The Exponential Smoothing Methods (Double-Triple) and Its Applications On Time Series Data

Pratikno, B [1], Niqatani, B.D. [2], Hasan, M. [3], and Erfiana, D. [4]
[1,2,3,4)] Department of Mathematics
Faculty of Mathematics and Natural Science
Jenderal Soedirman University, Purwokerto, Indonesia.
[1] bpratikto@gmail.com
[2] niqatani.belsa@gmail.com
[3] miftachulhasan07@gmail.com
[4] dinaerfiana6@gmail.com

Abstract - The research studied double and triple exponential smoothing methods and its applications on forecasting time series data. Due to the original of plot data (pattern) increasing seasonal, the exponential smoothing method and mean absolute percentage error (MAPE) are then used. Here, Zaitun and *Minitab* software are used to compute the result of the forecasting. The result showed that the smoothing parameters significant influenced the significant result of the forecasting with small MAPE around 0.09 and 0.1, in double smoothing exponential (DES) and triple smoothing exponential (TES), respectively.

***Keyword*:** Forecasting, parameters lag and smoothing exponential.

I. INTRODUCTION

Many authors have already studied some techniques of the forecasting, such as Najmudin [11], Salvatore [18], Nasution and Prasetyawan [12], and Indiyanto [6]. Furthermore, detail and in depth statistical methods for forecasting on time series data analysis is found in Abraham and Ledolter [1]. They expressed that time series tend to exhibit a cyclical pattern that has tendency to repeat itself on a fixed period. In term of e*xponential smoothing* methods, we noted some authors studied it on time series analysis data, such as Oktaviarina [14], Montgomery [9], Chase and Jacobs [4], Aritonang [3], Oktaviana and Wijaya [15], Pangestu et al. [16], and Stevenson and Sum [19].

Following Makridakis [7] there are three methods of the exponential smoothing, namely (1) single exponential smoothing (for random and stationary data), (2) double exponential smoothing (Brown and Holt methods). It tends to be suitable for trend increases, and (3) triple exponential smoothing (for seasonal of the trend). In the triple exponential smoothing (TES), there are three smoothing weighted, namely α, β and γ (Makridakis, et al., [8]). More detail with TES, single exponential smoothing (SES), and double exponential smoothing (DES) are found in Makridakis [7].

To measure the significant result of the forecasting for *p* period ahead, there are three measurements of error, namely (1) mean absolute deviation (MAD), (2) mean absolute percentage error (MAPE), and (3) mean squared error (MSE) (see Najmudin, [11]). Due to the values of MAPE is smaller than MAD and MSE, we then used it. Note that the formula of MAPE is given

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\frac{\left|Y_t - \hat{Y}_t\right|}{Y_t} \quad , \tag{1}$$

where Y_t is an actual data at period *t*, $\hat{Y}_t$ is a forecasting data at period *t*, and *n* is a number of data. Following Goh and Law [5], and Andini [2], the criteria of MAPE is noted as: (1) $0 < x < 10$ is very good, (2) $10 \leq x < 20$ is good, (3) $20 \leq x < 50$ is enough, (4) $x \geq 50$ is bad. We note here that the small value of MAPE is then chosen as a best indicator in determining the significant forecast.

There are several steps to produce the significant result of the forecasting on *p* period ahead: (1) plot and identify the trend of the data (time series data), (2) find the suitable method, (3) give a simulation data, (4) used the formula and software to find the result of the forecasting, and (5) check the significant result using MAPE.

In this paper, the introduction is given in Section 1. The double exponential smoothing (DES) and its simulation are presented in Section 2. The triple exponential smoothing (TES) and its simulation are obtained in Section 3. Then, Section 4 described the conclusion of the research.

II. A SIMULATION STUDY AND RESULT

2.1. The Double Exponential Smoothing Method

Following Makridakis [7] and Makridakis, et al., [8], the DES had multiple smoothing weighted, namely α, γ, a_t and b_t. The α and γ are chosen at the smallest values of MAPE (in several trials). Detail of the DES is found in Makridakis [7]. To make clear the concept of DES and TES, we refer to the first theory of smoothing exponential methods, that is the SES. Here, the formula of SES is given as

$$\hat{Y}_{t+1} = \alpha Y_t + (1-\alpha)\hat{Y}_t \quad (2)$$

where Y_t is an actual data at period *t*, $\hat{Y}_t$ is a forecasting data period *t*, n is a number of data, $\hat{Y}_{t+1}$ is the forecasting data period *t+1*, and $\alpha = \left(\frac{1}{n}\right)$ is the parameter smoothing (0 < α <1). Furthermore, following Najmudin [11], the DES's formula is then presented in term of the Brown is given as

$$\hat{Y}_{t+p} = a_t + b_t p \quad (3)$$

where $\hat{Y}_{t+p}$ is a forecasting data on *p* period of *t*, a_t is a differences values of smoothing at *t* period, b_t is a additional factor at period of *t*, p is a number of period ahead to forecast. To get the $\hat{Y}_{t+p}$, we must follow some steps: (1) compute the smoothing exponential using $L_t = \alpha Y_t + (1-\alpha)L_{t-1}$, (2) then we calculate double smoothing exponential with $L^*_t = \alpha Y_t + (1-\alpha)L^*_{t-1}$, (3) determine a_t using $a_t = 2L_t - L^*_t$, (4) and b_t is determined using $b_t = \frac{\alpha}{1-\alpha}\left(L_t - L^*_t\right)$. Here, L_t is an estimation value of single smoothing and L^*_t is an estimation value of double smoothing. Similarly, Following Najmudin [11], Holt method follows three steps, that are: (1) determine level of estimation using $L_t = \alpha Y_t + (1-\alpha)\left(L_{t-1} + T_{t-1}\right)$, (2) we also determine trend of estimation using $T_t = \gamma\left(L_t - T_{t-1}\right) + (1-\gamma)\left(T_{t-1}\right)$, and furthermore, we written the Holt formula as

$$\hat{Y}_{t+p} = L_t + pT_t, \quad (4)$$

where $\hat{Y}_{t+p}$ is a forecasting data on *p* period of *t*, a_t is a differences values of smoothing at *t* period, b_t is a additional factor at period of *t*, p is a number of period ahead to forecast, L_t is a level estimation, and T_t is a trend estimation. Note that L_0 and T_0 are intercept of linear estimation and slope, respectively (Montgomery [10]).

A simulation study is given for forecasting data of a number of motorcycle in Figure 1. Figure 1 showed that the trend of the plot of the data increases as the month increases (Year of data: 2017-2018).

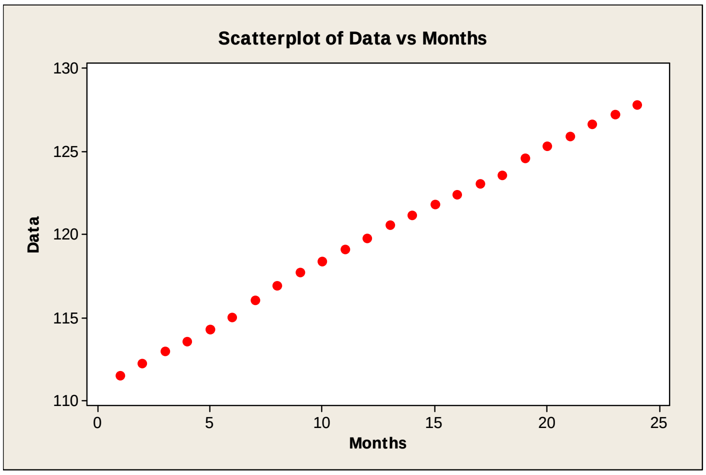

Figure 1. Trend of motorcycle data

It is clear that the data (Figure 1.) increases as the month increase (has a trend). We therefore decide to use DES as a suitable method. From the simulation of MAPE (smallest MAPE =0.09), we got $\alpha = 0.7$ for Brown method, and $\alpha = 0.9$ and $\gamma = 0.1$ for Holt. The forecasting data is then given in Table 1 and Table 2 for both methods (Brown and Holt).

Table 1. The Forecasting data and Its error (Brown)

Months (2019)	$\hat{Y}_{t+p}$	Y	Error (%)
January	128.4 x 10^3	128.5 x 10^3	0.08
February	129.1 x 10^3	129.14 x 10^3	0,07
March	129.7 x 10^3	129.68 x 10^3	0,02

Table 2. The Forecasting data and Its error (Holt)

Months (2019)	$\hat{Y}_{t+p}$	Y	Error (%)
Januari	128.5 x 10^3	128.55 x 10^3	0,01
Februari	129.2 x 10^3	129.1 x 10^3	0,06
Maret	129.9 x 10^3	129.7 x 10^3	0,17

From Table 1 and 2., we see that the average of error the Brown method is 0.04 and the average of error of the Holt is 0.19. We then conclude that Brown method is better than Holt (the error 0.04 < 0.19).

2.2. The Triple Exponential Smoothing Method

Similarly with the Section 2, we then studied the TES in this section. Following Makridakis [7] and Makridakis, et al., [8], the triple exponential smoothing (TES) had three smoothing weighted, namely α, β and γ. These parameters are chosen based on the smallest mean absolute percentage error (MAPE) on several trials. Detail TES is found in Makridakis [7]. Following Makridakis et al. [8] and Najmudin [11], the general formula of the TES is given as

Level : $$L_t = \alpha\left(Y_t - S_{t-s}\right) + (1-\alpha)\left(L_{t-1} - T_{t-1}\right)$$

Trend : $$T_t = \gamma\left(L_t - L_{t-1}\right) + (1-\gamma)\left(T_{t-1}\right)$$

Seasonal : $$S_t = \beta\left(Y_t - L_t\right) + (1-\beta)\left(S_{t-s}\right)$$

Forecasting : $$\hat{Y}_{t+p} = L_t + pT_t + S_{t-s+p} \quad (5)$$

where L_t is value of level, α, β and γ are and smoothing weighted, T_t is an estimation *trend*, γ is *smoothing constant* of trend estimation, S_t is an estimation of seasonal, β is smoothing constant of trend seasonal, s is length of seasonal, $\hat{Y}_{t+p}$ is a forecasting data on *p* period ahead, and p is period of forecasting.

A simulation study is given to the data of the executive class train ticket as below. The scatter plot is presented in Figure 2. (Niqatani [13] and Pratikno et al. [17]), we then chose the TES as a suitable method for this plot.

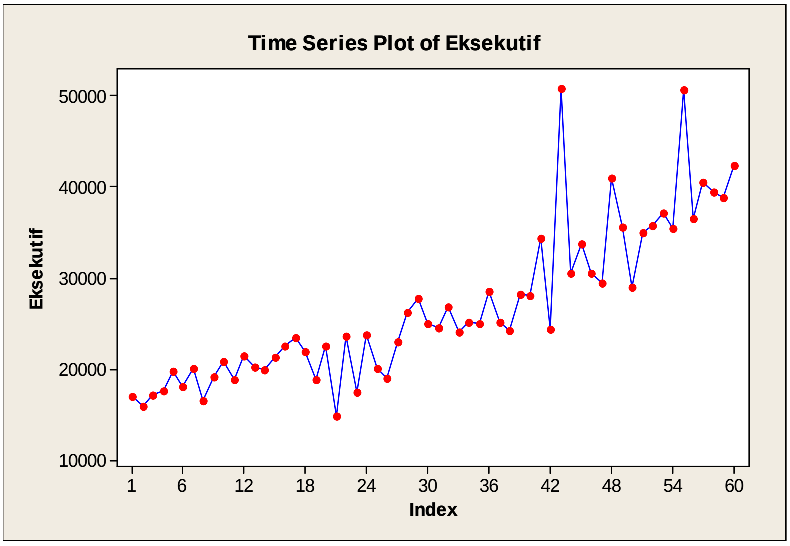

Figure 2. Scatter plot of the executive class train ticketing

From Figure 2., we see that the trend of the plot increases (seasonal). Therefore, we choose the TES as a suitable method for forecasting it. The first procedure, we firstly choose the values of α, β, and γ from the smallest MAPE that generate using Zaitun software. Here, we got the parameters smoothing $\alpha = 0.1$, $\beta = 0.6$, and $\gamma = 0.1$ on MAPE= 0.1 (as smallest MAPE). Using these parameters ($\alpha = 0.1$, $\beta = 0.6$, and $\gamma = 0.1$), and following Pratikno, et al. [17], we then get the values of the data forecasting (Figure 3.).

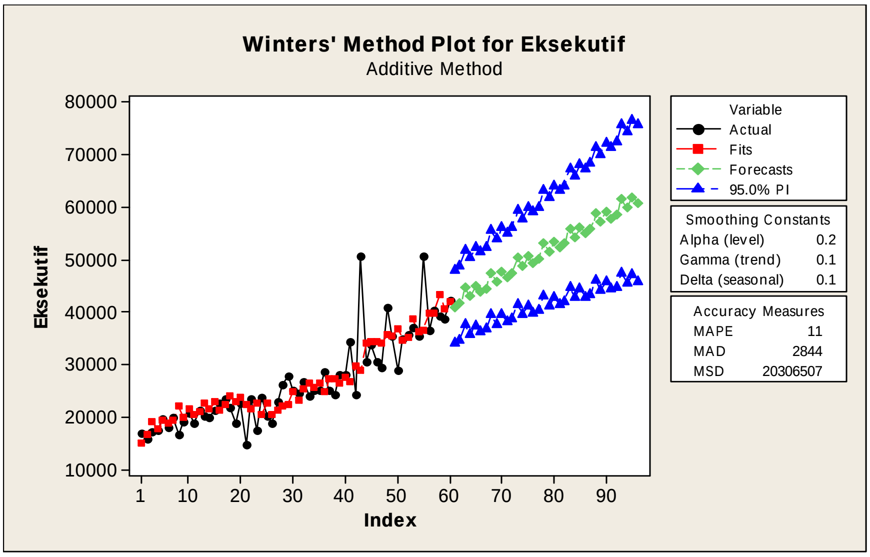

Figure 3. The forecasting of executive class train ticketing

From Figure 3. It is clear that the green line (forecasting) is in line with the trend of the actual data (red line). They both lie at 95% its confidence interval, with the accuracy measurement of the MAPE is 0.1 (small). The actual error is given in Table 3.

Table 3. The values of the forecasting data on January-March 2018

Year 2019	January	February	March
Y_t	39960	30678	32378
$\ddot{Y}_{t+p}$	36652	38364	40947
error	0.006	0.02	0.2

From Table 3. we see that the error of the forecasting is around 0.006 - 0. 2 (in average is 0.07). It means that the error is relative small. Thus, we conclude that it is an enough error for getting the good forecasting.

III. Conclusion

The research studied the double and triple smoothing exponential method in forecasting time series data. Both methods are suitable due to the trend of the plot increases (DES methods) and seasonal increases (TES). The MAPE is used to obtain the eligible forecasting. Zaitun and *Minitab* software are used to compute the result. The result showed that the forecasting data are really influenced by smoothing parameters, with the smallest MAPE are 0.09 (DES) and 0.1 (TES), respectively. For the DES, Brown method is better than Holt method.

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