{ij}\,(0{,}0) = V(a)d_{ij} \quad \ldots(10)$$

$$E^{ij}\,(1{,}1) = V(a)d_{ij} \quad \ldots(11)$$

$$E^{ij}\,(0{,}1) = V(-2\pi + a)d_{ij} \quad \ldots(12)$$

$$E^{ij}\,(1{,}0) = V(2\pi + a)d_{ij} \quad \ldots(13)$$

So, for each energy term E_h^{ij} and E_v^{ij}, authors construct an "elementary" graph with four vertices $\{\, s; t; v\,; v^I\}$, where $\{s; t\}$ represents source and the sink, common to all terms, and $\{\, v\, v^I\}$ represents the two pixels involved (v being the left (up) pixel and v^I the right (down) pixel). Finally as shown in figure 2, the directed edge $\{\, v\,, v^I\}$ is defined with the weight of $E(0{,}1) + E(1{,}0) - E(0{,}0) - E(1{,}1)$. Moreover, if $E(1{,}0) - E(0{,}0) > 0$ edge $\{\, s, v\}$ is defined with the weight of $E(1{,}0) - E(0{,}0)$ or, otherwise, edge $\{\, v, t\}$ is defined with the weight $E(0{,}0) - E(1{,}0)$.Once energy is mapped on to the graph, energy is easily minimized by using the max flow /min-cut on the constructed graph. Among the available max flow algorithms, authors of PUMA uses BK [10] Algorithm (alpha expansion graph cut) for finding the min cut/maxflow .PUMA runs for k iterations to unwrap the phase of a profile, where k is the number of 2π multiples.



(a) (b)

Fig. 2. (a) Elementary graph for a single energy term, where s and t represent source and sink, respectively, and $v\,;v^{I}$ represent the two pixels involved in the energy term. In this case $E(1,0)-E(0,0)>0$ and $E(1,0)-E(1,1)>0$. (b) The graph obtained at the end results from adding elementary graphs.

B. ENERGY MINIMIZATION BY A SEQUENCE OF BINARY OPTIMIZATIONS: NON-CONVEX POTENTIALS

The PUMA Algorithm of Convex potential does not fits to non-convex potential as it faces with the below two issues.

1. If the clique potential is non convex υ, it is not possible, in general, to reach the minimum through 1-jump moves only.

2. With a general non convex υ , the condition of regularity (9) does not hold for every horizontal and vertical pairwise clique interaction. This means, it's not possible to map energy terms onto a graph and energy cannot be minimized via graph cuts.

Energy minimization of a non-convex potential is a NP Hard problem. Even though, authors proved that the second issue can be resolved by applying majorize minimize (MM) concepts to energy function. That is where ever the pixel pairs does not satisfy the regularity condition (9) then the edge weight between pixel pairs are set to zero to satisfy the regularity condition .With respect to the first referred issue, authors extend the range of allowed moves.Instead of only 1-jumps they now use sequences of s–jumps .In simpler terms if the pixel belongs to Source (s) then it is increased by s instead of 1 as in convex algorithm .Finally authors[8] showed that the non convex potentials can also be solved via graph cuts and for uniformity sake they name the algorithm as PUMA .

III. OPTIMIZATION TECHNIQUE : IBFS GRAPH CUT

For Both Convex and Non Convex potentials, once the energy function (2) is mapped onto the Graph, then the max flow algorithm finds the energy minimization. In PUMA, authors used BK Graph cut [10] as a max flow solver and BK Algorithm attain this by using the concept of augmenting paths. The BK algorithm has three steps; those are Growth Step, Augmentation step and Adoption Step. The main drawback of BK is that we cannot make any assumption of the structure of the trees and thus no assumption on the length of the augmenting paths.

Incremental Breadth First Search (IBFS) [11] is an extension of the BK algorithm. As BK set out to always maintain the search trees (however arbitrarily), IBFS also maintains the trees and make sure that they first search breadth-wise. IBFS maintain two directed trees S and T and a set of free Nodes N .The first step is growth step where S-tree is grown by scanning all nodes whose distance Ds is equal to the current max distance . Any free node is found and it is added to the tree and gets the distance $ds(u)=Ds+1$. In such a search of a free node, if a T-node is found, the growth step is interrupted by the augmentation step. After the growth step, if there are no nodes with distance $Ds+1$ the algorithm terminates, otherwise Ds is incremented and the growth step begins anew. Growing the T-tree is done symmetrically.

Augmenting the path creates S and T orphans. The S- orphan process is started by searching a potential parent (u) which satisfies either $ds(v)=ds(u)+1$ or it should minimize $ds(u)$. If none such exists then free V and make all its children S-orphans. Otherwise, set $p(v)=u$ and $ds(v)=ds(u)+1$. Processing the T-orphans is done in the same way and free V only if it satisfy $(u)\geq Dt$, as it is not advisable to grow T at the growth stage of S.

During the adoption stage, in order to keep the distance labelling valid, a parent is found out at same distance labeling level .Otherwise, IBFS tries to re-attach the orphan as close to the root of the tree as possible. During this process, most likely the orphans will be re-adopted by their previous parent and it continues until their new distance exceeds the maximum distance of the tree. Empirical testing of IBFS versus BK has shown that in the vast majority of cases, IBFS outperforms BK with a speed increase of about 20 to 50% on a variety of vision instances.

IV. PHASE UNWRAPPING VIA IBFS GRAPH CUT : IPUMA

PUMA is an two-step phase unwrapping process .Firstly, the elementary graphs is constructed for a site by using the energy equation $E(\frac{K}{\psi})$ and secondly, it is minimized by the graph cut optimization techniques. These two steps together are run for k iterations to unwrap the phase.

PUMA [8] is one of the novel technique which unwraps phase even at dis-continuities for both convex and non-convex potentials. The only disadvantage of PUMA algorithm is that it will consumes a lot of time to unwrap phase for larger images. This often time limits the utilization of PUMA algorithm .In-order to overcome the difficulty and to faster the PUMA Algorithm, we have to reduce the complexity of the PUMA Algorithm. Computational complexity of the PUMA algorithm is $\mathrm{N_{bopt}} * \mathrm{N_{mf}}$ where $\mathrm{N_{bopt}}$ is number of binary optimization and $\mathrm{N_{mf}}$ stands for number of flops per max-flow computation .Authors of PUMA proved that the algorithm stops in k iterations where k is the number of 2π multiples and it is equal to $\mathrm{N_{bopt}}$ .Concerning $\mathrm{N_{mf}}$,authors used augmentation path type max flow and its worst case complexity $\mathrm{T(m.n)} = \mathrm{O(n^2m)}$. So, finally the complexity of PUMA is $\mathrm{T(m.n)}$, i.e. K times the worst case complexity of max flow algorithm. We have observed that in order to make PUMA faster and to reduce its complexity, we have to reduce the max flow algorithm's complexity, as its contribution in the complexity of PUMA is more. Different researchers have published different algorithms for max flow computation .Among them authors of IBFS [11] offers a faster and theoretically justified alternative to BK Algorithm. They have attained this by using the breadth-first search trees. We have accepted the hypothesis provided by the authors of IBFS, as there is a significant increase in running times has been observed. So we utilize the advantage of the IBFS Graph cuts over BK and we used IBFS as an optimization technique in our algorithm. Like PUMA algorithm, I PUMA algorithm also comprises of two steps .As a First step, we have to construct the elementary graphs by using the energy equations. The power of PUMA lies in its energy minimization framework and shows greater attenuation to noise , so we have not modify or changed the energy equation $E(\frac{K}{\psi})$ of PUMA .In I-PUMA the sequences of steps to unwrap the phase for both convex and non-convex will be retain same .Later on the minimization of energy equation with respect to δ is now mapped onto a max-flow problem .For construction graph we have followed the approach of PUMA, where the vertices and edges corresponding to each pair of neighboring pixels are build first , and then join these elementary graphs together based on the additive theorem .Once the energy is represented onto the graph , we have used the IBFS max flow algorithm for minimizing the equation . As we have followed the same energy minimization approach as in PUMA for both Convex and non-convex algorithms, our algorithm I-PUMA also stops for K iterations .Memory usage of I -PUMA is 7n bytes.

Also we have analyse the coding parts of PUMA and we have identified that there are some unnecessary calls between various programming parts of PUMA and by properly tunning it we can make the algorithm bit faster. As per [12] the run times will be faster if we use the optimization techniques while compiling the matlab mex's. So, we have also make use of full compiler optimizations technique (/Ox in Visual Studio) while compiling the Mex. We have in cooperated all these above mentioned changes in PUMA and from the experiments in Section VI, We have attained 20-40 % faster in run times .We have also observed that I PUMA and PUMA have nearly equal error norm . For some profiles I PUMA run for some more iteration's so that further energy is minimized. As PUMA, I-PUMA also deal well with discontinuity and unwraps the phase faster than PUMA.

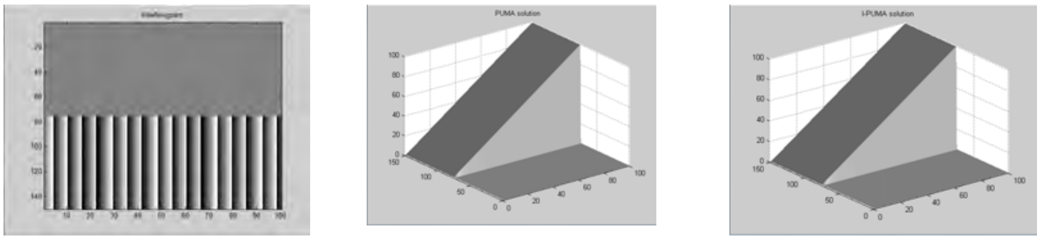
V. EXPERIMENTAL RESULTS

To check the uniqueness and power of I-PUMA verses PUMA Algorithm, we have tested both the algorithms on some simulation image and real data. Survey do to know effect of four factors are discontinuity (mask), noise, PSNR and elapsed time.

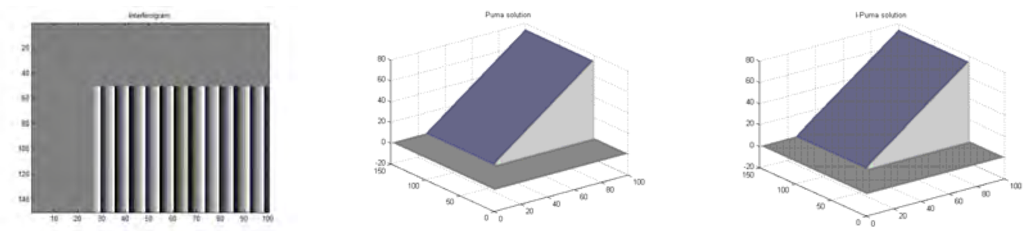
A. Discontinuity

Phase unwrapping at discontinuities is a critical one with the available information as in wrapped image .PUMA algorithm though belongs to Minimum norm method, it unwraps the phase properly at discontinuities. Because of the discontinuity preserving ability, PUMA is one of popular among all available algorithms. Our algorithm, I –PUMA also owns to discontinuity preserving ability and it is tested on interferogram's of shear and ramp, Gaussian and mask, as in Figure 3& 4 respectively.

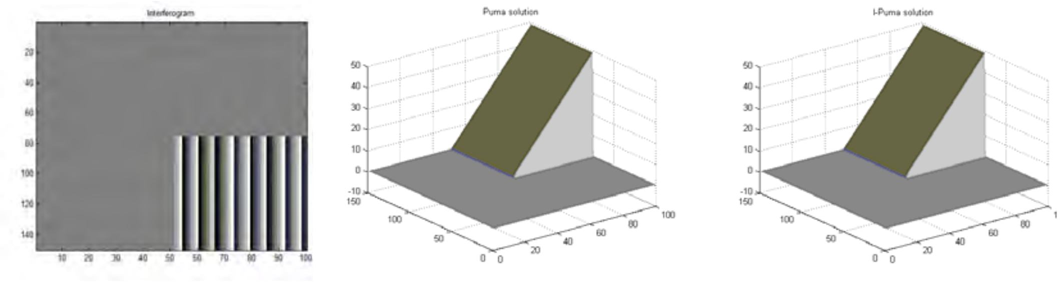
Execution of both algorithms to the interferogram's of shear and ramp, Gaussian and mask. We can see, for shear ramp ratio of 1/4 ,1/2, 3/4, 2*1/5 (as in figure 3(a),(d),(f) & (i)) both the algorithms are good and equally powerful while unwrapping at discontinuity. Even for High phase rate Gaussian Hill with a quarter set to zero as in figure 4(a), both results of interferogram's are good as in figures 4(b) & 4(c) respectively , especially while unwrapping at the centre of the interferogram's. As per [13], both PUMA and IPUMA performs well than CUNWRAP Algorithm at discontinuities.



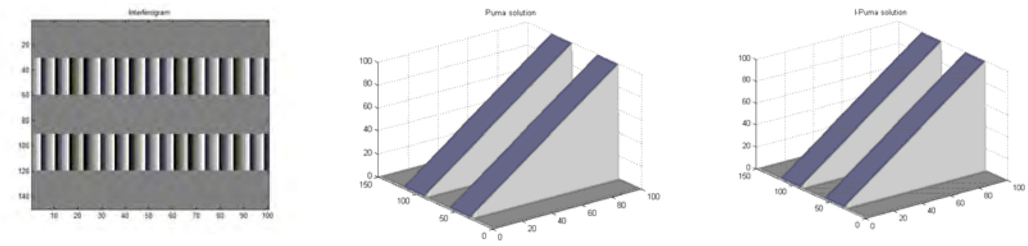
(a) (b) (c)



(d) (e) (f)

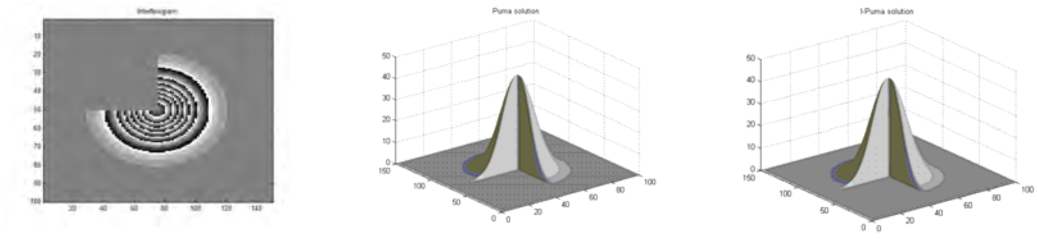


(f) (g) (h)



(i) (j) (k)

Fig. 3(a),(d),(f),(i) Interferogram's of shear ramp ratio of 1/4 ,1/2, 3/4, 2*1/5 respectively .3(b),(e),(g),(j) unwrapped by PUMA algorithm for interferogram's of (a),(d),(f),(i) respectively.3(c),(f),(h),(k) unwrapped by I-PUMA algorithm for interferogram's of (a),(d),(f),(i) respectively.



(a) (b) (c)

Fig. 4(a) Interferogram's of High phase rate Gaussian Hill with a quarter set to zero .4(b) unwrapped by PUMA algorithm for interferogram of (a) respectively.4(c) unwrapped by I- PUMA algorithm for interferogram's of (a) respectively.

B. NOISE

Random noise assigned with 3 coefficients 0.6; 0.8; 1.0 sequentially as in figure 5(a),(d),(g) respectively . Each of them did iterative running with both the algorithm's. We can see in Figure 5 (b) & (c) , (e) & (f) both the algorithms are same for random noise of variance 0.6,0.8 .But for noise variance of 1.0 ,I-PUMA as in figure 5(i) has more incorrect shape at the peak of the Gaussian hill than PUMA as in figure 5(h) ,but it is not worse than CUNWRAP[13] .So I-PUMA is as equal as PUMA regarding robustness to noise and unwraps the phase correctly .

C. PSNR,MSE

MSE (mean square error) & PSNR are two parameters which determine the robustness of the algorithm .Mean Square Error (MSE) is taken between the True Image and unwrapped images by both the algorithm's and listed in the table I. We have treated both the algorithms under different simulation profiles and noted down the MSR, PSNR Values. I-PUMA & PUMA provides approximately equal values of MSE & PSNR. I-PUMA is a bit low noise resistant than PUMA. The difference between the PUMA and I-PUMA is more pronouncing as variance of noise increases.

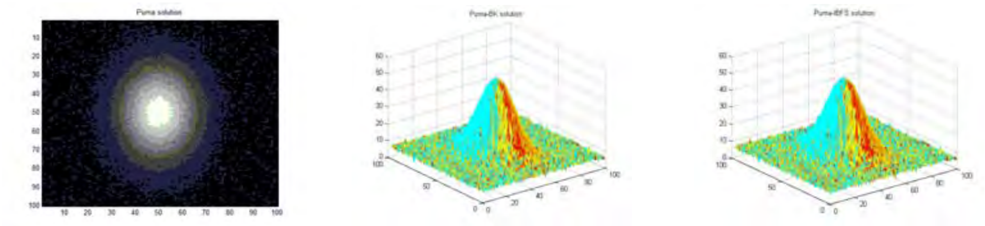
D. Elapsed Time

Specification of computer in experimental is processor Intel Core 2DUO CPU, 4 GB RAM, 500GB HDD, LCD 11.6".

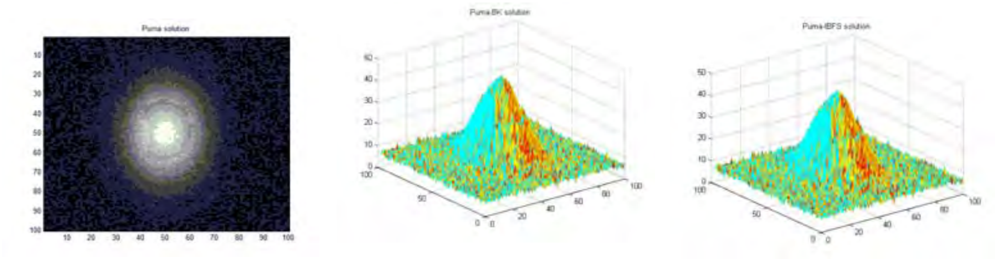
Image Simulation is

1. High phase rate Gaussian Hill with noise variances (0.6, 0.8, 1.0)
2. Shear Ramp
3. High phase rate Gaussian Hill with a non-vertical and non-horizontal aligned sector set to zero.

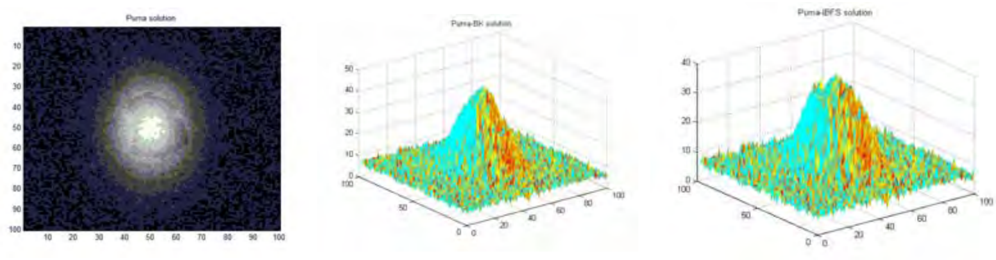
Recapitulation of ET (elapsed Time, with tic-toc function in Mat Lab) for example list in table II.



(a) (b) (c)



(d) (e) (f)



(g) (h) (i)

Fig. 5(a),(d),(g) Interferogram's of High phase rate Gaussian Hill a quarter set to zero treated with noise of coefficients 0.6,0.8 & 1.0 respectively .5(b),(e),(h) unwrapped by PUMA algorithm for interferogram of 5(a),(d),(g) respectively.5(c),(f),(i) unwrapped by I- PUMA algorithm for interferogram's of 5(a),(d),(g) respectively.

TABEL – I
PSNR AND MSE RESULTS

Interferogram	**I-PUMA (proposed)**		**PUMA**	
	MSE	**PSNR**	**MSE**	**PSNR**
Guassian Hill with zero noise variance	0.2714	5.6645	0.2713	5.6655
With Noise Variance 0.6	40.0342	-16.0243	40.0727	-16.0285
With Noise Variance 0.8	38.6637	-15.8730	38.6165	-15.8677
With Noise Variance 1.0	39.5363	-15.9700	38.5626	-15.8617
Shear Ramp	1.3128e-29	288.8182	1.3128e-29	288.8182
Gaussian Hill with a non-vertical and non-horizontal aligned sector set to zero	0.4053	3.9221	0.1737	7.6019

TABLE-II
Elapsed Time

Interferogram	**I-PUMA (in Sec)**	**PUMA (in Sec)**
High phase rate Gaussian Hill	0.232	0.338
Shear Ramp	0.086	0.226
High phase rate Gaussian Hill with both Sectors set to zero	0.243	1.338
High phase rate Gaussian Hill With noise variance 0.6	0.2435	0.3287
High phase rate Gaussian Hill With noise variance 0.8	0.2371	0.3041
High phase rate Gaussian Hill With noise variance 1.0	0.2523	0.4273
	Mean from 10 Records	

As listed in the table II, our algorithm I-PUMA is able to unwrap the interferogram's faster than PUMA. For all the interferogram's as shown in the Fig.7 I-PUMA records very low elapsed times. I –PUMA is faster even if the interferogram's is noisy. We have tested this for an interferogram's added with different noise variance (0.6, 0.8, and 1.0). As noise variance increases the difference between the elapsed times of PUMA and I-PUMA Increases .We have observed that I-PUMA is consistently faster even if noise variance increases as shown in Fig 7. Further delay in elapsed time as size of the image increases .For instance, as the size of the image doubles, the elapsed time gets delayed by 7-8 times .In such scenarios it is better to use the I- PUMA than PUMA as it unwraps faster than PUMA as listed in Table III below.

TABLE –III
Elapsed Time for different sizes of image

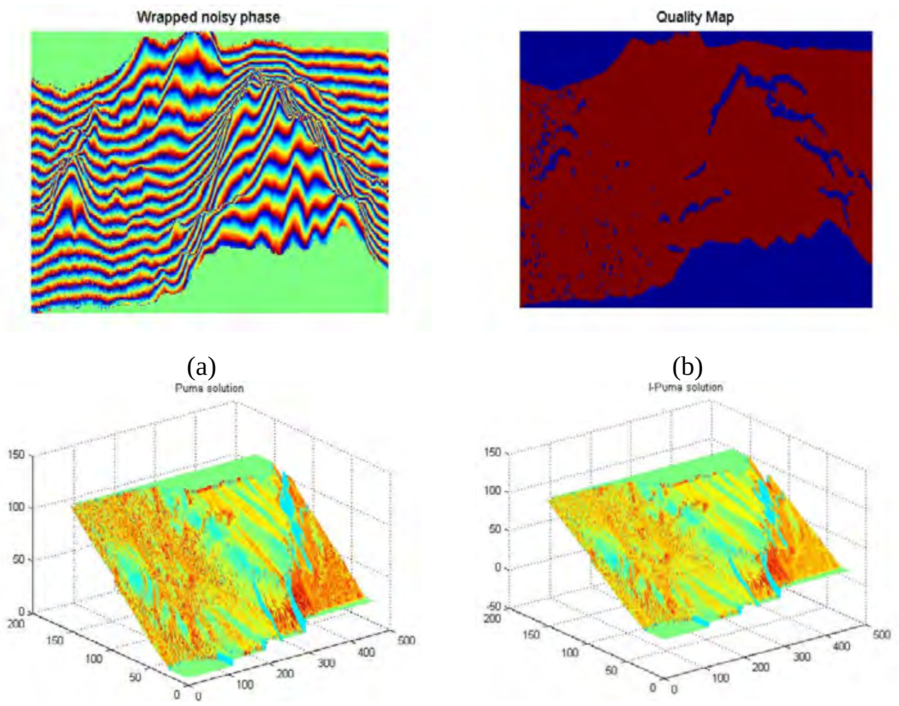
Interferogram's	**N=128**		**N=256**	
	IPUMA	**PUMA**	**IPUMA**	**PUMA**
Gaussian Hill With noise variance 0.6	0.6274	0.7656	4.9610	4.0499
Gaussian Hill With noise variance 0.8	0.5919	0.6615	7.3204	7.6301
Gaussian Hill With noise variance 1.0	0.5286	0.6390	4.6859	5.2185
	Mean from 10 Records			

E. Phase Unwrapping to Real Image

1. Area corresponds to Long's Peak, CO

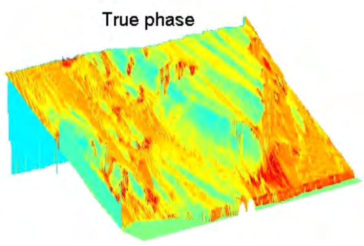
Fig 6(a) shows a phase image associated with noise (152 X 458 pixels) to be unwrapped. It was obtained from an original absolute phase surface that corresponds to a (simulated) InSAR acquisition for a real steep-relief mountainous area inducing, therefore, many discontinuities and posing a very tough PU problem. This area corresponds to Long's Peak, CO, and the data is distributed with book [1]. The wrapped image is generated according to an InSAR observation statistics, producing an interferometric pair; by computing the product of one image of the pair by the complex conjugate of the other and finally taking the argument, the wrapped phase image is then obtained. Fig. 6(b) shows a quality map (also distributed with book [1]) computed from the InSAR coherence estimate (see [1, Ch.3]) for further details). The unwrapping was obtained using the approximate version of PUMA, with m=2. The resulting phase unwrapped is "3-D" rendered for I-PUMA in Fig 6(c) and for PUMA in Fig 6(d) .True phase of the area corresponds to Long's Peak,CO is in Fig 6(e)

Both PUMA and I-PUMA have the same view as shown in Fig.6(c) &.6(d) .Elapsed time of PUMA is 9.1915 seconds whereas the I-PUMA unwraps the Real Image within 7.1410 seconds. I-PUMA outperforms PUMA with a speed increase of about 22% on a real image and the error norm is same for both algorithms.



(a) (b)

(c) (d)



(e)

Fig. 6(a) Wrapped phase image obtained from a simulated InSAR acquisition from Long's Peak, CO (data distributed with [1]). (b)Quality Map of (a) distributed with [1]. (c) Image in (a) unwrapped by PUMA (21 iterations). (d) Image in (a) unwrapped by I-PUMA (21 iterations).(e) True Phase of (a)

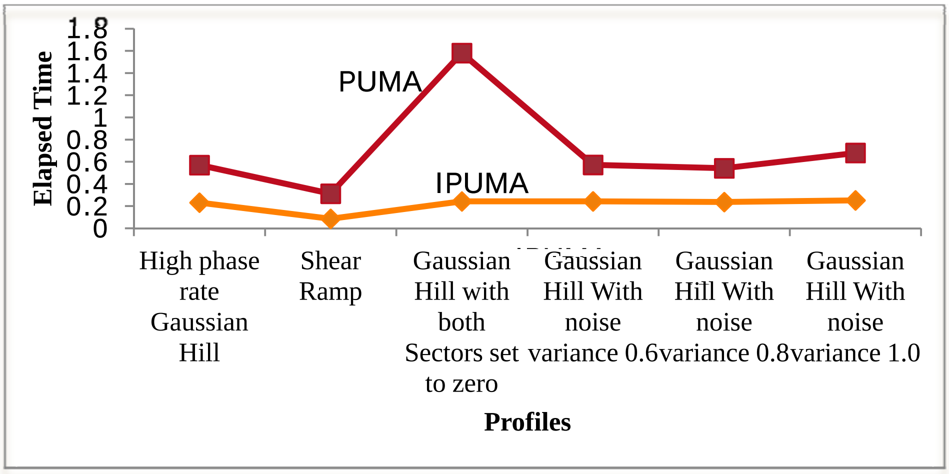
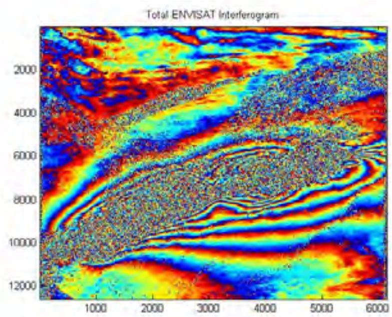


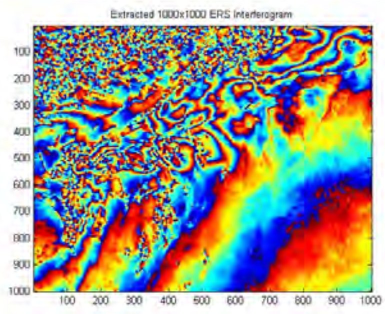
Fig. 7. Elapsed time of PUMA and I PUMA for different profiles

2. ENVISAT DATA (Area corresponds to Mexicali, Baja California)

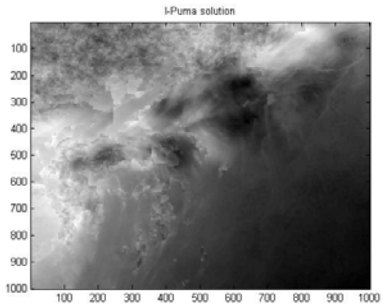
In this experiment, the original phase data of Interferometric Synthetic Aperture Radar provided by ENVISAT are unwrapped by both the algorithms and results are noted. The ENVISAT interferogram of size (6000 X 12000) is shown in Fig 8(a) .Its area corresponds to Mexicali, Baja California, USA .The mid region of the interferogram is the earthquake occurred area and its associated fringes . To process the interferogram on workstation we have extracted some part of the interferogram (1000 X 1000) and processed. As shown in Fig 8(b), the extracted area corresponds to the tip area of the earthquake and processed by both the algorithms. Fig 8(c) is the unwrapped phase by the I –PUMA Algorithm and Fig 8(d) is the unwrapped phase by the PUMA algorithm. The 3-D rendered image of Fig 8(c) & 8(d) is shown in Fig 8(e) & 8(f). Elapsed time of PUMA is 270 seconds whereas the I-PUMA unwraps the real phase within 96 seconds. The Correlation map of the corresponding area is shown in the Fig 8(g) and google image of that area is shown in Fig 8(h).

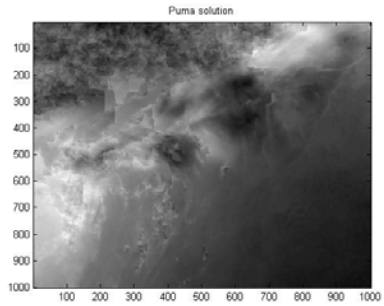


(a)

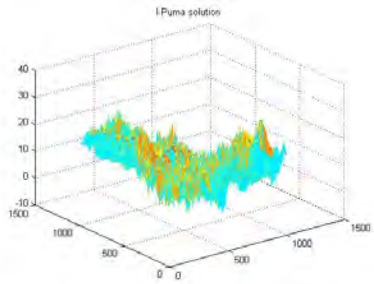


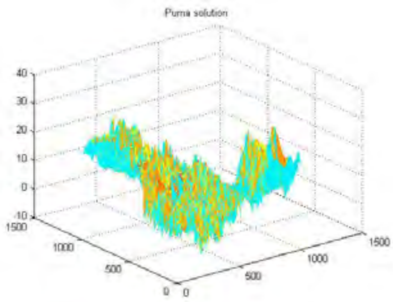
(b)



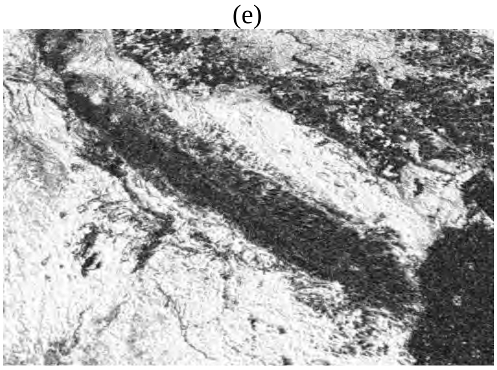


(c) (d)





(e) (f)



(g) (h)

Fig. 8(a) Wrapped phase image obtained from ENVISAT , area corresponds to Mexicali, Baja California, USA (b) Extracted image (1000x1000) of (a) .(c) Image in (b) unwrapped by I-PUMA. (d) Image in (b) unwrapped by PUMA. (e) 3-D image of (c). (f) 3-D image of (d). (g) Correlation map of (a). (h) Google image of (a).

VI. Conclusion

An unwrapping via IBFS Graph cut method abbreviated as I-PUMA is proposed in this letter. This method performs very well for both the Convex and Non Convex potentials .The proposed method does the unwrapping using IBFS Graph Cut as an Optimization step. The advantage is significant performance gain as well as a reduction of the complexity of the algorithm, which renders our method particularly useful for interferogram's of large size. Our experimental evaluation shows that the proposed method can achieve good result to the simulation phase map and the real interferometric synthetic aperture radar phase image.

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