ISSN (Print) : 2319-8613
ISSN (Online) : 0975-4024

Static Analysis of Tall Buildings with Combined System of Framed Tube, Shear Core, Outrigger and Belt Truss

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***Abstract*-In this paper, behaviour of tall structures with combined system of framed tube, shear core and outrigger-belt truss is investigated using continuum approach. Framed tube structure and shear core against lateral loads acts as a cantilever beam and the effect of outrigger and belt truss on the shear core are considered as a torsional spring in the position of outrigger-belt truss system at the height of the structure. The torsion spring acts in the opposite direction due to applied loads and reduces displacement and axial force of the structure at of bottom. Using compatibility relations of different parts of the combined system and replacing them in Euler–Bernoulli's theory and Timoshenko's theory, and applying boundary conditions, the displacement of floors and roof is obtained. The validity and effectiveness of the proposed method are examined using numerical examples and those obtained from computer analysis.**

Keyword-Framed Tube, Outrigger and Belt Truss, Euler–Bernoulli's Beam, Timoshenko's Beam, Shear deformation.

I. INTRODUCTION

In recent years, in Iran, as in other countries, tall buildings construction in cities has been taken into account in the context of population based on policies and land shortages. Today, construction is taking place for taller buildings, especially in major cities, and one of the most important issues in tall structures is choosing the suitable structural form sustaining of lateral loads. The susceptibility of tall structures to lateral loads is far greater than gravity loads and if the height of the structure increased, the conventional methods for resisting these structures are not enough. One of the most applied structural systems for tall structures is outrigger and belt truss in tall structures to reduce structure deformations and resistance to lateral loads. Outrigger and belt truss connects external columns to the inner shear core. As a result, the set of external columns and outrigger resists against the shear core rotation, reducing the lateral deformations and reducing the momentum on the structure's bottom. Addition to columns at end of the outriggers, usually other peripheral columns are also used to fix the outriggers. This type of structural form is outrigger and belt truss [1].Several methods have been proposed for analysing framed tubes. Coull and Bose presented a method based on theory of elasticity. In this method, the structure is modelled as equivalent orthotropic plates, and the equilibrium and compatibility equations are satisfied in the equivalent structure [2]. Coull and Ahmed presented a method for obtaining deflection of the framed tube [3]. Using equivalent orthotropic plates, energy relations and the relations of the theory of elasticity. Kwan provided equations for determining the stress in the columns, as well as obtaining lateral displacements of framed tube structure [4]. Connor and Pouangare proposed a five-member vertical method in which the structure is equated to vertical beams and vertical plates. By calculating the shear and bending stiffness of the members, relations are obtained for stresses in the columns [5]. Another way to improve the behaviour of framed tube is to add internal framed tubes to the original structure. In this case, distribution of stress and displacement can be significantly adjusted. Also, several other methods for analysing framed tubes have been presented by researchers such as Paulino [6], Mahjoub et al. [7]. Kamgar and Rahgozar studied vibration of tall structures using analytical methods [8]. Jahanshahi and Rahgozar determined the best position of a belt in a combined framed tube system using energy method [9]. Malekinejad and Rahgozar investigated free vibration analysis of tall structures that has a framed tube system [10]. In addition, Ramezani et al. conducted their research on analysis of non-uniform tall building structures considering axial force effects [11]. Tavakoli et al. evaluated the soil-structure interaction of outrigger-belt truss system of high-rise structures under seismic demand criteria [12]. Kim et al. studied optimization usage of outriggers in tall buildings in order to diminish lateral displacement and di□erential axial shortening [13].A dimensionless formula has been developed for flexural stiffness of tall buildings with objective of optimization problem by Alavi et al. in 2017 [14]. Kamgar and Rahgozar proposed a method for reducing the static roof displacement and axial forces in the columns of high-rise buildings [15]. In this paper, using Euler–Bernoulli and Timoshenko's theory, displacement of floors and roof of combined system of framed tube, shear core and outrigger-belt truss is obtained.

II. COMBINED SYSTEM OF FRAMED TUBE, SHEAR CORE AND OUTRIGGER-BELT TRUSS

A tall structure with outriggers includes a reinforced concrete shear core or a braced steel frame, which is attached to the outer columns by the horizontal cantilever. When the building is under the effects of a horizontal load, the outriggers prevent rotation of the core and cause lateral displacements and moments of the core to become a less compared to unbraced state (*Fig. 1*).

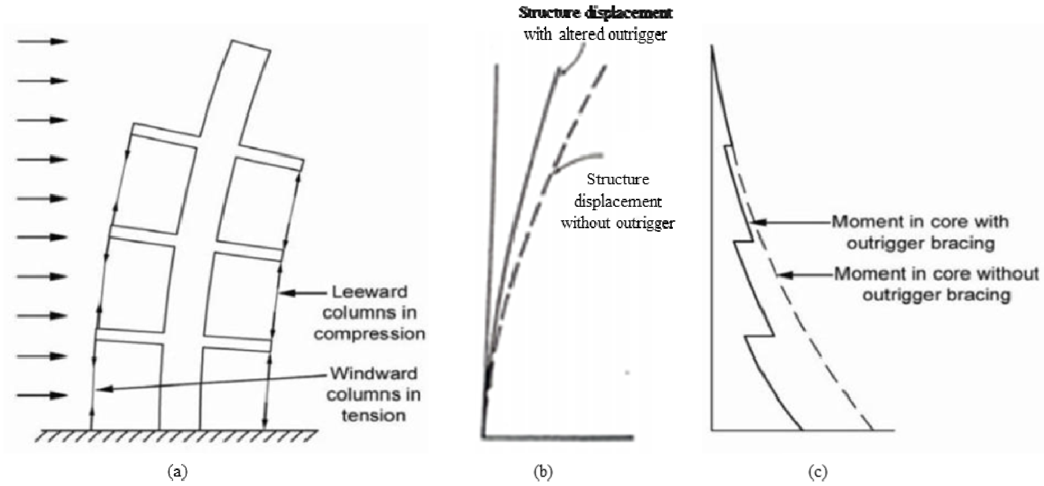

Fig. 1. a) Structure with altered outrigger subjected to lateral loads; b) Resultant of displacements; c) Resultant of core moments [14]

In this section, an approximate analysis method for structures with uniform outrigger, cores, columns, and outriggers with similar dimensions at each level is introduced .Although conventional structures are not usually uniform and their vertical members change in dimensional altitudes, this analytical method can be used in initial design to estimate the approximate displacements and forces.

In this section, structural displacement is obtainedusing an outrigger and belt truss, with following assumptions:

1) Joints of outriggers are pinnedto outer columns and only will transfer axial forces.
2) Connection of outriggers to shear core is rigid.
3) Outrigger and belt truss are quite rigid and prevent entire torsion of the structure (flexibility of outriggers and belt truss are ignored).
4) Axial stiffness of periphery columns andmomentum of the core inertia are constant and uniform along height of the structure.
5) Shear core at bottom of the structure is quite rigid and fixed.
6) Analysis method of the structureassumes be linear elastic.

III. ANALYTICAL MODELLING OF COMBINED SYSTEM WITH ONE RIGID OUTRIGGER AND BELT TRUSS BASEDON EULER–BERNOULLI THEORY

The use of outrigger and belt truss in tall structures is very common in reducing structural displacement and resistance to lateral loads. Outrigger with belt truss connects external columns to the inner shear core. As a result, the set of external columns and outrigger resists against the shear core rotation and reduces the lateral displacements and momentum at the structure's bottom. A tall structure with a framed tube system and shear core is modeled as a cantilever beam. Outrigger and belt truss is controlled along the structure and effects of outrigger on the main structure acts as a spring. As shown in *Fig. 2*, if this set is to be subjected to uniformly distributed lateral loading, the angle of rotation of the beam at point x is equal to θ_x.

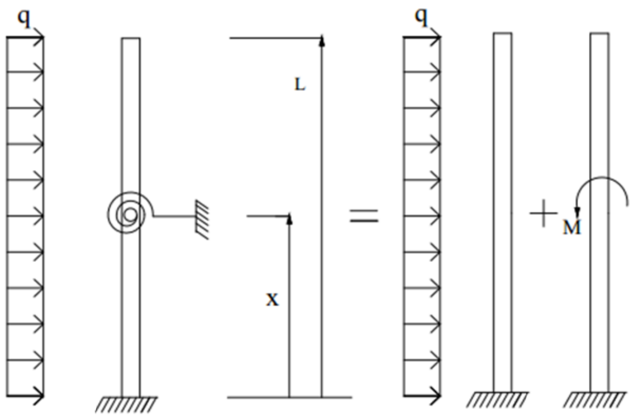

Fig. 2. Structural analytical model of outrigger subjected to loading based on Euler-Bernoulli's method

Given the rigidity of the outrigger and pinned connection of the outrigger to outer columns, as shown in *Fig. 3*, this moment becomes a force coupler in the outer columns:

$$M = Pd \tag{1}$$

Assuming that the cross-sectional area of the outer columns is constant at height of the structure:

$$P = \frac{AE\delta}{x} \tag{2}$$

Where δ is degree of axial deformation of outer columns and given the small angle of rotation of the outrigger, as:

$$\theta_x = \frac{2\delta}{d} \tag{3}$$

And:

$$K = \frac{AEd^2}{2x} \tag{4}$$

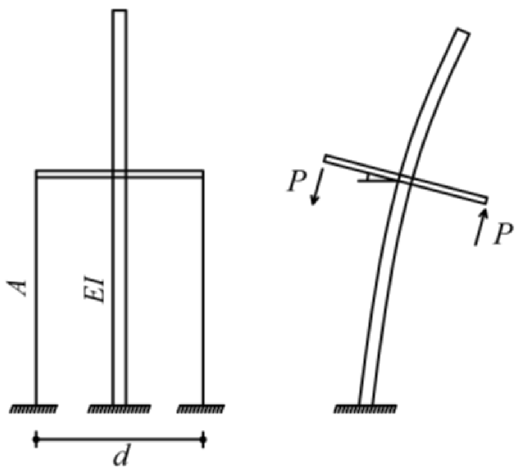

Fig. 3. Torsion spring moment matching with force coupler

The above relationship shows that spring stiffness is a function of position of the spring. To calculate θ_x according to *Fig. 2*, based on superposition principle, the sum θ_{x1} is resultant of a concentrated lateral load without spring and θ_{x2} obtained from moment,*M,* due to the spring:

$$\theta_x = \theta_{x1} + \theta_{x2} \tag{5}$$

$$\theta_{x1} = \frac{1}{EI}\int_0^x M_x d_x + \theta_0 \tag{6}$$

As can be seen, considering original coordinates at support, $\theta_0 = 0$ and the relation M_x as shown in*Fig. 2* is:

$$M_x = \frac{qL^2}{2} - qLx + \frac{qx^2}{2} \tag{7}$$

Consequently, usingEquation (*6*):

$$\theta_{x1} = \frac{1}{EI}\left(\frac{qL^2x}{2} - \frac{qLx^2}{2} + \frac{qx^3}{6}\right) \tag{8}$$

To calculate θ_{x2} from the moment M_x ,we will:

$$\theta_{x2} = \frac{-K\theta_x x}{EI} \tag{9}$$

As a result:

$$\theta_x = \frac{1}{EI}\left(\frac{qL^2x}{2} - \frac{qLx^2}{2} + \frac{qx^3}{6}\right) - \frac{K\theta_x x}{EI} \tag{10}$$

Equation (*10*) can also be written as follows:

$$\theta_x = \frac{\left(\frac{qL^2x}{2} - \frac{qLx^2}{2} + \frac{qx^3}{6}\right)}{EI + Kx} \tag{11}$$

In above relation,*q* is the intensity of the lateral load, *M*is moment at *x* point, *E* is modulus of elasticity of the braced shear core, *I* is moment of inertia of the structure, *A* is cross section of the peripheral columns in the structure, *L* is height of the structure, and *d* is distance between outer columns and *K* is calculated from Equation (*4*).

Displacement is also obtained according to superposition principle, including displacement due to loading of the structure, $y_1(x)$, and displacement due to the presence of a spring, $y_2(x)$, which are calculated according to the following relations.

$$y_1(x) = \frac{q}{EI}\left(\frac{L^2x^2}{4} - \frac{Lx^3}{6} + \frac{x^4}{24}\right) \tag{12}$$

$$y_2(x) = \begin{cases} \dfrac{-Mx^2}{2EI}, & 0 \le x \le a \\ \dfrac{-Ma(2x-a)}{2EI}, & a \le x \le L \end{cases} \tag{13}$$

$$M = K\theta_x \tag{14}$$

And finally, total displacement is equal to:

$$y(x) = y_1(x) + y_2(x) \tag{15}$$

IV. MODELLING OF RIGID OUTRIGGER AND BELT TRUSS SYSTEM BASED ON TIMOSHENKO'S THEORY

In the previous section, using Euler-Bernoulli method, displacement of the structure can be calculated at any desired point, but in this method, the shear effect is ignored. However, in Timoshenko's beam theory, effects of shear are taken into account and the relations are modified as follows.

To calculate θ_x as shown in Fig. 4, using superposition principle, θ_x will be equal to the sum of θ_{x1} from the concentrated lateral load without spring, and θ_{x2} resulting from the moment,*M*, due to the spring and θ_{x3} resulting from the shear effects:

$$\theta_x = \theta_{x1} + \theta_{x2} + \theta_{x3} \tag{16}$$

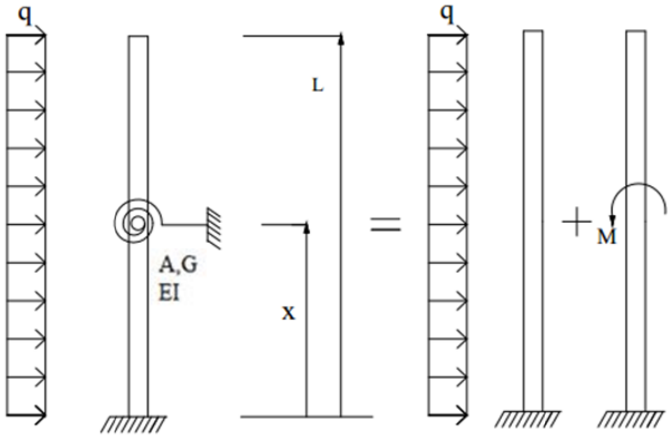

Fig. 4. An analytical model of the structure with an outrigger subjected to a uniformly distributed load by Timoshenko's method

In the case of a shear deformation, floors are displaced in parallel with each other so that the periphery columns are not tensioned or compressed and thus do not produce equivalent force in the spring. While, in bending mode, due to change in length of the peripheral columns, force is created in the spring (*Fig. 5*).As a result, in the compatibility relations, the effect of shear (θ_{x3}) is not considered and only θ_{x1} and θ_{x2} are taken into account.

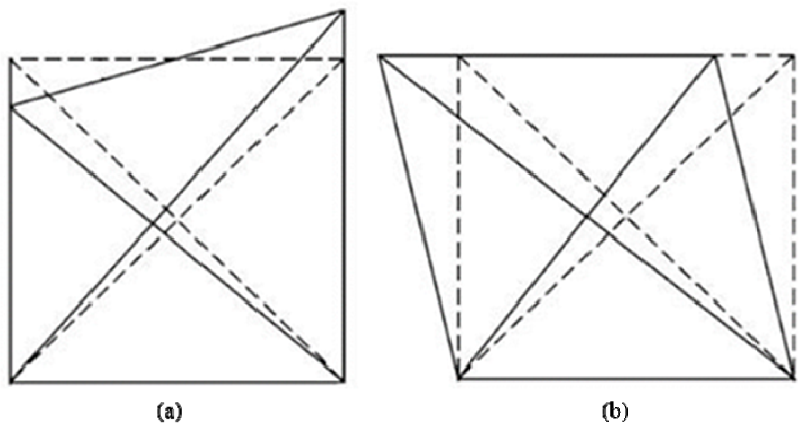

Fig. 5. Comparison of bending and shear deformation a) Bending deformation and b) Shear deformation.

As a result, the value of θ_x is equal to:

$$\theta_x = \theta_{x1} + \theta_{x2} \tag{17}$$

The amount of θ_{x1} obtained from a distributed lateral load without a spring, as in Equation (12), is obtained, and the value of θ_{x2} resulting from the moment,*M*, due to the spring, is also obtained as in Equation (13).

The assumptions used in Timoshenko's theory are as follows:

1) Each plate remains as a plate.

2) The cross section of beams is constant and symmetrical.

3) The stress-strain relationship follows Hooke's law.

As can be seen, the condition of plate's perpendicularity is removed in Timoshenko's theory beam (*Fig. 6*).

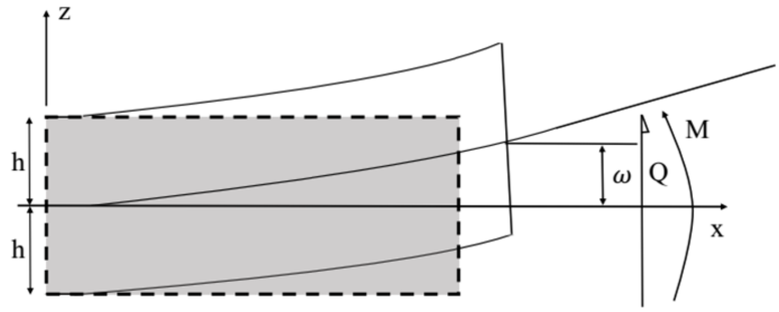

Fig. 6. Deformation of Timoshenko's beam.

Regarding the geometry of *Fig. 6*, we have:

$$u_x = -z\varphi(x) \tag{18}$$

$$\varepsilon_x = \frac{du_x}{dx} = -z\frac{d\varphi(x)}{dx}$$

On the other hand, according to Hooke's law:

$$\sigma_x = E\varepsilon_x \tag{19}$$

Based on Fig. 7, which displays the deformation of the beam section and using Equations (18) and (19):

$$\sigma_x = -Ez\frac{d\varphi(x)}{dx} \tag{20}$$

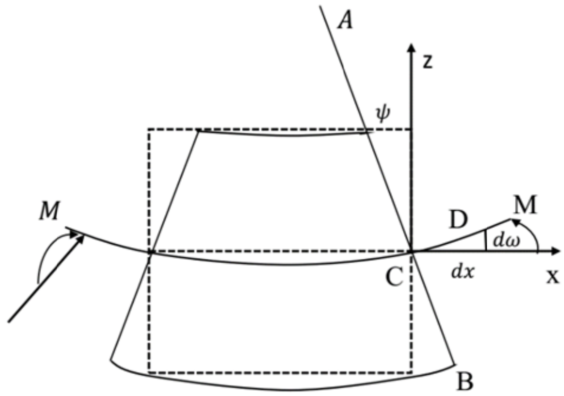

Fig. 7. Geometry of Deformation in Timoshenko's beamtheory.

On the other hand:

$$\gamma_{xz} = \frac{d\omega}{dx} - \varphi(x) \tag{21}$$

To obtain shear stress according to $\tau_{xz} = \gamma_{xz}G$, we have:

$$\tau_{xz} = \left(\frac{d\omega}{dx} - \varphi(x)\right)G \tag{22}$$

This theory is more accurate than Euler-Bernoulli's theory, but as it considers the shear strain distribution at constant altitude, the shear correction coefficient,*k*, is used, as:

$$\tau_{xz} = kG\left(\frac{d\omega}{dx} - \varphi(x)\right) \tag{23}$$

Using the bending moment equilibrium:

$$M_x = -\int z\,\sigma_x dA \tag{24}$$

Using Equations (20) and (24), bending moment is obtained as follows:

$$M_x = \int Ez^2 \frac{d\varphi(x)}{dx} dA = EI \frac{d\varphi(x)}{dx} \tag{25}$$

On other hand, the relation of shear in terms of τ_{xz} is as follows:

$$V_x = \int \tau_{xz} dA \tag{26}$$

Using Equation (23), we have:

$$V_x = \int kG\left(\frac{d\omega}{dx} - \varphi(x)\right) dA = kGA\left(\frac{d\omega}{dx} - \varphi\right) \tag{27}$$

In above relations, G is beam shear modulus and k is shear correction factor, which are presented in [*4*] and [*15*]. A is also an effective shear section in framed tube plan.

On the other hand, moment of inertia, I,is calculated equivalent to the framed tube plan as follows:

$$I = \frac{1}{12}(2b)(2a+2t)^3 - \frac{1}{12}(2b-2t)(2a)^3 \tag{28}$$

If, in recent relation, parameter $t$be neglected because of their smallness, then:

$$I = 4a^2bt + \frac{4}{3}a^3t \tag{29}$$

In this section, the structure is subjected to a uniformly distributed lateral loading with intensity$q$ accordance to*Fig. 8*.

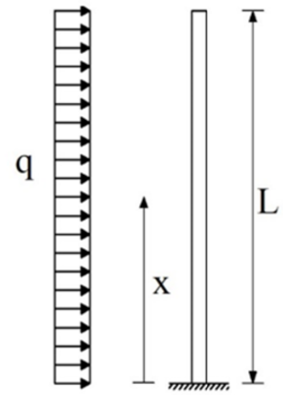

Fig. 8. Modeling of Timoshenko beam subjected to uniformly distributed load.

In this case, the moment relation at cross-section x from base of the structure, is as follows:

$$M_x = \frac{qL^2}{2} - qLx + \frac{qx^2}{2} \tag{30}$$

Using $M_x = EI\frac{d\varphi}{dx}$ relationship, results:

$$EI\frac{d\varphi}{dx} = \frac{qL^2}{2} - qLx + \frac{qx^2}{2} \tag{31}$$

By integrating from above relation, $\varphi(x)$ is obtained as follows:

$$\varphi(x) = \frac{q}{EI}\left(\frac{L^2x}{2} - \frac{Lx^2}{2} + \frac{x^3}{6}\right) + C_1 \tag{32}$$

Applying boundary conditions, the integral constant is obtained as:

$$\varphi(0) = 0 \qquad \Rightarrow \qquad C_1 = 0 \tag{33}$$

According to Equation (27), shear is obtained from the following relation:

$$V_x = kGA\left(\frac{d\omega}{dx} - \varphi(x)\right) \tag{34}$$

Using above relationship:

$$\frac{d\omega}{dx} = \frac{V(x)}{kGA} + \varphi(x) \tag{35}$$

By integrating the above relation, results:

$$\omega(x) = \int \varphi(x)dx + \int \frac{V(x)}{kGA}dx \tag{36}$$

$$\omega(x) = \frac{q}{EI}\left(\frac{L^2x}{2} - \frac{Lx^2}{2} + \frac{x^3}{6}\right) + C_1' + \frac{qLx - \frac{qx^2}{2}}{kGA}$$

Given the boundary conditions:

$$\omega(0) = 0 \Rightarrow C_1' = 0 \tag{37}$$

As a result:

$$\omega(x) = \frac{q}{EI}\left(\frac{L^2x^2}{4} - \frac{Lx^3}{6} + \frac{x^4}{24}\right) + \frac{qLx - \frac{qx^2}{2}}{kGA} \tag{38}$$

Equation (38) represents the relation $y(x) = y_1(x) + y_3(x)$.

In order to obtain the displacement, displacement caused by the loading of the structure $y_1(x)$ is due to presence of a spring $y_2(x)$ and the displacement caused by the shear effect, $y_3(x)$, according to the superposition principle, we have:

$$y(x) = y_1(x) + y_2(x) + y_3(x) \tag{39}$$

Displacement due to presence of a spring is in accordance with Equation (13), and the displacement due to loading of the structure and the shear effect is obtained in accordance with Equation (38).

V. Software Achievements and Numerical Reviews

In this section, in order to investigate accuracy and efficiency of the proposed method, static analysis of tall buildings with a system of framed tube, shear core, outrigger and belt truss with symmetric plan was performed. The building is a 40-story concrete building modeled with SAP2000 software. All beams, columns, outrigger and belt truss have a value of $0.8(m) \times 0.8(m)$. Height of the floors is 3 m, thickness of the slab is 0.25 m, and distance from the center to center of columns is 2.5 m. Modulus of elasticity and shear modulus are 20 GPa and 8 GPa, respectively. Shear core dimensions are equal to $5(m) \times 5(m)$ and thickness of shear wall is equal to 0.25 m. The structure is subject to a uniformly distributed lateral load with intensity120 KN/m [4]. Fig. 9 and Fig. 10 presents model of the structure, which has been modeled in SAP2000.

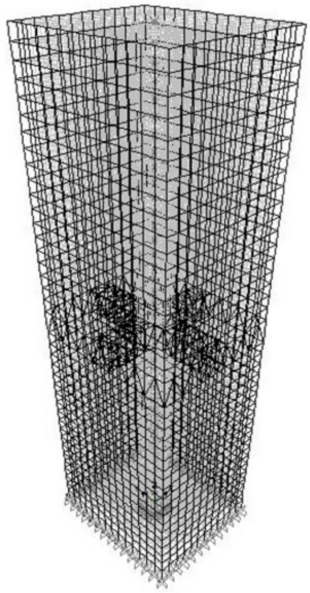

Fig. 9. Schematic model of 40-story structure with outrigger and belt truss at middle of the structure.

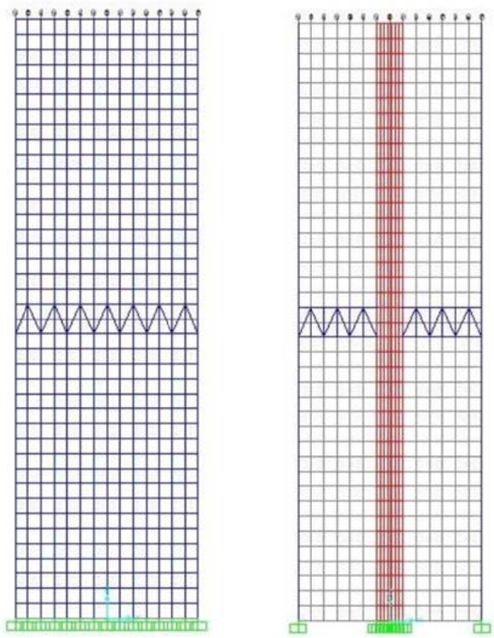

Fig. 10. The x-z view of the 40-story building with outrigger.

In *Fig. 11*, comparison of stories displacement obtained by finite element methods, Timoshenko and Bernoulli's beam theories for a 40-story building is shown. The second norm of displacement by finite element method about *79%* is different with the second norm of displacement by the Bernoulli method, and the second norm of displacement by the finite element method about *28%* is different with the second norm of displacement obtained by Timoshenko method.

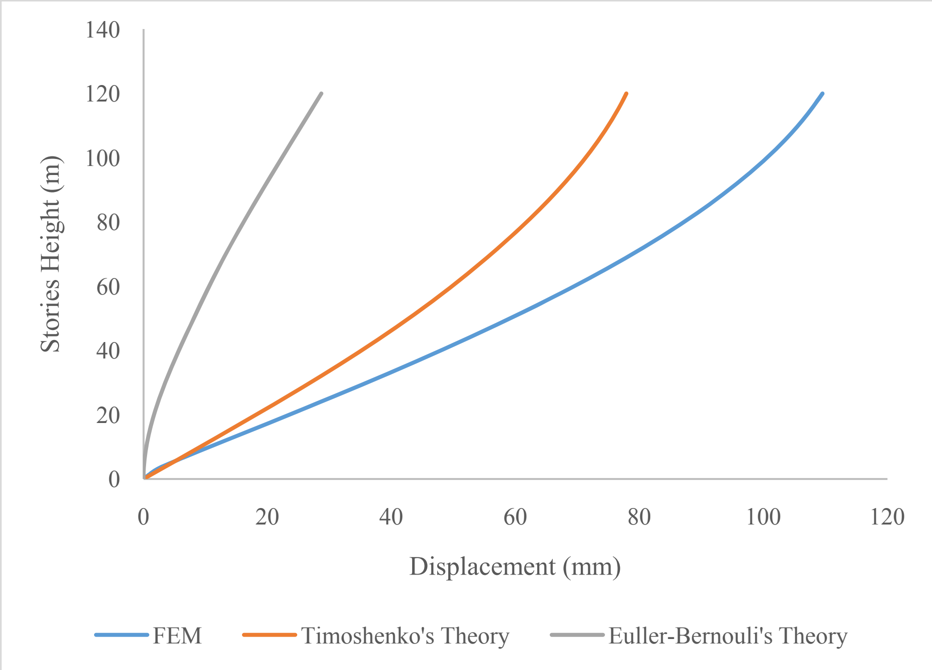

Fig. 11. Displacement of a 40-story building with an outrigger-belt truss.

Displacement at different location of heights of building based on Euler-Bernoulli method, in which shear effects has been is neglected, is very different from the proposed method (Timoshenko's beam theory). In this method, the shear effects are considered that the analysis results are much closer to reality

VI. Conclusion

The proposed model for static analysis of tall structures with symmetrical plan produces acceptable results. In particular, in the initial design stage, this method can provide a proper prediction of behavior of the structure and, based on it, obtains a basic design for the structure. Although the results obtained from the analytical method are different from those obtained by computer analysis, but the accuracy of the results can be increased by knowing the reasons for this phenomenon and fixing it. The main reasons for the discrepancy between results including the shear lag effect, the nature of the proposed method is based on the continuous model, while the actual structure has discrete elements. The combined system of framed tube consisting of periphery beams and columns is simplified with the simple model of the equivalent box section. In addition, the combined system of outrigger and belt truss and periphery columns has been replaced with a concentrated spring. The flexural stiffness of the outrigger and belt truss is assumed to be rigid, whereas it is not the same in real constructions.However, if the revised Timoshenko's theory has been used to analysis the structure, more accurate result will be concluded.

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